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A Boubaker-Turki polynomials solution to pancreatic islet blood flow biophysical equations in the case of a preset monitored spatial rotating field

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ABSTRACT

In this paper, an analytic solution to the flow equations inside blood vessels is proposed in a particular case. The solution is based on bringing together the initial set of Bloch equations and a recently established homogenous equation for the Boubaker-Turki polynomials, which was proposed as a support to canonical applied physics investigations. The main restriction, besides experimental constraints, was the imposition of a spatial uniform rotating field rF. The representative function of the magnetization response is compared to an experimental set of points. The mean error was less then 18%.

KEYWORDS

Integrodifferential equations ;
Mathematical methods;
Magnetic resonance (radiation measurement);
Flow through biological.

INTRODUCTION

Historically, the first use of the nuclear magnetic resonance NMR was limited to physics. Later, its resolution and sensitivity enhancement made it a useful method for biologists. The following introduction of mathematical transforms tools allowed the investigations of larger, more complex domains, as studies inside organic tissues. In the last decades, the NMR, as a spectroscopic method, has seen spectacular growth over both as a technique and in its simulations. Actually, the NMR applications span a wide range of scientific disciplines, from biophysics to medicine to physics[1-3]. The major part of the actually published results is numerical and not analytical, like the works of C. Callicott et al.[4] on blood physical properties alterations, and the investigations of X. Song et al. and N. Hashiya et al.[5-6].

In several NMR simulation works[7-11], the calculations are founded on the integration of the Bloch equations inside a particular geometrical model. They consist of derivation of a nonlinear first-order partial differential equation system.

As it has been investigated[12-14] that any obstacle to the normal flow of blood causes a malfunctioning in the body system that leads to cardiovascular related diseases, many studies aimed to implement the NMR as an mean to blood flow inside arteries and veins[13,14].

This paper is based on the idea of recasting the macroscopic fluid flow problem in the form of a second order differential equation[13,15].The solution is deduced thanks to noticed similarities with a recently established homogenous equation[16-20] for the Boubaker polynomials[16-20].

METHODS AND MATERIAL

The bloch NMR flow equations

The real physical model of the blood vessel is difficult to design since the constitution of the vessel envelop is heterogeneous and generally anisotropic. The movement of blood particles[12,14] is also hard to simu-

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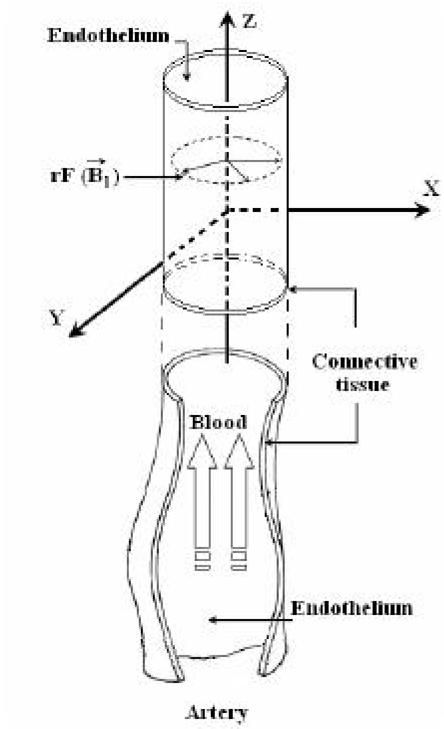

Figure 1(a) : A simplified model of the pancreatic islet blood artery

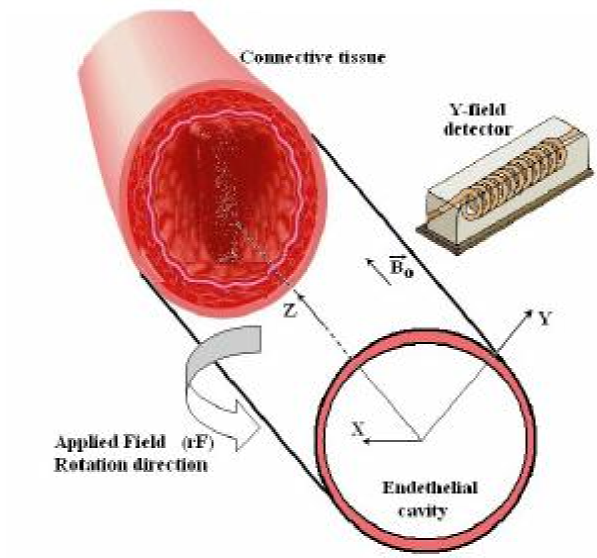

Figure 1(b) : NMR principle scheme

late because of its dependence on several parameters including, viscosity, temperature, pH etc.

We established a physical simplified model of a blood artery on a defined short length alongside with a prospecting device (figure 1a).

In the system described by the Bloch NMR equations, a particle, spins with angular speed ω, in a rotating coordinate system. When the rF $\vec{H}_1$ field is applied on a microscopic volume of mass m of the red cell, at equilibrium, the total forces on m must be zero[15]. The forces are the contact force, Coriolis force and the centrifugal force. The coriolis and centrifugal forces seem quite real in a rotating frame.

It is a commonly known fact that, among the various atomic nuclei, many possess a magnetic moment μ expressed by (eq.1) :

$$\mu = \hbar I\gamma \tag{1}$$

where $\hbar I$ is the intrinsic angular momentum or spin, and γ is the gyromagnetic ratio.

The Larmor theorem states that the motion of a magnetic moment in a magnetic field $\vec{H}_0$ is a precession around that field. The precession frequency, also called Larmor frequency , is given by (eq. 2):

$$f_0 = \frac{\gamma H_0}{2\pi} \tag{2}$$

In our model (figure 1.b), the external static field $\vec{H}_0$ is applied along the z-axis and the field detector stands along with the y-axis. After the sample has reached its equilibrium, the system shows a magnetisation vector $\vec{M}$ along the z-axis. In this state, no NMR signal is observed, since there is no transverse rotating magnetization. By applying an additional pulsed rotating magnetic field $\vec{H}_0$ in the horizontal plane (figure 1a), the orientation of $\vec{M}$ can be shifted into this plane as the precession of is always around the total magnetic field. To investigate the variations of magnetisation vector $\vec{M}$ in the presence of the field $\vec{H}_1$, it is convenient to use a rotating coordinate system instead of a static one. The rotating coordinate system is chosen to rotate at the same frequency than $\vec{H}_1$, the manner that both $\vec{H}_1$ and $\vec{H}_0$ become time-independent. The Bloch equations[1-14] in this coordinate system are expressed by the system (3).

$$\begin{cases} \frac{dM_x}{dt} = V.gradM_x + \frac{\partial M_x}{\partial t} = -\frac{M_x}{T_2} \\ \frac{dM_y}{dt} = V.gradM_y + \frac{\partial M_y}{\partial t} = \gamma M_z H_1(x) - \frac{M_y}{T_2} \\ \frac{dM_z}{dt} = V.gradM_z + \frac{\partial M_z}{\partial t} = -\gamma M_y H_1(x) - \frac{M_0 - M_z}{T_1} \end{cases} \tag{3}$$

where γ is the gyromagnetic ratio of fluid spins, $\omega/2\pi$ is the rF excitation frequency, f_0/γ is the off-resonance field in the rotating frame of reference, M_o is the equilibrium magnetization, T_1 and T_2 are respectively the spin-lattice and the spin-spin relaxation times and finally V is the flow velocity .

Bloch NMR simplified equation

In order to calculate the transverse magnetisation component $\overrightarrow{M_y}$, two conditions[14,15] were chosen:

1. $M_o \neq M_z$ a situation which holds good in general and in particular when the rF $B_1(x)$ field is enough strong.
2. before entering signal detector coil, fluid particles has magnetization M_x =0 and M_y = 0

Under these conditions and for steady flow we have the consideration: $\frac{\partial M_y}{\partial t} = 0$

We can write, from the system (3) :

$$V^2T_1\frac{d^2M_y}{dx^2} + VT_1\left(\frac{1}{T_1}+\frac{1}{T_2}\right)\frac{dM_y}{dx} + T_1\left(\gamma^2H_1^2(x)+\frac{1}{T_1T_2}\right)M_y = M_o\gamma H_1(x) \tag{4}$$

Equation (4) is the main equation to be solved.

The Boubaker-Turki polynomials

1. Historical summary

The Boubake-Turki polynomials[16-20] are an enhanced form of the earlier defined Boubaker polynomials[16-20] which emerged from an attempt to yield a solution to heat equation[16]. In fact, in a calculation step during resolution process, an intermediate calculus sequence raised an interesting recursive formula leading to a class of polynomial functions that performs difference with common classes[19].

The serial of the Boubaker polynomial functions $B_m(X)$ is defined by the explicit expression (5):

$$B_n(X) = \sum_{p=0}^{\xi(n)}\left[\frac{(n-4p)}{(n-p)}C_{n-p}^{p}\right].(-1)^p.X^{n-2p} \tag{5}$$

where

$\xi(n) = \left\lfloor \frac{n}{2} \right\rfloor = \frac{2n+((-1)^n-1)}{4}$ ($\lfloor\ \rfloor$ symbol represents the floor function[19])

with the ordinary generating function (6):

$$f_B(X,t) = \frac{1+3t^2}{1+t(t-X)} \tag{6}$$

2. The characteristic homogenous differential equations

2.1 Case of the modified Boubaker polynomials

The Boubaker-Turki polynomials $\tilde{B}_n(X)$ [16-20], have the explicit expression :

$$\tilde{B}_n(X) = 2^n.X^n - 2^{n-2}(n-4).X^{n-2} + \sum_{p=2}^{\xi(n)}\left[\frac{(n-4p)}{p!}\prod_{j=p+1}^{2p-1}(n-j)\right].2^{n-2p}(-1)^p.X^{n-2p}$$

Calculations start out from an already established result (7):

$$\tilde{B}_n(X) = \frac{4X}{n}\frac{dT_n(X)}{dx} - 2T_n(X) \tag{7}$$

which gives (8):

$$\frac{d\tilde{B}_n(X)}{dX} = \frac{4X}{n}\frac{d^2T_n(X)}{dX^2} - \left(\frac{4}{n}-2\right)\frac{dT_n(X)}{dX} \tag{8}$$

and (9)

$$\frac{d^2\tilde{B}_n(X)}{dX^2} = \frac{4X}{n}\frac{d^3T_n(X)}{dX^3} - \left(\frac{8}{n}-2\right)\frac{d^2T_n(X)}{dX^2} \tag{9}$$

Using the third order differential equation (10):

$$(1-x^2)\frac{d^2T_n(X)}{dX^2} - 3X\frac{d^3T_n(X)}{dX^3} - (n^2-1)\frac{dT_n(X)}{dX} \tag{10}$$

We obtain finally the second order differential equation (11) :

$$\begin{aligned} &4X(1-X^2)y''+(-4X^2+2nX-2n+8)y'+\\ &(-4X^2n+6n-n^2-32)y = 2(-4X^2n+6n-n^2-32)T_n(X) \end{aligned} \tag{11}$$

2.2. Case of the Boubaker polynomials

According to the relations that defined the modified Boubaker polynomials[16-20], we could establish a differential equation (12), verified by the original Boubaker polynomials:

$$\begin{aligned} &4X(1-4X^2)y''+(-8X^2+2nX-n+2)y'+(-16X^2n+\\ &6n-n^2-32)y = 2(-16X^2n+6n-n^2-32)T_n(2X) \end{aligned} \tag{12}$$

Bloch equation solution

Comparison between equations (4) and (11) gives :

$$V^2T_1^2 = 4X(1-4X^2) \tag{13}$$

$$VT_1^2\left(\frac{1}{T_1}+\frac{1}{T_2}\right) = (-8X^2+2nX-n+2) \tag{14}$$

$$(-16X^2n+6n-n^2-32) = \frac{1}{T_2}\text{, under the condition}\quad \gamma^2H_1^2(X) \ll \left(\frac{1}{T_1T_2}\right) \tag{15}$$

For obtaining a true solution of equation (4), T_1 is

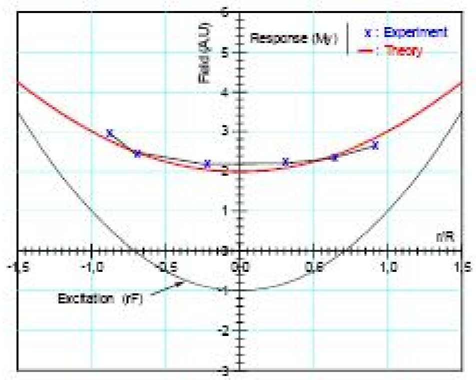

Figure 2: The experimental and theoretical spatial variations of the functional NMR transverse magnetization My(X), for a second order polynomial excitation term

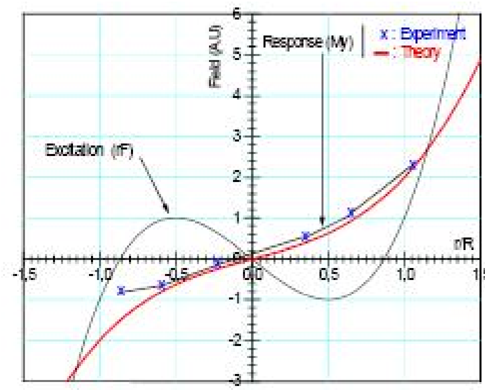

Figure 3: The experimental and theoretical spatial variations of the functional NMR transverse magnetization My(X), for a third order polynomial excitation term

set equal to unity. Typically[21], the values of T_1 and T_2 for human blood are $0.8s<T_1<1.2s$ and $0.0s<T_2<0.5s$ respectively.

In previous studies[12,15], we tried unsuccessfully to give an analytical solution to (eq. 10), by attributing some arbitrary expressions to the right term of the equation. In fact this term corresponds to the X-dependant expression to the rF field, which is an exogenous and controllable parameter. The existing setup could easily yield many standard functions such as : $B_1(X)=$ cst., $B_1(X)= aX+b$ (linear), $B_1(X)= a.\cos(\omega t+\phi)$ and $B_1(X)=aX^2+bX+c$ (Parabolic).

The last function (parabolic) was the most useful, as long as the right term of (eq. 4) is a polynomial function. The existing rF field generator has been consequently managed so that it can generate appropriated preset polynomial functions (16).

$$\left\|\vec{B}_1(X)\right\| = \frac{1}{M_o\gamma T_2}.(2X^2-1) \quad (16)$$

Consequently, a solution to the equation (4) could be yielded.

RESULTS AND DISCUSSION

By choosing a monitored spatial expression to the rF field $\vec{B}_1(X)$ which varies according to the expression (11) and which is indexed on the value of the spin-spin (transverse) relaxation time T_2, the general form of the equation (4) becomes identical to the equation (11). This feature allows deriving modal solutions to main equation (4). The authors have discussed the validity of the solutions with many specialized researchers, and made comparison with several proposed studies[10,11,22-25].

The figures 2 and 3 represent the gathered experimental and theoretical results for respectively , a second-order and a third-order polynomial rF field $\vec{B}_1(X)$ monitored according to equation (16).

I these cases, and thanks to the recently yielded proprieties, an analytical solution could be performed rather than the empirical solutions already proposed in to previous publications[12,15,22-25].

It was noticed that the measured transverse magnetization amplitude (figures 2 and 3.) follows the spatial parity of the excitation field as confirmed in precedent studies like the results of K. Sasaoka et al.[22] and J.Simbrunnera et al.[24]. Calculation of the Pearson coefficient of the second order polynomial regression in relation with the experimental spatial distribution (figure 2) yielded a mean error about 18.5% and an ordinary standard deviation close to 0.3.

CONCLUSION

In this study, we have investigated a solution to the Bloch equation in the case of blood vessel NMR device subjected to particular preset conditions. The main advantage to this study lays in giving analytical spatial expressions of the magnetic transverse response to a preset rotating field. Further investigations are oriented toward analyzing the responses to higher orders or multimodal excitation terms.

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