# Bio Technology 

# A simplified diagram multiplication method for displacement calculation of variable cross-section beam 

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#### Abstract

Because there are several times of graphics Segmentation with too tedious work using Diagram Multiplication to solve some problems, we put forward a new method which can simplify this process. The process is simple to improve computational efficiency, even more, it can be widely applied in the Variable Cross-Section Rob and Bending Moment Diagram which is a broken line graph. Students can easily understand and accept it, since it is a valuable way to promote.


## KEYWORDS

Diagram multiplication; Displacement calculation; Simplified method; Structural mechanics; Optimize.

## INTRODUCTION

The unit-load method is a common way in structure mechanics. When calculating the bending moment virtual work of member, we often run into a integral problem, and when the member is calculated by constant cross-section rods, and a section of the bending moment graph is a straight line, we can use diagram multiplication operation to replace the expression of integral operation, greatly improving the computing efficiency of students, and making teaching process more clear. So Diagram Multiplication is pretty welcomed ${ }^{[1-8]}$. However, it has some limiting conditions: 1. the rod must be straight. 2. EI must be constants. 3 . There is at least a straight line graph in $\mathrm{M}_{\mathrm{p}}$ and M . In common, the three conditions cannot be met at the same time causing lots of lengthy counting process, thus, it is more than important to decline the amount of calculation. According to this situation, we put forward a new method which has a great value.

## PRESENTATION OF QUESTIONS

As the following picture(Fig.1), calculate the vertical displacement of B with uniform distribution load.


## Figure 1: Example of variable cross-section beam

When solving this problem, you can impose unit force at $B$, then, the bending moment diagram is as following picture (Fig.2-3).


Figure 2: Bending moment diagram under uniformly distributed load


Figure 3: Bending moment diagram under unit load
It is a typical problem of piecewise calculation which is prone to error as Fig.4.


Figure 4: Wrong diagram multiplication

The reason is that the graphics after segmentation is not a standard parabolic form.

## TRADITIONAL ALGORITHM

Segmentation of the bending moment diagram of AC is shown as Fig.5.


Figure 5: Graphics segmentation of AC-beam
After that, it can be divided into 4 parts, as Fig. 6 shown.


Figure 6: Traditional algorithm
This time, the 4 parts are all standard graphics which can use ready-made formula to calculate as follows:
$\Delta_{B V}=\frac{1}{2 E I}\left(\mathrm{~A}_{1}{ }^{*} \mathrm{y}_{1}+\mathrm{A}_{2} * \mathrm{y}_{2}-\mathrm{A}_{4}{ }^{*} \mathrm{y}_{4}\right)+\frac{1}{E I}\left(\mathrm{~A}_{3} * \mathrm{y}_{3}\right)(1)$

Where:
$\mathrm{A}_{1}=\frac{1}{2} \frac{l}{2} \frac{q l^{2}}{2}=\frac{q l^{3}}{8}$
$\mathrm{A}_{2}=\frac{1}{2}-\frac{l}{2} \frac{q l^{2}}{8}=\frac{q l^{3}}{32}$
$\mathrm{A}_{3}=\frac{1}{3}-\frac{q}{2} \frac{q l^{2}}{8}=\frac{q l^{3}}{48}$
$\mathrm{A}_{4}=\frac{2}{3} \frac{l}{2} \frac{q l^{2}}{32}=\frac{q l^{3}}{96}$
$y_{1}=\frac{2}{3} \square+\frac{1}{3} \square \frac{l}{2}=\frac{5 l}{6}$
$y_{2}=\frac{1}{3} \square+\frac{2}{3}-\frac{l}{2}=\frac{2 l}{3}$
$y_{3}=\frac{3}{4}-\frac{l}{2}=\frac{3 l}{8}$
$y_{4}=\frac{1}{2} \square+\frac{1}{2}-\frac{l}{2}=\frac{3 l}{4}$
We can obtain $\Delta_{B V}=\frac{17 q l^{4}}{256 E I}$.

## PROPOSED ALGORITHM

We need to calculate 4 times using the traditional method with different yi corresponding to each graphics. For simplifying the counting process, we put forward a new method.

This method calculates it as the whole first, and then subtracts the part with different stiffness, after that; add the part with its own stiffness. It has simple calculation which only needs to square three times with standard graphics. (Fig.7)

(a) (b) (c)

Figure 7: Proposed algorithm
The result is equal to Fig7(a)-(b)+(c), calculation process is deduced as follows:
$\Delta_{B V}=\frac{1}{2 E I}\left(\mathrm{~A}_{1} * \mathrm{y}_{1}-\mathrm{A}_{2} * \mathrm{y}_{2}\right)+\frac{1}{E I}\left(\mathrm{~A}_{3} * \mathrm{y}_{3}\right)$
Where:

$$
\begin{aligned}
& \mathrm{A}_{1}=\frac{1}{3} \square \frac{q l^{2}}{2}=\frac{q l^{3}}{6} \\
& \mathrm{~A}_{2}=\frac{1}{3} \frac{l}{2} \frac{q l^{2}}{8}=\frac{q l^{3}}{48} \\
& \mathrm{~A}_{3}=\mathrm{A}_{2}=\frac{q l^{3}}{48} \\
& y_{1}=\frac{3}{4} \\
& y_{2}=\frac{3}{4}-\frac{l}{2}=\frac{3 l}{8} \\
& y_{3}=y_{2}=\frac{3 l}{8} \\
& \quad \text { We can also obtain } \Delta_{B V}=\frac{17 q l^{4}}{256 E I} .
\end{aligned}
$$

## COMPARISON OF TWO METHODS

Comparing the two methods, the features are as Table.1.
TABLE 1: Comparison of two methods

| Algorithm | Calculation times |  | Process |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | Diagram Multiplication | $\boldsymbol{A i}$ | $\boldsymbol{y i}$ | $\boldsymbol{A i}$ | $\boldsymbol{y i}$ |
| Traditional | 4 | 4 | 4 | hard | hard |
| Proposed | 3 | 2 | 2 | easy | easy |

We can see that the areas are not standard graphics which need 4 times' graphical multiplication in the traditional method. What's more, we need to use different formula including the difficulty of the calculation. Comparing with it, the new method needs only 3 times' graphical multiplication and one formula. And the calculation of Ai and yi just need 2 times which is also easier than the traditional method.

## COMPARISON OF TWO METHODS

This article uses two different methods to solve the displacement problem of varying-section bar. We can see that although the traditional method can simplify the process to some degree, it is still a little complicated. But the method this article put forward is more than easy and has less amount of calculation when it is used in the structural displacement. Since this new method is far more convenient when solving the structural displacement of varying-section bar. It is worthy to be promoted in teaching structural mechanics.

## ACKNOWLEDGEMENT

This research was supported by grants from The Henan Province foundation and front engineering research project (No. 122102210091)

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