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## A novel T-FGM(1,1) forecasting model based on rbf neural network for water demand forecasting

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### ABSTRACT

In order to forecast the water demand and enhance the utilization of water resources, based on the basic principle of Grey Model with First Order Differential Equation and one Variable (GM(1,1)), in this paper, a novel First-entry traversal Grey Model with First Order Differential Equation and one Variable (T-FGM(1,1)) was established by minimum total residual sum of square. Furthermore, A T-FGM(1,1) (First-entry traversal Grey Model with First Order Differential Equation and one Variable)-RBF (radial basis function) neural network model is established. The proposed model not only reduces the unstable factors that influence the forecast, but also can interfuse the advantages in the uncertainty domain in neural network.

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### KEYWORDS

GM(1,1);  
Grey model;  
First order differential equation  
and one variable;  
Wind speed forecasting;  
RB (radial basis function)  
neural network.

### INTRODUCTION

Water is essential for human survival and well-being, socio-economic development and for maintaining healthy ecosystems. With the development of social economy, the increase of population, human demand for water resources and the growing shortage of water resources, the water resources supply and demand contradiction is more outstanding. Therefore, it is very necessary and urgent to predict how accurate, efficient use of water, avoid the wrong investment, alleviate the tense situation of water resource. The correct prediction of water demand is of great significance for the harmonious development of our society, economy and environment, to the correct decision-making and implementation of major water resources engineering, water management which under the conditions of the market

economy. Water demand prediction is an important goal of water conservancy investment, can provide the necessary basis for water resources planning and management. In recent decades, many advanced theories have been produced in studies on water demand forecasting<sup>1-7</sup>.

Motivated by<sup>3-4</sup>, a novel First-entry traversal Grey Model with First Order Differential Equation and one Variable (T-FGM(1,1)) was established by minimum total residual sum of square. Furthermore, a T-FGM(1,1) (First-entry traversal Grey Model with First Order Differential Equation and one Variable)-RBF (radial basis function) neural network is established. The proposed model not only reduces the unstable factors that influence the forecast, but also can interfuse the advantages in the uncertainty domain in neural network.

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## BASIC THE FIRST-ENTRY GREY ARMA-EGARCH-M COMBINED FORECASTING MODEL

### GM (1, 1) models

The grey system theory, which was firstly brought forward by Deng in [5], has developed rapidly in recent years, has been successfully applied and is playing a more and more role in many fields. The Grey Model with First Order Differential Equation and one Variable (GM(1,1)) is a first-order linear dynamic model of single sequence and is a generally used grey sequence forecast model. The establishment process of the Grey Model with First Order Differential Equation and one Variable (GM(1,1)) is as follows:

Let  $x^{(0)}$  be the original data sequence of the existing water demand:  $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$ .

Then the first-order accumulated generating operation (1-AGO) of  $x^{(0)}$  is that:

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}. \quad (1)$$

Where the first-order accumulated generating operation sequence as

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, \dots, n.$$

The differential equation of (1) constituted by the one-order grey model can be expressed as:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b, \quad (2)$$

where  $a$  and  $b$  are the grey developmental coefficient and the grey control parameter respectively. The parameters  $a$  and  $b$  can be solved by means of the least-

square method as follows:  $\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N$ , where

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix}$$

and

$$Y_N = [x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)]^T.$$

The prediction value of the grey model with respect to the data sequence  $x^{(1)}$  is given by

$$\hat{x}^{(1)}(k) = (x^{(0)}(1) - \frac{b}{a})e^{-a(k-1)} + \frac{b}{a}, k = 2, 3, \dots, \quad (3)$$

from (3), the modeling value  $\hat{x}^{(0)}$  can be derived to be  $\hat{x}^{(0)}(1) = \hat{x}^{(1)}(1) = x^{(0)}(1)$ , and  $\hat{x}^{(0)}(k)$  analogue value will be:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) = (x^{(0)}(1) - \frac{b}{a})(1 - e^{-a})e^{-a(k-1)}, k = 2, 3, \dots$$

### T-FGM(1,1) model:

Now, We select an optimal weight vector  $\omega$  for the linear combination of the  $n$  P(k)-FGM(1,1)( $k = 0, 1, \dots$ ) models, and then the traversal grey model (T-FGM(1,1)) can be obtained by taking weighted mean value of prediction value of each model as the prediction result.

Suppose the traversal grey model (T-FGM(1,1))  $X = \Phi(\hat{x}^{(0)}(0), \hat{x}^{(0)}(1), \dots, \hat{x}^{(0)}(n))$ , which consists of  $n$  P(k)-FGM(1,1)() and weight vector, is

$$f_T = \sum_{k=0}^n \omega_k f_{P(k)} \quad (4)$$

Where is the T-FGM(1,1) forecasting model, is the P(k)-FGM(1,1) forecasting model.

By means of the  $n$  the  $n$  P-GM(1,1) prediction residual series

( $e(0,0), e(0,1), \dots, e(0,n), e(1,0), e(1,1), \dots$ ), the prediction residual errors information matrix can be expressed as

$$E = \begin{bmatrix} E(0,1) & E(0,1) & \dots & E(0,n) \\ E(1,0) & E(1,1) & \dots & E(1,n) \\ \vdots & \vdots & \ddots & \vdots \\ E(n,0) & E(n,1) & \dots & E(n,n) \end{bmatrix}$$

Where

$$E(k, j) = e(k,0)e(j,0) + e(k,1)e(j,1) + \dots + e(k,n)e(j,n), k, j = 0, 1, 2, \dots, n.$$

On general, E is positive definite matrix.

On the basis of traversal grey model residual error square and minimum standards, we can construct the following a mathematical programming model:

$$\begin{aligned} \min J &= \sum_{k=0}^n \left( \sum_{j=0}^n \omega_j e(k, j) \right)^2 \\ &= \sum_{k=0}^n \sum_{j=0}^n \omega_k \omega_j E(k, j) \\ &= \omega^T E \omega \end{aligned} \tag{5}$$

$$s.t. \sum_{k=0}^n \omega_k = 1, \omega_k \geq 0. \tag{6}$$

By means of the simultaneous solution of (5) and (6), we can calculate the weight vector . By substituting of the calculated weight vector into (4), the T-FGM(1,1) can be modeled.

**Tests of The model**

Usually, after-test residue checking and small error probability is used as a standard of quality of the model[25].

Residual error:

$$e(i) = x^{(0)}(i) - \hat{x}^{(0)}(i) (i = 0, 1, \dots, n),$$

Residual errors mean:  $\bar{e} = \frac{1}{n} \sum_{i=0}^n e(i),$

Mean:  $\bar{x} = \frac{1}{n} \sum_{i=0}^n x^{(0)}(i),$

Mean square deviation of raw series:

$$S_1^2 = \frac{1}{n} \sum_{i=0}^n (x^{(0)}(i) - \bar{x})^2,$$

Mean square deviation of residual error:

$$S_2^2 = \frac{1}{n} \sum_{i=0}^n (e(i) - \bar{e})^2,$$

posterior error ratio:  $C = \frac{S_2}{S_1},$

small error probability:

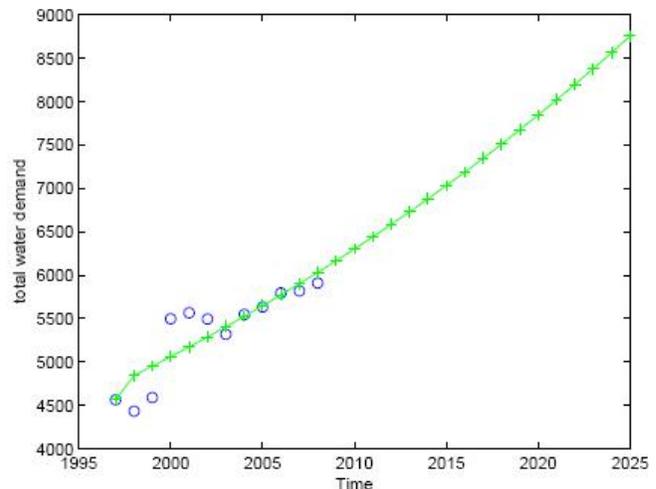
$$P = p [|e(i) - \bar{e}| < 0.6745S_1].$$

The less *C* value is, the better model is, and the bigger *P* value is, the better model is.

**Tests of the model**

Based on a series of the T-FGM(1,1) models, we can get a series of predictive values. But the predictive values generally have a certain deviation and the original data, and also have some relevance between the original sequence, and the association is unknown. Therefore, we use the RBF neural network model to simulate the relationship between the predicted value and the deviation between actual value and sequence, the predicted values as input data of RBF neural network, the actual value as the output samples, take some structure, then the RBF neural network training, each layer of each node can be obtained the weights and thresholds the GM ( 1, 1) to predict the next hour or more moments of the model values as the input of neural network, get the corresponding output as the final prediction for the next moment or a plurality of time value. RBF neural network learning algorithm selects the nearest neighbor clustering algorithm, which is an online adaptive clustering algorithm, the number of nodes in the hidden layer unit without ahead to determine.

The following Figure 1 presents the predicted amount of the national wide total water demand from 1997 to 2025.



**Figure 1 : Amount of the national wide total water demand**

**CONCLUSION**

In this paper, a novel First-entry traversal Grey Model with First Order Differential Equation and one Variable (FT-GM(1,1)) was established by minimum total residual sum of square. Furthermore, a FT-GM(1,1)( First-entry traversal Grey Model with First

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Order Differential Equation and one Variable) RBF (radial basis function) neural network is established. The proposed model not only reduces the unstable factors that influence the forecast, but also can interfuse the advantages in the uncertainty domain in neural network.

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