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A tutorial to solve the 'free' two-body binary pulsar celestial mechanics problem

Abstract

The 'captured' 2-body Kepler problem usually requires bodies that have a large difference between their separate masses. The larger body is usually assumed centrally located that influences a smaller satellite body. In binary pulsar orbits, the two bodies may have similar masses as 'free' bodies to generate separate orbits. These bodies with similar weights that may produce highly elliptical trajectories while other pulsar binaries with different weights produce near circular orbits; clearly this behavior is counter-intuitive. Consequences of these orbits are either premature or that the neutron star may alter gravity due to excessive axis rotation. Closed-form solutions are presented for the 'free' two-body problem. Results indicate that the eccentricities for binaries may be different with each of these 'free' orbits. Moreover, there is a correlation that relates eccentricities, to the mass of the binaries, the type of trajectories as well as a function of the neutron star's rotation rate. If this is the case, there is a possibility that angular momentum may play a role in gravitation.

INTRODUCTION

When an engineering student normally learns about the 'captured' 2-body problem[1-3], there are several restrictive assumptions for the analysis. One basic assumption is that there is a large difference between the weights of the two bodies. Moreover, the larger body, say the Earth is immovable and the second lighter body moves similar to a small satellite in orbit around the Earth. These solutions result in an orbit that is defined as either a circular, elliptical, or a hyperbolic orbit depending upon the satellite's kinetic energy and initial conditions. Accuracy for these problems is rather straightforward and results are satisfactory to predict satellite orbits with these assumptions. Changes to the analytical solution for the satellite-Earth problem can be varied to consider more detailed information regarding the characterization of the gravitational effects on the Earth. For example, the sphere of the Earth can consider mass density variations that result in changes in the gravitational attraction due to the presence of a mountain range, an iron ore deposit, the ocean or other effects as a function of the surface location on a spherical model.

The problem of concern is to evaluate effects within a binary pulsar. Based upon an analysis by Murad[4], a tabulation was formulated that provided information about performance on several binary pulsar orbits. These binary pulsars usually have a neutron star and a companion body or another neutron star that results in separate distinctive elliptical or circular orbits. The problem is that the two bodies have relatively similar masses that travel in different or disconcerting orbits that may be centered about a common point of the elliptical orbit's foci points. Clearly this makes the 'free' celestial mechanics problem more complicated. Based upon the knowledge of the first problem, the uneducated would make an initial assumption that the pole or focal point of the elliptical orbit must have another gravitational effect or possibly contain a third body. The effort in this paper is to explain how to gain some important insights about the orbits and find a relationship between the masses of the body, rotation rate of the neutron star and the resulting trajectories or their eccentricities.

DISCUSSION

Based upon the masses of the neutron star and its companion in a binary pulsar, astronomers made some judgments about such orbits. However, their judgments may be counter intuitive in that when the masses are the same as the two bodies, one would expect that the orbits would be circular and when the masses are greatly different, they would expect it to be highly elliptical, parabolic or even hyperbolic. In reality, they are found most likely highly elliptical orbits for nearly equal masses in lieu of producing circular orbits while bodies with mass differences produce near circular orbits.

These consequences based upon the conventional wisdom are contrary or counterintuitive toward these expectations. A possible rationale for these differences is that gravity, in addition to having a Newtonian attraction, might have an impact upon angular momentum per Jefimenko[5,6], as well as the notions from Winterberg[4] that suggests highly rotating rates of a body, for example a neutron star, could reduce gravitation. These significances suggest that there is a coupling effect between the rotation rate of the neutron star and the masses between the two bodies to explain these differences in their orbits. This analysis obviously assumes that the initial masses are correct. Thus, there has to be a balance within the neutron star's high rotation rate and these trajectories. Neutron stars can rotate at speeds from ω values of 10 to 600 revolutions per second using masses that are anywhere from a mass that is 1 to 1.4 times greater than our own sun. Interestingly, the sun rotates about once a month that is extremely slow by comparison to these values.

ANALYSIS

Before looking at specific orbits and orientations, several notions are required. Trajectories dependent upon several bodies seem to obey a particular law in lieu of resulting in significant collisions. For example, in looking at two orbits where the bodies are close to each other, say one orbit is clockwise while the other is counter clockwise, it becomes obvious that the two bodies should be attracted in a collision. Thus there has to be some assumptions for this analysis.

An initial assumption will be that both orbits are in the same motion plane so that the problem can be reduced to a two-dimensional analysis. One major assumption is based upon the notion that if gravity does not have only a gravitational attraction but produces angular momentum by considering that both orbits must be either clockwise or both counter-clockwise.

If this does not occur, the momentum at each orbit results in both a radial and azimuthal force based upon the gravitation from the other body. This becomes a difficult geometric situation that results in motion that would leave the orbits ending in a collision. These assumptions are discussed in this section.

Standard terminology

The basic problem of the 'captured' two-body model is that one body is relatively light in terms of mass while the other body has a significant mass in the same plane. With this premise[1-3], the body with the larger mass is assumed to be immovable compared to the first body. The issue is to determine the initial momentum conditions and energy conservation problem essentially based upon the premise that the lighter body performs the dynamics that are of interest since the larger body is assumed stationary. The coordinate system uses a polar coordinate system where a unique point is determined based upon a vector length that has a distance and an angular orientation to completely specify the coordinate location related to a reference coordinate origin. The distance between the two bodies is from a center of the reference coordinate system that includes a focal point on the smaller mass' orbit. The center of this reference coordinate system is assumed to also be some negligible distance from the center of the larger body. In reality, there is some small distance treated as inconsequential between the actual locations for the barycenter.

The radial and angular momentum equation is defined as:

$$\bar{F}_r = m\bar{a}_r = m\left(\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\hat{r},$$
$$\bar{F}_\theta = m\bar{a}_\theta = m\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\hat{\theta} = \frac{m}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\hat{\theta}. \quad (1)$$

The subscripts in the LHS are not derivatives but actually the radial and azimuthal force directions respectively. Derivatives are functions of time. The radial force is based upon the gravitational attraction between the two bodies. Moreover, the second equation assumes the azimuthal force vanishes for each of these bodies.

In this problem, the radial dimension is changed as the difference between the distances to the two objects. The problem can be reduced to one dimension with some definitions where μ is the total mass of both bodies ($G(m_1+m_2)$). This is considered as the gravitational attraction for this problem as follows:

$$\left(\frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right) = -\frac{\mu}{r^2},$$
$$m\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right) = \frac{m}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0., \quad h = r^2\left(\frac{d\theta}{dt}\right). \quad (2)$$

Clearly the azimuthal gravitation disappears with a constant, h, that is the angular momentum per unit mass used to satisfy this equation for the azimuthal acceleration. Thus the second equation vanishes. At this point, a variable is selected based upon an inverse function of the radius to simplify the problem and removing the time derivatives with substitutions from the problem. This results in:

$$r = \frac{1}{u}, \quad \frac{d\theta}{dt} = hu^2, \quad \frac{dr}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt} = -h\frac{du}{d\theta};$$
$$\frac{d^2 r}{dt^2} = -h\frac{d^2 u}{d\theta^2}\frac{d\theta}{dt} = -h^2u^2\frac{d^2 u}{d\theta^2}. \quad (3)$$

When these are substituted into the above equation for the radial momentum, the results are:

$$-h^2u^2\frac{d^2u}{d\theta^2}-h^2u^3=-\mu u^2 \quad , \tag{4}$$

Or with some simplifications:

$$\frac{d^2u}{d\theta^2}+u=\frac{\mu}{h^2} \tag{5}$$

The solution of this ordinary differential equation considering a geometric length *l* and eccentricity *e* is:

$$u=\frac{\mu}{h^2}+C\cos(\theta-\theta_o) \quad \text{or} \quad r=\frac{1}{(1+e\cos\theta)}. \tag{6}$$

The importance of this equation is that the eccentricity *e* plays a significant role. Basically the smaller body rotates about the larger body with a circular orbit (if *e* is zero) or if the eccentricity is positive and less than *1.0*, the orbit is an elliptical orbit with the major body located at one of the focal points in the elliptical orbit. If eccentricity is greater than *1.0*, the orbit is hyperbolic and it leaves or escapes the gravitational pull of the larger body. Obviously, this result depends upon initial velocity conditions and kinetic energy before the interaction.

'Free' body orbits

Let us do some tutorial thinking for two sections. The concern is that the mass fractions between the bodies are so different, what occurs if the masses or weights are comparable and how does that impact the trajectories of both bodies? Figure 1 shows a typical situation that if it were possible, the two bodies would have to include an azimuthal gravitational force. The only allowable situation would be if the bodies were moving away from each other at high enough speed but even here, the final orbit would also become questionable. The other possibility is that a larger third body may exist at the central focal point. The concern is to define some conditions about stable orbits.

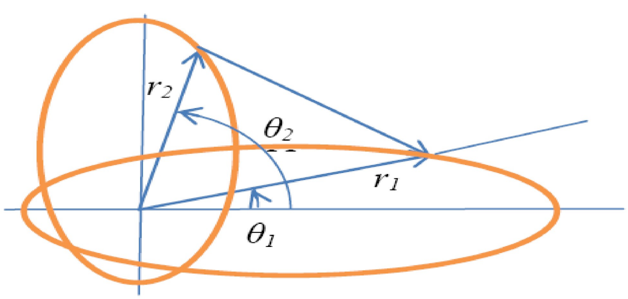

Figure 1 : If the two bodies are oriented at these orbits, they will undergo azimuthal acceleration toward each other. Here the desire of the bodies may leave the orbits and directly move toward each body possibly resulting in a collision.

Unstable orbits

There are several things that could destroy this methodology or require considerable complications where the problem is no longer solvable. For example, let us assume two bodies that move in separate elliptical orbits. Under these circumstances, the bodies would be as in Figure 1 and potentially change these elliptical orbits to result in a collision. When crossing orbits, the probability increases for a collision as well as orbital changes due to gravitational overload that traverses on one side of the elliptical orbit over the other. The fact that the rotation rate is the same using rotation of the line between the two bodies also implies that there is some possibility that angular momentum is a component of gravitation. This is specified by Jefimenko and an effort by Lavrentiev et al[7] that suggests all of the moons in the solar system operate in the same rotational direction that face the same portion of the moon toward their main planets. Thus, we only see the same side of the Moon while on the Earth. This is true throughout the solar system. Moreover, it is highly probable that the rotation of each mass can be established in a binary pulsar system. The counter-argument is that each of these moons has a gravitational offset that allows this phenomenon.

Let us look at what might appear to be a reasonable stable situation. Let us place both orbits at the same azimuth orientation as shown in Figure 1. An unstable situation may occur if the bodies are in an orientation that depends upon the separation distance as $r_1 - r_2$ that results in the simplest form of the radial direction in a simple form:

$$\left(\frac{d^2r_1}{dt^2}-r_1\left(\frac{d\theta_1}{dt}\right)^2\right)=-\frac{\mu_1}{(r_1-r_2)^2},$$
$$\left(\frac{d^2r_2}{dt^2}-r_2\left(\frac{d\theta_2}{dt}\right)^2\right)=-\frac{\mu_2}{(r_1-r_2)^2} \tag{7}$$
$$\text{where } \mu_1=Gm_2 \text{ and } \mu_2=Gm_1.$$

And in the simplest form of the azimuthal direction:

$$m_1\left(r_1\frac{d^2\theta_1}{dt^2}+2\frac{dr_1}{dt}\frac{d\theta_1}{dt}\right)=\frac{m_1}{r_1}\frac{d}{dt}\left(r_1^2\frac{d\theta_1}{dt}\right),$$
$$h_1=r_1^2\left(\frac{d\theta_1}{dt}\right),$$
$$m_2\left(r_2\frac{d^2\theta_2}{dt^2}+2\frac{dr_2}{dt}\frac{d\theta_2}{dt}\right)=\frac{m_2}{r_2}\frac{d}{dt}\left(r_2^2\frac{d\theta_2}{dt}\right), \tag{8}$$
$$h_2=r_2^2\left(\frac{d\theta_2}{dt}\right).$$

In these equations, the *h* values are no longer constants but rather complex geometric relations. However, let us just examine the radial terms. These radial equations reveal that if the satellites can cross each other or asymptotically reaches the same radius length, the gravitational attraction when the bodies approach each other will become a singularity or infinite gravitation that results in a possible collision. This obviously is an unstable orbit.

Stable orbits

Let us assume that both bodies will move in the same rotation direction simultaneously either as clockwise or counter-clockwise. This assumes that Jefimenko is probably correct that gravity has a radial attraction similar as Newton as well as produces angular momentum. We will also assume that both bodies move with mixed orbits as described in Figure 2 where one is circular and the other is elliptic. Moreover, the focus is at the closest point to the orbit at its closest approach (e.g., perigee).

(a) The barycenter requirement

The issue is if a line between the two bodies at the barycenter coincides with a focal point or not. Obviously this determines the types of orbits. In the initial problem of the captured 2-body problem, the barycenter is located close to the focal point of the orbit. For this issue, can a mixed orbit exist where you have one body in an elliptical orbit and the second body at a circular orbit as seen in Figure 2? The mathematics for the latter are greatly simplified but if they exist, the aerial area per Kepler's law might be violated. This raises another set of conditions if you have elliptical orbits where their long axis are, say 45 or 90 degrees apart as shown in Figure 1. In this case, the bodies at specific locations would require excessive speed to meet these requirements. Looking at Figure 3, the intuitive wisdom is that the heavier body should be located at the central location or have a smaller radius as it asymptotically approaches the captured 2-body problem.

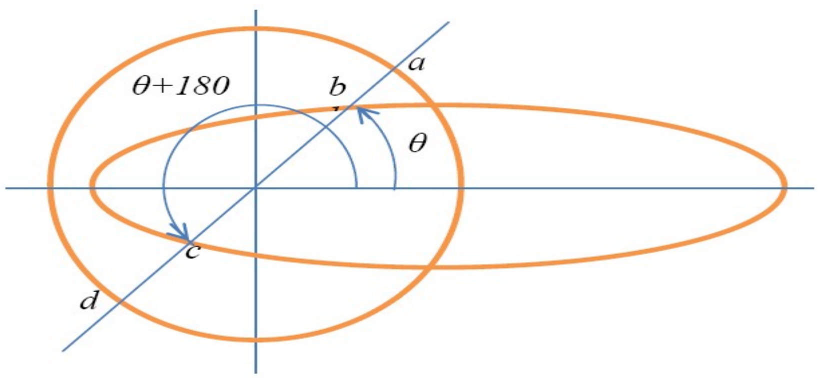

Figure 2 : This involves an elliptical and circular orbit combination. Both orbits use common focal point. If the bodies are located in *a* and *b*, they are unstable because they can easily leave the orbits and lead to a collision. If, however, they are located at say, *a* and *c*, or *b* and *d*, the two orbits may represent a stable set of orbits but may violate the Kepler aerial area theorem.

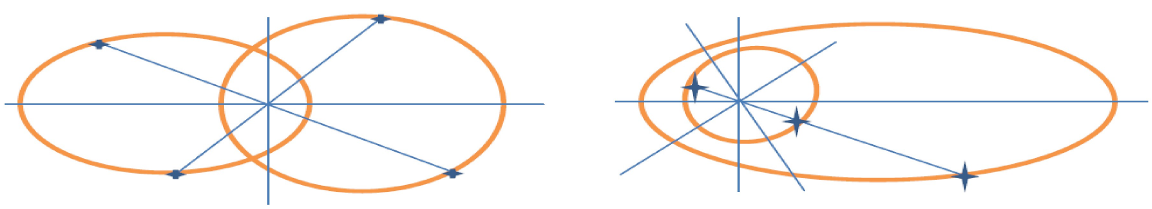

Figure 3 : These are representative stable orbits for different orientations. Stability occurs if the line joining the two bodies is joined by a focal point that is common to each orbit. In the second case, an unstable orbit occurs at the stars closest to each other and if each axis is not collinear.

Let us assume that there are two masses alone in the Universe. They form a system with a center of mass. That center of mass is without acceleration and moves uniformly along some velocity vector – an unchanging velocity vector. We can choose our coordinate inertial system to have its origin anywhere as long as it moves uniformly without any acceleration that moves with constant velocity. We can, therefore, choose our inertial system's origin at one of the two bodies and have that inertial-frame origin move at exactly the constant velocity of that body at *a particular instant of time*. That velocity vector and the observer (sitting on the chosen mass) at that *instant of time* define a plane (from geometry a point and a line defines a plane – the line being the instantaneous velocity vector in this particular case). The usual idea is to select the center of mass, CM, of the two masses (barycenter) that acts as the center of coordinates or origin of the coordinate system. Thus a three dimensional framework to a single two-dimensional plane is collapsed. We are in a two-dimensional plane inertial framework that is moving uniformly alone in the Universe. In essence, the binary orbits remain in one inertial plane moving without rotation and with constant velocity through the universe. There are no other external forces to move any mass out of that plane. If the observer is on a mass (of any value relative to the other mass) then this mass exerts a gravitational force on the other mass. The observer on this mass calculates the motion relative to the other body (mass) based upon the inverse-square-law gravitational force and the initial condition (instantaneous velocity vector) of that other mass —- this mass finds it to be a conic section as the solution to the classical two-body problem. This is the result based upon the forces acting against the barycenter. Let us suppose they are at a distance r_1 for m_1 and r_2 for m_2. Again by definition of the CM the distances must always maintain the same ratio that is: $r_1 / r_2 = m_2 / m_1$ and that is defines the CM defined so that a large mass must be closer to the CM fulcrum than a small mass that is located further to maintain a balance. And as time marches on we can watch the masses approach and receive from us in unison. Thus we have a simple 1D linear coordinate system if no other forces are involved as shown in equations 1-6.

Thus the barycenter is an important constraint. Orbits would have to be orientated that fall in the same line for the long axis of the elliptical orbits as well. If there are mixed orbits or elliptical orbits that are oriented at an angle, then there is a strong possibility that the two bodies are influenced by either a third or fourth body(s), a region with unusual gravitational fields representing a possible singularity, or that there are solar wind-like effects where particles shower the two bodies in a specific directional bias. If not, then these orbits will not be stable. Thus we shall assume that the barycenter is unmovable without the presence of other forces.

(b) Elliptical orbits

Let us place the bodies in a two-dimensional plane inertial framework that is moving uniformly alone in the Universe. In essence, the pulsar binary orbits remain in one inertial plane moving without rotation and with constant velocity throughout the universe. There are no other external forces to move any mass out of that plane. This reduces as in the previous, a three-dimensional situation to a two-dimensional plane.

Let us assume that the two bodies move in the same rotational direction with two different elliptical orbits in the same plane. Additionally, let us assume that there is a line between both bodies that also crosses through the focal point or barycenter that is common to either elliptical or circular orbits. If the line is such then the distance between both bodies are equal to $r_1 + r_2$ where the first radius is from the first body to the foci and the second radius is the second body to the same foci which is collinear on the same line as the first body. Thus, the distance between the two bodies is easy to establish. Let us assume that the angular orientation is at an azimuth angle at θ for the first body where the orientation for the second body is located at $\theta + \pi$ or *180* degrees further in a counter-clockwise direction as a perihelion. Let μ_1 be $G\, m_2$ and μ_2 be $G\, m_1$ to account for the mass terms for gravity.

The governing radial and azimuthal equations for these two bodies under these conditions now become:

$$\begin{aligned}
&\left(\frac{d^2 r_1}{dt^2} - r_1\left(\frac{d\theta_1}{dt}\right)^2\right) = -\frac{\mu_1}{(r_1+r_2)^2},\\
&\left(\frac{d^2 r_2}{dt^2} - r_2\left(\frac{d\theta_2}{dt}\right)^2\right) = -\frac{\mu_2}{(r_1+r_2)^2},\\
&\left(r_1\frac{d^2\theta_1}{dt^2} + 2\frac{dr_1}{dt}\frac{d\theta_1}{dt}\right) = \frac{1}{r_1}\frac{d}{dt}\left(r_1^{\,2}\frac{d\theta_1}{dt}\right) = 0., \quad h_1 = r_1^{\,2}\frac{d\theta_1}{dt},\\
&\left(r_2\frac{d^2\theta_2}{dt^2} + 2\frac{dr_2}{dt}\frac{d\theta_2}{dt}\right) = \frac{1}{r_2}\frac{d}{dt}\left(r_2^{\,2}\frac{d\theta_2}{dt}\right) = 0., \quad h_2 = r_2^{\,2}\frac{d\theta_2}{dt}.
\end{aligned} \tag{9}$$

These are nonlinear equations regarding radial momentum. On this basis, if there is a relationship between the two bodies, whether they both move in elliptical or circular orbits, the common focal point must fall along the line between the two points.

Using the above methodology in equations 1-6, the equations to be solved are as follows:

$$\frac{d^2u_1}{d\theta_1^2} + u_1 = \frac{\mu_1\, u_2^2}{h_1^2\left(u_1+u_2\right)^2} \quad \text{and} \quad \frac{d^2u_2}{d\theta_2^2} + u_2 = \frac{\mu_2\, u_1^2}{h_2^2\left(u_1+u_2\right)^2}, \qquad \text{where} \quad u_1 = 1/r_1 \quad \text{and} \quad u_2 = 1/r_2. \tag{10}$$

With these equations, θ_2 is equal to $\theta_1 + \pi$. In addition the rotation rates for both orbits are identical. This also influences some relationships with h_1 and h_2.

Clearly these are coupled nonlinear equations where one orbit depends upon the second orbit and vice versa. The solution has to have periodic initial and boundary conditions. Note that the angle for the second orbit simultaneously includes 180 degrees. This is as follows:

$$u_1 = C_1\cos\left(\theta-\theta_o\right) + \mu_1\left\{\int_{\theta_o}^{\theta}\frac{u_2^2(\xi)}{h_1^2\left(u_1(\xi)+u_2(\xi)\right)^2}\sin\left(\theta-\xi\right)d\xi + \right.$$
$$\left. + \int_{\theta_o}^{\theta_o+2\pi}\frac{u_2^2(\xi)}{h_1^2\left(u_1(\xi)+u_2(\xi)\right)^2}\frac{\cos\theta\cos\left(\xi-\pi/2\right)}{2\sin\pi/2}d\xi + \int_{\theta_o}^{\theta_o+2\pi}\frac{u_2^2(\xi)}{h_1^2\left(u_1(\xi)+u_2(\xi)\right)^2}\frac{\sin\theta\sin\left(\xi-\pi/2\right)}{2\sin\pi/2}d\xi\right\}, \text{ and}$$

$$u_2 = C_2\cos\left(\theta+\pi-\theta_o'\right) + \mu_2\left\{\int_{\theta_o'}^{\theta}\frac{u_1^2(\xi)}{h_2^2\left(u_1(\xi)+u_2(\xi)\right)^2}\sin\left(\theta+\pi-\xi\right)d\xi + \right.$$
$$\left. + \int_{\theta_o'}^{\theta_o'+2\pi}\frac{u_1^2(\xi)}{h_2^2\left(u_1(\xi)+u_2(\xi)\right)^2}\frac{\cos\left(\theta+\pi\right)\cos\left(\xi-\pi/2\right)}{2\sin\pi/2}d\xi + \int_{\theta_o'}^{\theta_o'+2\pi}\frac{u_1^2(\xi)}{h_2^2\left(u_1(\xi)+u_2(\xi)\right)^2}\frac{\sin\left(\theta+\pi\right)\sin\left(\xi-\pi/2\right)}{2\sin\pi/2}d\xi\right\} \quad (11)$$

where $\theta_o' = \theta_o + \pi$.

Obviously, these are complex nonlinear Volterra Integral equations. Assumptions can be made based upon the relationships between the two different distances and rotation rates that define the *h* terms based upon the mass fractions. Moreover, the rotation rates should also be the same. The latter is a limiting assumption. These are:

$$\frac{r_1}{r_2} = \frac{\mu_1}{\mu_2} = \frac{u_2}{u_1}, \quad \text{and} \quad \frac{h_1}{h_2} = \frac{r_1^2\frac{d\theta_1}{dt}}{r_2^2\frac{d\theta_2}{dt}} = \frac{\mu_1^2}{\mu_2^2}. \quad (12)$$

The second and third integrals should result in a constant term. The resulting problem is elliptical orbits about each other:

$$u_1 = C_1\cos\left(\theta-\theta_o\right) + \mu_1\left\{\mu_1^2\int_{\theta_o}^{\theta}\frac{1}{h_1^2}\sin\left(\theta-\xi\right)d\xi + \right.$$
$$\left. + \frac{\mu_1^2}{2\sin\pi/2}\int_{\theta_o}^{\theta_o+2\pi}\frac{1}{h_1^2}\cos\theta\cos\left(\xi-\pi/2\right)d\xi + \frac{\mu_1^2}{2\sin\pi/2}\int_{\theta_o}^{\theta_o+2\pi}\frac{1}{h_1^2}\sin\theta\sin\left(\xi-\pi/2\right)d\xi\right\}, \text{ and}$$
$$u_2 = C_2\cos\left(\theta+\pi-\theta_o'\right) + \mu_2\left\{\mu_2^2\int_{\theta_o'}^{\theta}\frac{1}{h_2^2}\sin\left(\theta+\pi-\xi\right)d\xi + \right. \quad (13)$$
$$\left. + \frac{\mu_2^2}{2\sin\pi/2}\int_{\theta_o'}^{\theta_o'+2\pi}\frac{1}{h_2^2}\cos\left(\theta+\pi\right)\cos\left(\xi-\pi/2\right)d\xi + \frac{\mu_2^2}{2\sin\pi/2}\int_{\theta_o'}^{\theta_o'+2\pi}\frac{1}{h_2^2}\sin\left(\theta+\pi\right)\sin\left(\xi-\pi/2\right)d\xi\right\}.$$

Adding some more changes results in:

$$u_1 = C_1\cos\left(\theta-\theta_o\right) + \frac{\mu_1^3}{h_1^2}\left\{\int_{\theta_o}^{\theta}\sin\left(\theta-\xi\right)d\xi + \frac{\cos\theta}{2\sin\pi/2}\int_{\theta_o}^{\theta_o+2\pi}\cos\left(\xi-\pi/2\right)d\xi + \right.$$
$$\left. + \frac{\sin\theta}{2\sin\pi/2}\int_{\theta_o}^{\theta_o+2\pi}\sin\left(\xi-\pi/2\right)d\xi\right\}, \text{ and}$$
$$u_2 = C_2\cos\left(\theta+\pi-\theta_o'\right) + \frac{\mu_2^3}{h_2^2}\left\{\int_{\theta_o'}^{\theta}\sin\left(\theta+\pi-\xi\right)d\xi + \frac{\cos\left(\theta+\pi\right)}{2\sin\pi/2}\int_{\theta_o'}^{\theta_o'+2\pi}\cos\left(\xi-\pi/2\right)d\xi + \right. \quad (14)$$
$$\left. + \frac{\sin\left(\theta+\pi\right)}{2\sin\pi/2}\int_{\theta_o'}^{\theta_o'+2\pi}\sin\left(\xi-\pi/2\right)d\xi\right\}.$$

Note that a constant term appears for all of the integral equations based upon the initial conditions for the two masses. If we are dealing with a satellite moving about the Earth where μ_1 is very small, the integral expressions for the first orbit has a considerably different value for the multipliers of the second integral.

Integrating these terms using that the second angle is added by 180 degrees results in:

$$u_1 = C_1\cos\left(\theta-\theta_o\right) + \frac{\mu_1^3}{h_1^2}\left[1-\cos\left(\theta-\theta_o\right)\right], \text{ and } u_2 = C_2\cos\left(\theta+\pi-\theta_o\right) + \frac{\mu_2^3}{h_2^2}\left[1-\cos\left(\theta+\pi-\theta_o\right)\right]. \quad (15)$$

The solution assuming that the initial angle is at zero degrees results in:

$$r_1 = \frac{1}{+\left(C_1 - \frac{\mu_1^3}{h_1^2}\right)\cos\theta + \frac{\mu_1^3}{2h_1^2}}, \text{ and } r_2 = \frac{1}{+\left(C_2 - \frac{\mu_2^3}{h_2^2}\right)\cos(\theta+\pi) + \frac{\mu_2^3}{2h_2^2}}. \tag{16}$$

Where the constant terms are defined for the initial radius at an angle at the initial angle as subscripts measured from the coordinate reference system, the value of *r* is the initial distance and the subscript is for a particular orbit with:

$$C_1 = +\frac{\mu_1^3}{h_1^2} + \left[\frac{1}{r_1^o} - \frac{\mu_1^3}{2h_1^2}\right]\frac{1}{\cos\theta_o}, \text{ and } C_2 = +\frac{\mu_2^3}{h_2^2} + \left[\frac{1}{r_2^o} - \frac{\mu_2^3}{2h_2^2}\right]\frac{1}{\cos(\theta_o+\pi)}. \tag{17}$$

Combining these terms yields:

$$r_1 = \frac{2h_1^2 / \mu_1^3}{1 + \left(\frac{2h_1^2}{\mu_1^3 r_1^o} - 1\right)\frac{\cos\theta}{\cos\theta_o}}, \text{ and } r_2 = \frac{2h_2^2 / \mu_2^3}{1 + \left(\frac{2h_2^2}{\mu_2^3 r_2^o} - 1\right)\frac{\cos(\theta+\pi)}{\cos(\theta_o+\pi)}}. \tag{18}$$

Finally, the eccentricities for each of these orbits are:

$$e_1 = \left(\frac{2h_1^2}{\mu_1^3 r_1^o} - 1\right) \Big/ \cos\theta_o, \text{ and } e_2 = \left(\frac{2h_2^2}{\mu_2^3 r_2^o} - 1\right) \Big/ \cos(\theta_o+\pi) = \left(\frac{2h_1^2 \mu_2}{\mu_1^4 r_2^o} - 1\right) \Big/ \cos(\theta_o+\pi). \tag{19}$$

Note that for situations where the mass fraction is almost the same, the result is driven by the differences in the initial angles for the different orbits. This results in the two-body problem regarding elliptical, circular or hyperbolic trajectories but with two separately moving bodies. Astronomers only call out only a single eccentricity for both bodies in a binary. There is no real reason why one would assume that especially if the masses are not the same; hence, we are showing two different values for each trajectory. If these values are the same, one could have a direct relationship ratioing the *h* values as a function of the initial radius at each celestial body. Thus the second term shows that if these relationships were the same, they would only be true for the same mass fraction terms. Hence there should be different eccentricity for each trajectory. In any extent, the normal convention would have never established these different eccentricities in these trajectories unless a more complete trajectory evaluation is performed as here.

TABLE 1 : Pulsar information

Pulsar	Eccentricity e	P(ms)	$M_{primary}$	M_{comp}	Rotation Rate	Orbit Period
8. PSR B1913+16	0.61713	37.904	1.4414 ± .0002	1.3867 ± .0002	26.3Hz	.3229
13. PSR J1903+0327	0.437	2.15	1.74	1.0	465Hz	95.17d

RESULTS

The question is if there are any suitable results that could be established. Using a portion of the pulsar paper[4] by the author, some data reveals binary pulsars as follows.

These are shown in Figure 4. The eccentricities of both of these binaries are elliptical and one should expect to see such a trajectory. In a sketch of the first, these orbits are clearly oriented along the long axis. Note that the first sketch shows that the opposite bodies are directly opposite to each other. One would argue that the two bodies are fairly similar in terms of total mass. This tends to substantiate the values shown by looking at the length between the two bodies and measuring the distance to the foci of the elliptical orbits. The lengths are respectively μ_1 and μ_2. This value looks very close to being about .40 or 40%.

In Figure 5, assuming that this is an adequate representation of the above shown in Figure 4, the μ value varies from this graphic interpretation with no numerical information of .367 to .38. The value here is .364 or about 0.83% different or less than 1 percent. Obviously this result could be altered to reach a similar value. Also note the similar point of view that the longitudinal axis of both orbits is collinear. Moreover, the larger body is located on the smaller or inside trajectory. Note that this last binary is closer to obtaining circular motion. This also implies that when a highly eccentricity orbit exists, it could represent an immature scheme in that the orbit did not 'settle' down and was still in the situation to make additional corrections. Moreover, neither of these orbits has a trajectory that has a major axis at an

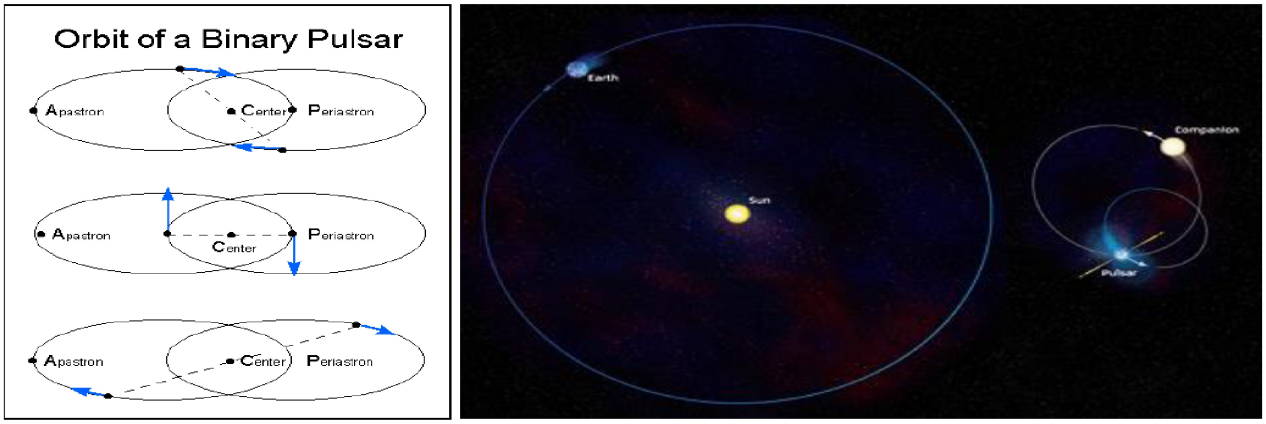

Figure 4 : The orbits of the binary pulsar 1913+16 and a comparison of the orbits of the pulsar J1903+0327 with its possible sun-like companion star with the orbit of the Earth around the sun[11].

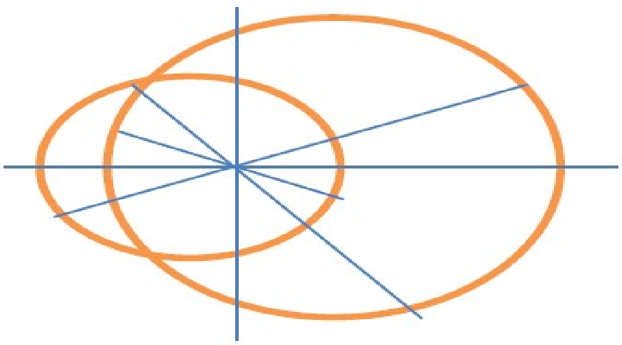

Figure 5 : This is a breakdown of the above with J1903+0327. Look at the relations between the distances from the focal point to the separate bodies and the total length between the bodies to form a mass relationship of the bodies.

angle similar to what is shown in Figure 1 but like the second drawing in Figure 3.

Early in the discussion, the point was raised about angular momentum effects. In a previous reference[4], the author used Winterberg's ideas regarding the rotational effect of the neutron star that would decrease its gravitational attraction to the companion. The approach would be to alter the gravitational attraction that would include some normalized quantity that includes a rotational rate term (ω^2) for the gravitational attraction to both bodies. In this format, the trajectory definition, mass fraction and neutron star rotation rate may provide some correlations directly to establish the value of eccentricity. These inputs will need:

$$\mu_1 = G m_2 \quad \text{and} \quad \mu_2 = G m_1 \left(1 + \frac{\omega^2}{2\pi G \rho_c}\right). \qquad (20)$$

These terms for the mass fractions would be adjusted to account for the normalization of the angular rotation rate of the neutron star. Of these values, G is the gravitational constant, ρ_c is the gravitational density of the companion and ω is the rotational rate of the neutron star. Note that as the neutron star rotation rate will drive eccentricity going toward an asymptotical circular orbit. This may be the rationale that demonstrates a correlation with gravity and angular momentum.

CONCLUSIONS

The basic requirements to identify conditions for creating a binary pulsar indicate that the two bodies will have a common focal point at the barycenter, which is expected under the conventional wisdom. Moreover, the orbits will have the largest axis of an elliptical orbit that would be collinear for both bodies. These bodies will rotate both in the same direction either as clockwise or counter-clockwise that by itself implies some angular momentum component as part of gravity. A simple graphic solution indicates that the mass ratio between the two bodies could be established. These conclusions that support the counter intuitive situation about the choice of masses for binary pulsars may indeed be valid. Thus these general orbits appear to be premature in most of these situations that require the continuous need for normalization of their orbits with extensive time.

By using the approach defined above, separate orbits are defined with separate eccentricities for each of the bodies. These results also indicate that there is a coupling between the neutron star's rotation rate and the type of trajectory that results based upon mass fraction. This was the original intention of this analysis. If we look at this further, we can only see *causes* but not the direct *effects*. If these are really mature orbits, then these masses may be incorrect based upon angular momentum as a consequence of gravity that is altered due to the spinning neutron star impact. Thus we may never really measure these masses and their *effects* since we cannot measure these *causes*.

NOMENCLATURE

a = Reference length
e = Eccentricity
F = Thrust or force

g = Gravity
G = Gravitational constant
m = Mass
r = Radial coordinate
t = Time

Symbols

ρ = Density
ω = Rotation rate
θ = True anomaly
μ = Mass fraction

Subscripts

o = Initial value
$1, 2$ = Individual bodies

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