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A Generalization of Quantum Theory

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Abstract

We generalize matter wave theory by replacing Planck's constant with a general parameter. Then we solve for the existence of standing wave solutions in the hydrogen atomic system to recover Planck's constant. The method can be applied to other physical systems to find their corresponding constants. For solar system, we use the corresponding constant to substitute Planck's constant in Schrodinger equation and solve it to obtain the radius to the sun of the planets, which are consistent with empirical data. Quantum theory can be generalized to different physical systems by substituting Planck's constant with with their characteristic constants.

Keywords: Planck's constant, matter wave, generalized quantum theory

1. Introduction

Quantum theory started with Planck's constant. Subsequent developments, such as Bohr's atomic model, de Broglie's matter wave theory, matrix mechanics and Schrodinger equation, all hinge on Planck's constant. Is Planck's constant indispensable for any quantum theory?

In the process of solving Schrodinger equation, we may realize that the existence of quantum phenomena does not depend on the size of Planck's constant. Rather, it is due to the requirement of periodicity and finiteness of the solutions of the wave functions. In other words, quantum phenomena are scale independent.

Mathematical methods of quantum theory depend heavily on Fourier analysis, which has broad applications in both micro and macro worlds. Uncertainty principle, a classical result originated from quantum mechanics, is valid for systems of all scales [1]. Mathematical methods originated from quantum mechanics, such as path integral, have been successfully applied

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to many problems in engineering, finance, economics and life sciences, problems in very different scales [2-4]. These results suggest that quantum theory may be extended to scales beyond subatomic world.

We can also explore the scale issue from generalizing matter wave theory. De Broglie assumed energy and momentum of a particle to be represented by hf and h/λ, where h is Planck's constant, f is the frequency and λ is the wavelength of the matter wave. We may assume energy and momentum of a particle to be represented by xf and x/λ, where x is a general parameter whose value to be determined by the characteristics of specific physical systems. In a hydrogen atomic system, when the wavelength of the matter wave is equal to the circumference of the orbit of an electron at its ground state, the parameter of x takes the value of h, Planck's constant. In other words, Planck's constant corresponds to the physical systems of electrons and the associated electromagnetic waves. When we apply the standard matter wave theory (E= hf, P = h/λ) to large systems, the magnitude of quantum effects declines to insignificance.

However, we can apply generalized matter wave theory to find different constants for different physical systems. Do quantum phenomena exist in other physical systems, such as gravitational systems? Solar system is a gravitational system. Recently, Cao discovered that the orbits of the solar planets exhibit quantum properties [5]. For example, the ratios of the square roots of the radius of Mercury, Venus, Earth and Mars to the sun are close to 3:4:5:6. This parallels to what was found in an atomic system for electrons orbiting at different energy levels. Then he proposed a group of quasi Planck constants, with scaling factors of no physical meaning, to calculate planetary orbital movements. Finally, he scaled the solutions of Schrodinger equation to obtain the energy levels and radius to the sun of different planets.

From generalized matter wave theory, we can obtain a new physical constant for the solar system, parallel to Planck's constant for the atomic system. We use this constant to replace Planck's constant in Schrodinger equation. Then we solve this new equation to obtain energy levels and radius to the sun of different planets. In this way, we obtain a new physical constant with clear physical meaning. We also solve the new equation with the new physical constant related to the solar system directly, without depending on a scaling factor with no physical meaning. This generalizes quantum theory with less ad hoc manners. The fundamental theories about quantum phenomena, such as matter wave theory and Schrodinger equation, are extended naturally to physical systems of different scales and under the influence of different forces.

In the following, we will go over detailed derivations and provide more detailed discussion.

A generalization of matter wave theory and Planck's constant

From de Broglie's theory of matter wave, the energy and momentum of a particle can be represented as,

$$E = hf \qquad (1)$$

$$P = \frac{h}{\lambda} \qquad (2)$$

where h is Planck's constant, f is the frequency and λ is the wavelength of the matter wave. Instead of using h, Planck's constant, we will use x, an unknown parameter to represent matter wave relations.

$$E = xf \qquad (3)$$

$$P = \frac{x}{\lambda} \qquad (4)$$

For different physical systems, x may take different values that correspond to the specific characteristics of the systems. We will derive the value of x for an electron circling the nucleus of a hydrogen atom,. From the matter wave theory, what we observe are mostly stationary states of standing waves. For a standing wave to occur, an integral multiple of the wave length will equal the circumference of the circle, or

$$n\lambda = 2\pi r \qquad (5)$$

where n is an integer, r is the radius of the circle. From (4)

$$\lambda = \frac{x}{P} = \frac{x}{mv} \qquad (6)$$

From (5)

$$n\lambda = n\frac{x}{mv} = 2\pi r \qquad (7)$$

From (7), we obtain

$$x = \frac{2\pi rmv}{n} \qquad (8)$$

We now try to determine v, the velocity of the electron on this orbit. From Coulomb's law and Newton's second law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = ma = m\frac{v^2}{r} \qquad (9)$$

where m is the mass of an electron, ε_0 is permittivity of vacuum, e is the electron charge. So

$$v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr}$$

or

$$v = e\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{1}{mr}} \qquad (10)$$

Substituting (10) into (8), we have

$$x = \frac{2\pi rm}{n} e\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{1}{mr}} = \frac{e}{n}\sqrt{\frac{\pi mr}{\varepsilon_0}} \qquad (11)$$

Let n =1, the smallest possible positive integer. This corresponds to the ground state of the hydrogen electron. Equation (11) becomes

$$x = e\sqrt{\frac{\pi m r_1}{\varepsilon_0}} \qquad (12)$$

where r_1 is Bohr's radius. With these input, the value of x is 6.626*10^(-34). This is Planck's constant. So Planck's constant corresponds to the situation when the wavelength of electron matter wave equals to the length of the circle of the ground state of hydrogen nucleus.

From (11) and (12), we can determine the radius of nth orbits.

$$r_n = r = \frac{\varepsilon_0}{\pi m}\left(\frac{xn}{e}\right)^2 = \frac{\varepsilon_0}{\pi m}\left(\frac{x}{e}\right)^2 n^2 = r_1 n^2 \qquad (13)$$

where r_1 and r_n represents the radius of the first orbit and the nth orbit.

Historically, Bohr's atomic model was inspired by solar planetary system. We will look for a constant corresponding to Planck's constant in the solar system. There are two main differences between the atomic systems and the solar system. First, the main force in the solar system is gravitational force while the main force in atomic systems is electric force. Second, the scales of the solar system and the atomic systems are very different. We will go over the same process as in de Broglie's matter wave theory, this time for the solar system. From formula (1) to (8), the derivation is identical. The force in formula (9) is electric force. We will replace it with gravitational force and continue with the last several steps of derivations.

From gravitational law and Newton's second law,

$$F = \frac{GMm}{r^2} = ma = m\frac{v^2}{r} \qquad (14)$$

So

$$v^2 = \frac{GM}{r}$$

or

$$v = \sqrt{\frac{GM}{r}} \qquad (15)$$

Substituting (15) into (8), we have

$$x = \frac{2\pi rm}{n}\sqrt{\frac{GM}{r}} = \frac{2\pi m}{n}\sqrt{GMr} \qquad (16)$$

Let n =1, the smallest possible positive integer. Equation (16) becomes

$$x = 2\pi m\sqrt{GMr} \qquad (17)$$

Of these parameters, *m* is the mass of a planet, *r* is the radius from the planet to the sun, *G* is the gravitational constant, *M* is the mass of the sun. As the mass of each planet varies, a more general result would be obtained if we divide both side with m. This leads to a new variable *y*

$$y = x/m = 2\pi\sqrt{GMr} \qquad (18)$$

What value should we use for *r*? We might be tempted to use the distance from the sun to Mercury, the innermost planet to the sun, as the value of *r*. However, Cao found the ratios of the square roots of distances of Mercury, Venus, Earth, Mars to the sun are close to 3,4,5,6 [5]. From (13), this indicates that Mercury, Venus, Earth, Mars, are the third, fourth, fifth and sixth planet orbiting around the sun. For now we set aside the obvious question where the first and second planets are. They may never evolve in the first place, or they have been absolved by the sun in a later period. We can use the radius of Mercury, Venus, Earth, Mars to the sun to extrapolate the radius of the hypothetical first planet to the sun.

The radius of Mercury, Venus, Earth and Mars to the sun are

Mercury	5.790E+10
Venus	1.082E+11
Earth	1.496E+11
Mars	2.279E+11

All units are in meters.

A simple method to estimate the radius of the hypothetical first planet to the sun is to use the average of 1/3^2 of Mercury's radius, 1/4^2 of Venus's radius and so on.

$$r_1 = \frac{1}{4}\left(\frac{5.790 \times 10^{10}}{3^2} + \frac{1.082 \times 10^{11}}{4^2} + \frac{1.496 \times 10^{11}}{5^2} + \frac{2.279 \times 10^{11}}{6^2}\right) = 6.378 \times 10^9 \qquad (19)$$

G is 6.673*10^(-11), *M*, the mass of the sun, is 1.989*10^30. Together, we can calculate

$$y = \frac{x}{m} = 2\pi\sqrt{GMr_1} = 5.781 \times 10^{15} \qquad (20)$$

for the solar system. For hydrogen atom, the corresponding number would be

$$y = \frac{h}{m_e} = 7.274 \times 10^{-4} \qquad (21)$$

Formula (17) provides a generalization of Planck's constant to the solar system. With this new constant, the energy and momentum of a planet can be represented by by (3) and (4). It is a generalization of the de Broglie matter wave theory.

A generalization of Schrodinger equation

The most fundamental equation in quantum mechanics is Schrodinger equation. It takes the form of

$$i\hbar \frac{\partial}{\partial t}\Psi(r,t) = \left\{-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right\}\Psi(r,t) \qquad (22)$$

Planck's constant is used in Schrodinger equation. It is used to solve problems in subatomic world. Can we solve problems from a similar equation with other constants? We will propose a similar equation generalized from Schrodinger equation. The constant, h-bar, in Schrodinger equation will be replaced by a general constant. The specific value of this constant will be determined by specific physical systems. When applying to the solar system, we will replace Planck's constant with the constant we obtained from the last section. We will see if the new equation can solve the orbital problems in the solar system.

From (20),

$$x = 2\pi m\sqrt{GMr_1} \qquad (23)$$

where m is the mass of a planet. The corresponding x-bar is

$$\bar{x} = m\sqrt{GMr_1} \qquad (24)$$

The generalization of Schrodinger equation will be

$$i\hbar\frac{\partial}{\partial t}\Psi(r,t)=\left\{-\frac{\hbar^2}{2m}\nabla^2+V(r)\right\}\Psi(r,t) \qquad (25)$$

The time independent version will be

$$\left\{-\frac{\hbar^2}{2m}\nabla^2+V(r)\right\}\Psi(r)=E\Psi(r) \qquad (26)$$

where E is the energy level of a planet in the solar system. We will find radial solutions of solar planetary trajectories and the energy levels of each planet. In the solar system, the potential energy from the gravitational force is

$$V(r)=-\frac{GMm}{r} \qquad (27)$$

We will follow the standard procedure, such as in Section 11.3.1 of [*6*], to find the energy levels of the nth radial solutions. They are

$$E_n=-\frac{m}{2}\frac{(GMm)^2}{\hbar^2}\frac{1}{n^2} \qquad (28)$$

Substituting the value from (24) into (28), we get

$$E_n=-\frac{m}{2}\frac{(GMm)^2}{m^2(GMr_1)}\frac{1}{n^2}=-\frac{m}{2}\frac{GM}{r_1n^2} \qquad (29)$$

Here M is the mass of the sun, m is the mass of a particular planet rotating around the sun. This is the same as the total energy of a planet when we calculate from classical mechanics. From (29)

$$r_n=r_1n^2 \qquad (30)$$

Cao obtained the same results of (29), (30) by substituting corresponding terms in the solution of Schrodinger equation for the hydrogen atom with parameters for the solar system (*5*).

With formula (30), we can calculate the distance of planets to the sun. r_1 takes the value from (19) and n =3, 4, 5, and 6 represent Mercury, Venus, Earth and Mars respectively. The following **Table 1** listed the calculated and measured values of distances from a planet to the sun.

TABLE 1. **The calculated and measured values of distances from a planet to the sun.**

Radius to the sun of a planet	**Calculated value**	**Measured value**
Mercury	5.740E+10	5.790E+10
Venus	1.020E+11	1.082E+11
Earth	1.594E+11	1.496E+11
Mars	2.296E+11	2.279E+11

More detailed discussion about solar system and beyond can be found from (5).

Conclusion

We generalize quantum theory in several ways. First, we generalize matter wave theory. It provides a method to find constants parallel to Planck's constant in different physical systems. Then we use these constants to substitute Planck's constant in Schrodinger equation. These new equations correspond to quantum phenomena in different systems. In the case of solar system, we solve the new equation to obtain energy levels and radius to the sun of the planets, which are consistent with empirical data. A generalization of quantum theory to systems of different scales will have deep implications to our understanding of the world.

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