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## A Model To Calculate Sauter Diameter ( $d_{32}$ ) In Pulsed Sieve-Plate Columns



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### ABSTRACT

In this research, according to Pilhofer method, a developed model and an algorithm for the calculation of sauter or average drop diameter ( $d_{32}$ ) in pulsed sieve-plate columns is reported. The principal of this model is based on balancing between forces exerted on the drops when they pass through column length. The validity of the model is evaluated by two different extraction systems in a 22cm diameter column. The experimental results proved that model can predict Sauter diameter of drops, especially for high surface tension systems, with acceptable accuracy.

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### KEYWORDS

Pulsed sieve-plate column;  
Sauter diameter;  
Average diameter.

### INTRODUCTION

Diameter of drops is one of the crucial parameters to evaluate hydrodynamic of extraction columns. Therefore, many equations have been developed for drop size estimation. Most of the represented methods are empirical<sup>[1]</sup>, and some of them are only valid within a limited range of operating parameters<sup>[2]</sup>.

Among the existing models, Pilhofer<sup>[3]</sup> has presented a model which has an acceptable accuracy to predict Sauter diameter of pulsed sieve-plate columns, but it has been examined for only a few ex-

traction systems.

In this study, a method based on Pilhofer model and a developed algorithm have been presented to calculate Sauter ( $d_{32}$ ) or average diameter in pulsed sieve-plate columns. The advantage of the algorithm is that it can be developed for every agitated column.

On the other hand, drop size diameter not only does effect on hydrodynamic behavior of column but also it determines maximum load, flooding point and mass transfer rate of it. It is obvious that there is a wide distribution of drop sizes in extraction towers. Thus, the Sauter diameter has been used as a

parameter which shows average diameter of drops in extraction columns. This parameter is calculated from the following equation:

$$d_{32} = \frac{\sum n_i d_i^3}{\sum n_i d_i^2} \quad (1)$$

Sauter diameter which is the ratio of volume to surface area is a representative average diameter.

The following equation is established between Sauter diameter ( $d_{32}$ ), Hold up of dispersed phase ( $\phi$ ) and Interfacial area ( $a$ ):

$$a = \frac{6\phi}{d_{32}} \quad (2)$$

Consequently, it is possible to determine interfacial area, a crucial parameter in calculating mass transfer rate, by use of Sauter diameter.

## NOMENCLATURE

- a : Interfacial area, m<sup>2</sup>
- A : Total amplitude of pulsation, m
- b : Deceleration, m/s<sup>2</sup>
- C<sub>D</sub> : Drag coefficient
- d<sub>N</sub> : Diameter of tray hole, m
- d<sub>p</sub> : Drop diameter, m
- d<sub>32</sub> : Sauter diameter
- e : Free area of tray
- f : Frequency, 1/s
- F<sub>A</sub> : Buoyancy force, N
- F<sub>w</sub> : Inertial force, N
- F<sub>D</sub> : Drag force, N
- F<sub>t</sub> : Interfacial tension force, N
- g : Gravitational acceleration, m/s<sup>2</sup>
- t : Hole Pitch, m
- V : Velocity of drop, m/s
- V<sub>p</sub> : Velocity of pulsation, m/s
- V<sub>t</sub> : Terminal Velocity, m/s
- ΔS : Half the distance of two holes, m
- ρ<sub>C</sub> : Density of continuous Phase, kg/m<sup>3</sup>
- ρ<sub>D</sub> : Density of dispersed Phase, kg/m<sup>3</sup>
- Δρ : Density difference between dispersed and continuous phases, kg/m<sup>3</sup>
- γ : Interfacial Tension, N/m

## A model to calculate sauter diameter

If we assume that there is only one droplet in extraction column and the shape of it is spherical, the following forces are exerted on ascending drop

when it flows from beneath to above of tray.

$$1 - \text{Buoyancy Force} : F_A = \Delta\rho \cdot g \cdot \frac{\pi}{6} \cdot d_p^3 \quad (3)$$

$$2 - \text{Inertial Force} : F_w = \rho_D \cdot b \cdot \frac{\pi}{6} \cdot d_p^3 \quad (4)$$

$$3 - \text{Drag Force} : F_D = C_D \cdot \frac{\rho_C}{2} \cdot V^2 \cdot \frac{\pi}{4} \cdot d_p^2 \quad (5)$$

$$4 - \text{Interfacial tension force} : F_t = \pi \cdot \gamma \cdot d_p \quad (6)$$

The first three forces tend to destroy the drop; the last one stabilizes the drop. If there is a balance of forces, a stable drop exists. From eqs. (3) to (6) one gets:

$$d_p = \sqrt{\frac{6\gamma}{\Delta\rho \cdot g + b \cdot \rho_D} + \frac{9}{64} \cdot \left(\frac{C_D \cdot V^2 \cdot \rho_C}{\Delta\rho \cdot g + b \cdot \rho_D}\right)^2} - \frac{3}{8} \cdot \frac{C_w \cdot V^2 \cdot \rho_C}{\Delta\rho \cdot g + b \cdot \rho_D} \quad (7)$$

The calculation of the drop size by eq. (7) requires the knowledge of the velocity of a drop V, the deceleration b and the drag coefficient C<sub>D</sub>. These parameters will be calculated as follows:

## Calculation of the drop velocity

The velocity of drops in a pulsed sieve-plate column consists of two parts:

a- Velocity which is consequence of the pulsation (V<sub>p</sub>).

b- Velocity which is consequence of terminal velocity (V<sub>t</sub>).

To evaluate pulsation velocity, Pilhofer equation<sup>[3]</sup> has been used as follows:

$$V_p = \frac{2A \cdot f}{0.6e} \quad (8)$$

In the above equation, 2Af represents the mean velocity of the continuous phase resulted from the total amplitude A and the frequency f of the pulsation. The velocity increases in the holes of the plate because of the fractional free area e and the jet contradiction. By the factor 0.6, the jet contradiction is considered.

The terminal velocity of drop (V<sub>t</sub>) can be evaluated by the Hu-Kintner equation<sup>[4]</sup>. Therefore, velocity of drop is calculated as follows:

$$V = V_p + V_t = \frac{2A \cdot f}{0.6e} + V_t \quad (9)$$

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### Calculation of deceleration

To calculate deceleration, Pilhofer<sup>[3]</sup> idealized the region in the vicinity of a tray in the following manner (see Figure 1):

The continuous phase forms jets when flowing through the holes. Two vortices are formed between two neighbor jets. The velocity in their center is zero named dead zone. The deceleration is exerted on the drop when it passes from the center of jet with the maximum velocity, and the dead zone.

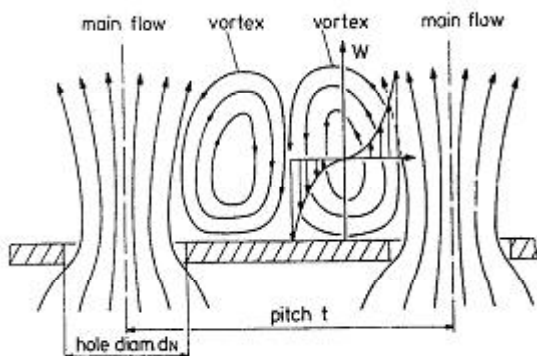


Figure 1: Model of flow behind a sieve-plate

Hence, the deceleration,  $b$ , holds:

$$b = \frac{V^2}{2 \cdot \Delta S} \quad (10)$$

The distance ( $\Delta S$ ) is half the distance of two holes for which from geometry of the tray holds:

$$t = \sqrt{\frac{0.9065 \cdot d_N^2}{e}} \quad (11)$$

Therefore, the deceleration  $b$  is calculated as follows:

$$b = \frac{V^2}{\sqrt{\frac{0.9065 \cdot d_N^2}{e}}} \quad (12)$$

### Drag coefficient

Because the drop is spherical, we can calculate drag coefficient as follows<sup>[5]</sup>:

$$C_D = \frac{4g \cdot \Delta\rho \cdot d_p}{3 \cdot \rho_C \cdot U_t^2} \quad (13)$$

### Developing an algorithm to calculate $d_{32}$

If we consider drops in the column as uniform

shapes, coalescence and dispersion could be neglected. Therefore, the drop diameter calculated by the described model, represents Sauter or average diameter in the extraction column.

Hence, to calculate Sauter diameter we can use the following procedure:

1. We assume a value for the Sauter diameter.
2. Terminal velocity is calculated by the Hu-Kintner.
3. Calculate Drag coefficient by equation (13).
4. Calculate pulsation velocity by equation (8).
5. Calculate drop velocity by equation (9).
6. Calculate deceleration by equation (12).
7. Estimate a new drop size by equation (7).
8. If the difference between assumed and estimated drop diameter is more than 0.01mm, return to step 2 and do the procedure by estimated diameter.

Based upon a column specification presented in TABLE 1, for Toluene-Acetone-Water and Butanol-Succinic acid-Water systems<sup>[6]</sup> which their physical properties are presented in TABLE 2 and TABLE 3, results are tabulated in TABLE 4 and TABLE 5. In figure 2 to 7. variation of Sauter diameter with Pulse intensity which are estimated by model and experimental data, are figured.

TABLE 1: Characteristics of pulsed sieve-plate column (PSE)

| Characteristic       | Value |
|----------------------|-------|
| Column Diameter (cm) | 22    |
| Free area            | 0.169 |
| Number of Plates     | 26    |
| Plate Spacing (cm)   | 6     |

TABLE 2: Physical properties of toluene-acetone-water system

| Physical Property                                | Value |
|--|-------|
| Surface Tension (N/m)                            | 0.032 |
| Density of Continuous Phase (kg/m <sup>3</sup> ) | 1000  |
| Density of Dispersed Phase (kg/m <sup>3</sup> )  | 860   |

TABLE 3: Physical properties of butanol- succinic- water system

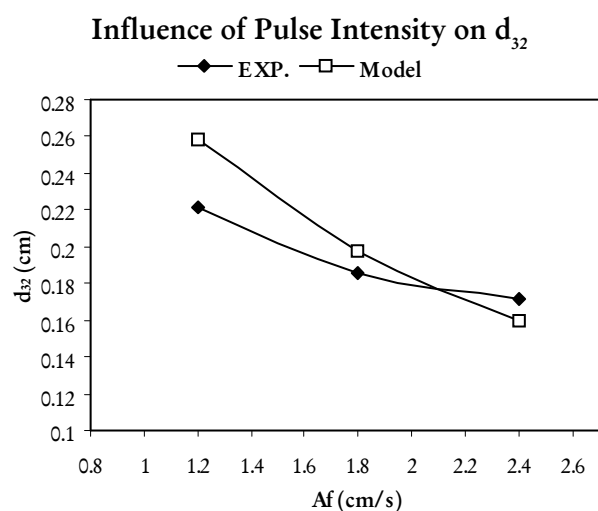
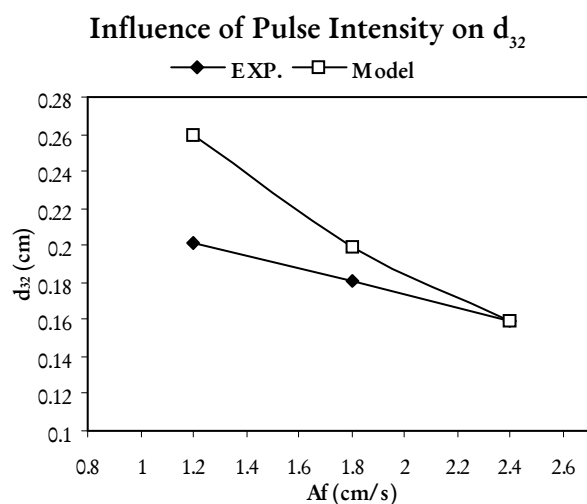
| Physical Property                                | Value  |
|--|--------|
| Surface Tension (N/m)                            | 0.0014 |
| Density of Continuous Phase (kg/m <sup>3</sup> ) | 993.5  |
| Density of Dispersed Phase (kg/m <sup>3</sup> )  | 835    |

TABLE 4: Results from model in comparison with experimental data for toluene-acetone-water system

| Temp (°C) | $Q_C$ (m <sup>3</sup> /s) | $Q_D$ (m <sup>3</sup> /s) | $A_f$ (cm/s) | $d_{32}$ (cm)<br>Exp. | $d_{32}$ (cm)<br>Model | Error% |
|-----------|---------------------------|---------------------------|--------------|-----------------------|------------------------|--------|
| 20.7      | 20.83                     | 31.47                     | 1.2          | 0.221                 | 0.2579                 | 16.7   |
| 24.5      | 20.83                     | 31.47                     | 1.8          | 0.186                 | 0.1977                 | 6.3    |
| 21        | 20.83                     | 31.47                     | 2.4          | 0.172                 | 0.1592                 | -7.4   |
| 25        | 34                        | 52                        | 1.2          | 0.201                 | 0.2597                 | 29.2   |
| 29.5      | 34                        | 52                        | 1.8          | 0.181                 | 0.1990                 | 9.9    |
| 19        | 34                        | 52                        | 2.4          | 0.159                 | 0.1588                 | -0.1   |
| 21.8      | 52                        | 78                        | 1.2          | 0.205                 | 0.2583                 | 26     |
| 27        | 52                        | 78                        | 1.8          | 0.181                 | 0.1984                 | 9.6    |
| 20.7      | 52                        | 78                        | 2.4          | 0.153                 | 0.1591                 | 4      |

TABLE 5: Results from model in comparison with experimental data for butanol- succinic -water system

| Temp (°C) | $Q_C$ (m <sup>3</sup> /s) | $Q_D$ (m <sup>3</sup> /s) | $A_f$ (cm/s) | $d_{32}$ (cm)<br>Exp. | $d_{32}$ (cm)<br>Model | Error% |
|-----------|---------------------------|---------------------------|--------------|-----------------------|------------------------|--------|
| 24.2      | 20.82                     | 21.12                     | 0.3          | 0.116                 | 0.1355                 | 16.81  |
| 27        | 20.82                     | 21.12                     | 0.6          | 0.097                 | 0.1053                 | 8.56   |
| 27        | 20.82                     | 21.12                     | 0.9          | 0.074                 | 0.0837                 | 13.11  |
| 26        | 20.82                     | 21.12                     | 1.2          | 0.064                 | 0.0686                 | 6.25   |
| 21.5      | 27.2                      | 27.06                     | 0.3          | 0.116                 | 0.1339                 | 17.76  |
| 24        | 27.2                      | 27.06                     | 0.6          | 0.09                  | 0.1039                 | 14.22  |
| 26        | 27.2                      | 27.06                     | 0.9          | 0.068                 | 0.0833                 | 20.88  |
| 21.8      | 27.2                      | 27.06                     | 1.2          | 0.057                 | 0.0674                 | 19.3   |
| 24.5      | 36.62                     | 36.46                     | 0.3          | 0.113                 | 0.1357                 | 20.18  |
| 27        | 36.62                     | 36.46                     | 0.6          | 0.083                 | 0.1053                 | 26.14  |
| 24.8      | 36.62                     | 36.46                     | 0.9          | 0.07                  | 0.0829                 | 19.43  |
| 22.7      | 36.62                     | 36.46                     | 1.2          | 0.059                 | 0.0677                 | 14.75  |

Figure 2: Influence of pulse intensity on Sauter diameter in toluene-acetone-water system ( $Q_D = 31.47$  m<sup>3</sup>/s,  $Q_C = 20.83$  m<sup>3</sup>/s)Figure 3: Influence of pulse intensity on Sauter diameter in toluene-acetone-water system ( $Q_D = 52$  m<sup>3</sup>/s,  $Q_C = 34$  m<sup>3</sup>/s)

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### Influence of Pulse Intensity on $d_{32}$

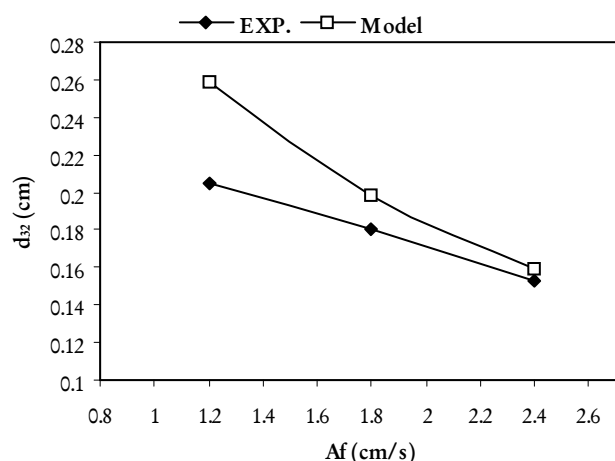


Figure 4: Influence of pulse intensity on Sauter diameter in toluene-acetone-water system ( $Q_D=78 \text{ m}^3/\text{s}$ ,  $Q_C=52 \text{ m}^3/\text{s}$ )

### Influence of Pulse Intensity on $d_{32}$

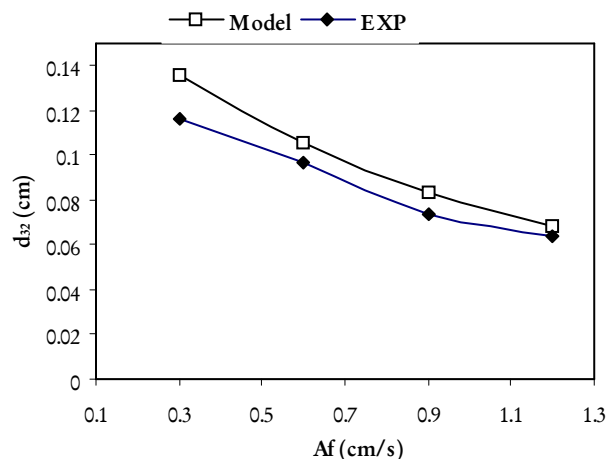


Figure 5: Influence of pulse intensity on Sauter diameter in butanol-succinic-water system ( $Q_D=21.12 \text{ m}^3/\text{s}$ ,  $Q_C=20.82 \text{ m}^3/\text{s}$ )

### Influence of Pulse Intensity on $d_{32}$

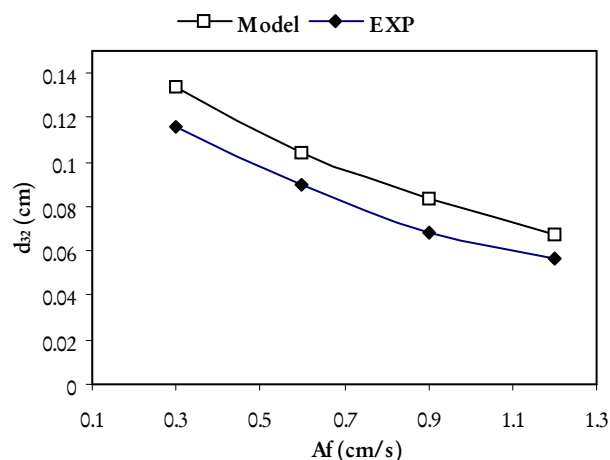


Figure 6: Influence of pulse intensity on Sauter diameter in butanol-succinic-water system ( $Q_D=27.06 \text{ m}^3/\text{s}$ ,  $Q_C=27.2 \text{ m}^3/\text{s}$ )

### Influence of Pulse Intensity on $d_{32}$

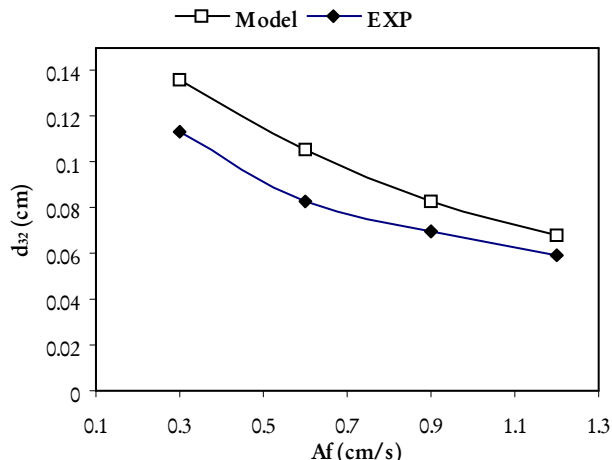


Figure 7: Influence of pulse intensity on Sauter diameter in butanol-succinic-water system ( $Q_D=36.46 \text{ m}^3/\text{s}$ ,  $Q_C=36.62 \text{ m}^3/\text{s}$ )

## RESULTS AND DISCUSSION

The results show that the model can predict Sauter or average diameter of drops with an accuracy of  $\pm 20\%$ .

It can be construed from figure 2 to 7, the Sauter diameter predicted by model, is more than the experimental data. It is because of neglecting the effect of shear stresses which are exerted on drops by dispersed and continuous phases.

Moreover, the model has estimated Sauter di-

ameter for Toluene-Acetone-Water more precisely than Butanol-Succinic acid-Water. As it mentioned before, if dispersion and coalescence of drops could be discounted, the predicted diameter by model is close to real. For Toluene-Acetone-Water, surface tension is higher, so that dispersion and coalescence effects are lower. Therefore, the accuracy of model is more precise in comparison with the other system.

**REFERENCES**

- [1] A.Aravamudan, M.Baird; *Ind.Eng.Chem.Res.*, **38**, 1596-1604 (1999).
- [2] J.C.Godfrey, M.J.Slater; 'Liquid-Liquid Extraction Equipment', John Wiley and Sons, New York, (1994).
- [3] P.H.Pilhofer, W.Pietzch; *Chemical Engineering Science*, **39**, 961-965 (1994).
- [4] S.Hu, R.C.Kintner; *A.I.Ch.E.J.*, (1955).
- [5] W.L.McCabe, J.C.Smith, P.Harriott; 'Unit Operations of Chemical Engineering', 4<sup>th</sup> Ed., McGraw-Hill, International Series.
- [6] Q.M.Shiekh; Ph.D. Thesis, 'Evaluation of Forward and Axial mixing Parameters for a 22cm Diameter Pulsed Sieve-Plate Column', Victoria University of Manchester, UMIST, (1991).
- [7] Y.Qian; 'Interface Mass Transfer Process in Pulsed Sieve-Plate Extraction Column', Ph.D.Thesis, Tsinghua University, Beijing, R.P.China, (1987).
- [8] A.E.Handlos, T.Baron; *A.I.Ch.E.J.*, **3**, 127-136 (1957).
- [9] R.Yarpalvi, P.K.Das, R.Kumar; *Chemical Science*, **10**, 2547-2553, (1986).
- [10] M.Otha, M.Suzuki, *Chem.Eng.Sci.*, **50(18)**, 2923-2931 (1995).
- [11] R.E.Treybal; 'Liquid-Liquid Extraction', 2<sup>nd</sup> Ed., McGraw-Hill, New York, (1963).