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ACCELERATING BIANCHI TYPE-VI_0 STRING COSMOLOGICAL MODEL IN BARBER'S SECOND SELF-CREATION THEORY

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ABSTRACT

Field equations in the presence of cosmic string source are obtained in Barber's [2] (Gen. Relativ. Gravit.14 : 117, 1982) second self-creation theory of gravitation with the aid of Bianchi Type VI_0 metric. An exact string cosmological models are presented which represents Reddy string [43] (Astrophys. Space Sci. 286, 2003b) in self-creation theory. Some physical and kinematical properties of the models are also discussed.

Key words: Bianchi Type VI_0, Self-creation theory, String source.

INTRODUCTION

Various cosmological problems are being studied by cosmologists to reveal the evolution of the universe. Einstein's general theory of relativity[1] has provided a sophisticated theory of gravitation. It has been very successful in describing gravitational phenomena. It has also served as a basis for models of the universe. Since Einstein himself pointed out that general relativity does not account satisfactorily for the inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. In recent years, there have been some interesting attempts to generalize the general theory of relativity by incorporating certain desired features which are lacking in the original theory. Many authors have proposed various alternative theories by modifying Einstein's general theory of relativity. Barber[2] proposed two theories known as self-creation theories. His first theory is a modification of the Brans and Dicke[3] theory and the second theory is a modification of the general theory of relativity. His first theory is both inconsistent with experiment as well as internally inconsistent[4]. The second theory of Barber is a modification of general relativity to a variable G theory and predicts local effects within the observational limits. In view of the consistency of Barber's second self creation theory of gravitation many authors[5-15] investigated various aspects of different space-time. Venkateswarlu and Pavan Kumar[16] studied the role of higher dimensional FRW models in Barber's second self creation theory when the source of gravitation is a perfect fluid.

Cosmic strings are hypothetical 1-dimensional (spatially) topological defects which may have formed during a symmetry breaking phase transition in the early universe when the topology of the vacuum manifold associated to this symmetry breaking is not simply connected. It is expected that at

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least one string per Hubble volume is formed. Their existence was first contemplated by the theoretical physicist Tom Kibble in the 1970s. The phase transitions leading to the production of cosmic strings are likely to have occurred during the earliest moments of the universe's evolution, just after cosmological inflation, and are a fairly generic prediction in both Quantum field theory and String theory models of the early universe. Sen[17], Barros et al.[18], Sen et al.[19], Gundalach[20], Barros and Romero[21], Bhattacharjee and Baruah[22], Rahaman et al.[23] and Reddy[24,25] have presented string cosmological models in Brans-Dicke and other alternative theories of gravitation. In particular, Reddy[26-27] has obtained string cosmological models in Brans-Dicke and Saez-Ballester scalar tensor theories of gravitation when the sum of the energy density and the tension density of the cosmic string source vanishes.

String cosmological models in scalar-tensor theories of gravitation have been investigated by Reddy et al.[28], Reddy and Naidu[29], Venkateswarlu et al.,[30] have investigated Bianchi type-I,II,VIII and IX string cosmological solutions in self-creation theory of gravitation .Recently Rao et al.,[31] have obtained exact Bianchi type-II,VIII & IX string cosmological models .

In this paper, we study Bianchi Type VI_0 universe in Barber's second self-creation theory in the presence of cosmic string.

Field equations & the model

We consider the spatially homogenous Bianchi type VI_0 metric in the form

$$ds^2 = - dt^2 + A^2\, dx^2 + B^2\, e^{-2ax}\, dy^2 + C^2\, e^{2qx}\, dz^2 \qquad ...(1)$$

where A, B, C, are the functions of t only and q is a non-zero constant. The field equation in Barber's[2] Second self-creation theory is -

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\phi^{-1} T_{ij} \ , \qquad …(2)$$

and $$\phi^{k}_{;k} = \frac{8\pi}{3}\mu T \ , \qquad …(3)$$

where T is the trace of the energy-momentum tensor, μ is a coupling constant to be determined from the experiment ($|\mu| \leq 0.1$) and semi-colon denotes covariant differentiation. In the limit as $\mu \to 0$ this theory approaches the standard general relativity theory in every respect and $G = \phi^{-1}$. The energy-momentum tensor T_{ij} for cosmic strings is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \qquad …(4)$$

Here ρ is the rest energy density of the system of strings with massive particles attached to the strings and λ the tension density of the system of strings. As pointed out by Latelier[22], λ may be positive or negative, u^i describes the system of four-velocities and x^i represents a direction of anisotropy, i.e. the direction of strings.

We have -

$$u^i u_i = -x^i x_i = 1 \text{ and } u^i x_i = 0 \qquad …(5)$$

We consider $\rho = \rho_p + \lambda$ is the rest energy of cloud of strings with particles attached to them, λ is the tension density of the string and ρ_p is the rest density of the particles, u^i the cloud four-velocity and x^i to be along x-axis.

The field equations (2), (3) for the metric (1) with the help of equations (4) and (5) can be written as

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4}{B}\frac{C_4}{C}+\frac{q^2}{A^2}=-8\pi\phi^{-1}\lambda\,, \quad \ldots(6)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4}{A}\frac{C_4}{C}-\frac{q^2}{A^2}=0\,, \quad \ldots(7)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4}{A}\frac{B_4}{B}-\frac{q^2}{A^2}=0\,, \quad \ldots(8)$$

$$\frac{A_4}{A}\frac{B_4}{B}+\frac{B_4}{B}\frac{C_4}{C}+\frac{A_4}{A}\frac{C_4}{C}-\frac{q^2}{A^2}=-8\pi\phi^{-1}\rho\,, \quad \ldots(9)$$

$$\frac{B_4}{B}-\frac{C_4}{C}=0\,, \quad \ldots(10)$$

$$\phi_{44}+\phi_4\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=\frac{8\pi}{3}\mu(\lambda+\rho), \quad \ldots(11)$$

where the suffix 4 indicates differentiation with respect to t.

Equation (10) readily gives

$$B=\mu C \quad \ldots(12)$$

where μ being an integration constant.

Without loss of generality, we take μ=1.Hence, we have

$$B=C \quad \ldots(13)$$

Using the transformations

$A=e^{\alpha}$, $B=e^{\beta}$, $dt=AB^2\,dT$

Equations (7)- (12) reduce to

$$2\beta''-2\alpha'\beta'-(\beta')^2+q^2e^{4\beta}=-8\pi\phi^{-1}\lambda\exp[2(\alpha+2\beta)]\,, \quad \ldots(14)$$

$$\alpha''+\beta''-2\alpha'\beta'-(\beta')^2-q^2e^{4\beta}=0\,, \quad \ldots(15)$$

$$2\alpha'\beta'+(\beta')^2-q^2e^{4\beta}=-8\pi\phi^{-1}\rho\exp[2(\alpha+2\beta)]\,, \quad \ldots(16)$$

$$\phi''=\frac{8\pi\mu}{3}(\lambda+\rho)\exp[2(\alpha+2\beta)]\,, \quad \ldots(17)$$

where a dash denotes' differentiation with respect to T. In view of the highly non-linear character of the field equations (14)-(17), we obtain an exact solution of field equations when the sum of rest energy density and tension density of the cloud of string vanishes i.e. when

$\lambda + \rho = 0$ (Reddy string). …(18)

Using equation (18), set of equations (14) – (17) admit an exact solution

$$A = \exp\left\{C_1 + C_2 e^{2aT} - \frac{aT}{2} + \frac{q^2 \exp[4(aT+b)]}{8a^2}\right\}, \quad \text{…(19)}$$

$$B = C = \exp(aT + b) \quad \text{…(20)}$$

$$\phi = cT + d \quad \text{…(21)}$$

$$\rho = -\lambda = \frac{-a^2 C_2 \exp(2aT)(cT+d)\exp\left\{-2\left[C_1 + C_2 e^{2aT} - \frac{aT}{2} + \frac{q^2 e^{4(aT+b)}}{8a^2} + 2(aT+b)\right]\right\}}{2\pi}, \quad \text{…(22)}$$

where a, b, c, d are constants of integration.

After a proper choice of co-ordinates and constants, the metric (1) in a scalar – tenser theory of gravitation proposed by Barber [2] becomes –

$$ds^2 = -dT^2 + \exp 2\left[C_1 + C_2 e^{2aT} - \frac{aT}{2} + \frac{q^2 \exp(4T)}{8a^2}\right]dx^2 + \exp 2(T - qx)dy^2 + \exp 2(T + qx)dz^2 \quad \text{…(23)}$$

Some physical and kinematical properties

The model represented by (23) represents spatially homogeneous Bianchi type VI cosmological model in the scalar-tensor theory of gravitation proposed by Barber[2].

For the model (23), the physical and kinematical variables which are important, in cosmology, are

$$\text{Spatial volume} = \sqrt{-g} = \exp\left[C_1 + C_2 e^{2aT} + \frac{3aT}{2} + \frac{q^2 \exp(4T)}{8a^2} + 2b\right] \quad \text{…(24)}$$

Scalar expansion:

$$\theta = \frac{1}{3}u^i_{;i} \quad \theta = \frac{1}{3}\left[\frac{3a}{2} + 2aC_2 e^{2aT} + \frac{q^2 \exp(4T)}{2a^2}\right]\exp - \left\{C_1 + C_2 e^{2aT} + \frac{3aT}{2} + \frac{q^2 \exp(4T)}{8a^2}\right\} \quad \text{…(25)}$$

$$\text{Shear scalar: } \sigma^2 = \frac{7}{162}\left[\frac{a^2(3 + 4C_2 e^{aT})q^2 \exp(4T)}{2a}\right]\exp - 2\left\{C_1 + C_2 e^{2aT} + \frac{3aT}{2} + \frac{q^2 e^{4T}}{8a^2}\right\} \quad \text{…(26)}$$

For large values of T, we get

$$\frac{\sigma}{\theta} \neq 0 \quad \text{…(27)}$$

As the time increases the spatial volume also increases which implies the anisotropic expansion with time. From equation (31), the model does not approach isotropy for large values of *T*.

The model (23) is of the inflationary type.

By the straight forward evaluation of deceleration parameter

$$q = -3\theta^{-2}\left[\theta_{;\alpha}u^{\alpha} + \frac{1}{3}\theta^{2}\right], \quad \ldots(28)$$

For the model (23), it is observed that q turns out to be negative which confirms the fact that model (23) represents inflation.

CONCLUSION

In this paper, we have obtained Bianchi type-VI_0 cosmological models in the presence of cosmic string source which corresponds to Reddy string ($\lambda + \rho = 0$) in the frame work of self-creation theory. It should be noted that the rest energy density and tension density of the cloud of strings have no initial singularity. The model, thus obtained, is found to be inflationary type and free from initial singularity. The model is anisotropic and is expanding with time. It is well known that the interacting scalar fields play a vital role in studies of inflationary cosmology. Hence inflationary models based on scalar-tensor theories are now fairly active fields of investigations. This study will throw some light on the structure formation of the universe, which has astrophysical significance.

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