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TWO FLUID PLANE SYMMETRIC COSMOLOGICAL MODELS IN GENERAL THEORY OF RELATIVITY

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ABSTRACT

We have presented anisotropic, homogeneous two-fluid cosmological models using a plane symmetric metric. Here one fluid represents the matter content of the universe and another fluid is chosen to model the CMB radiation. The radiation and matter content of the universe are in interactive phase. Also we have discussed the behaviour of fluid parameters and kinematical parameters.

Key words: A plane symmetric metric·Two Fluids.

INTRODUCTION

In recent years there has been a lot of interest in cosmological models of the universe which are important in understanding the mysteries of the early stages of its evolution. In the universe, the role of inflationary models are very important to solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. Two fluid FRW models of the universe have been investigated where one fluid is the radiation field corresponding to the observed cosmic background radiation, while a perfect fluid is chosen to represent the matter content of the universe[1,2].

The discovery of 2.73 K isotropic cosmic microwave background radiation (CMBR) motivated many authors to investigate FRW model with a two fluid source[2-4]. The observations of high red shift type-Ia supernova and supernova cosmology project[5,6-10] confirmed that the universe is expanding with a positive acceleration, implying the existence of the total negative pressure of the universe. As compared to the homogeneous and isotropic FRW models, Bianchi space-times provide spatially homogeneous and isotropic models of the universe. The solutions of Einstein's field equations are obtained by Kalligas et al.,[11] Arbab[12], Beesham et al.,[13] and Kilinc[14] with varying G and Λ was investigated by Vishwakarma[15]. The Bianchi type cosmological models for perfect fluid are studied by Beesham[16], Chakraborty and Roy[17], Coley and Dunn[18] has been investigated Bianchi type VI_0 model with a two fluid source. By using Bianchi type-II space-time, Pant and Oli[19] examined two fluid cosmological models. Recently Adhav et al.,[20] have investigated Bianchi

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type-V model with two fluid source. Motivating with this work, we have presented two-fluid plane symmetric cosmological model. The physical behavior of the model has been discussed in detail.

Field equations

A plane-symmetric metric is given by -

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2\right) - B^2 dz^2 \qquad \dots(1)$$

The Einstein's field equations for a two fluid source in natural unit (gravitational units) are written as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij} \qquad \dots(2)$$

The energy momentum tensor for a two fluid source is given by -

$$T_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)} \qquad \dots(3)$$

where $T_{ij}^{(m)}$ is the energy momentum tensor for matter field and $T_{ij}^{(r)}$ is the energy momentum tensor for radiation field [18] which are given by

$$T_{ij}^{(m)} = \left(p_m + \rho_m\right) u_i^m \, u_j^m - p_m g_{ij} \qquad \dots(4)$$

$$T_{ij}^{(r)} = \frac{4}{3}\rho_r u_i^r \, u_j^r - \frac{1}{3}\rho_r g_{ij} \qquad \dots(5)$$

with

$$g^{ij} u_i^m \, u_j^m = 1, \qquad g^{ij} u_i^r \, u_j^r = 1 \qquad \dots(6)$$

The off diagonal equations of (2) together with energy conditions imply that the matter and radiation are both co-moving, we get,

$$u_i^{(m)} = (0,0,0,1), \qquad u_i^{(r)} = (0,0,0,1) \qquad \dots(7)$$

Using (1), (3), (4), (5) and (6) the field equations (2) reduces to :

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\left(-p_m - \frac{\rho_r}{3}\right) \qquad \dots(8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\left(-p_m - \frac{\rho_r}{3}\right) \qquad \dots(9)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -8\pi\left(-p_m - \frac{\rho_r}{3}\right) \qquad \dots(10)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi\left(\rho_m + \rho_r\right) \qquad \dots(11)$$

By comparing (8), (9) and (10), we get

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} \qquad \dots(13)$$

Solving (13), we get

$$A = B = at + k \ ,$$

where *a* is arbitrary constant and *k* is constant of integration.

Using above, a plane symmetric metric (1) can be written as

$$ds^2 = dt^2 - (at + k)^2\left[dx^2 + dy^2 + dz^2\right] .$$

Some physical and kinematical properties

We assume the relation between pressure and energy density of matter field through the "gamma-law" equation of state which is given by

$$p_m = (\gamma - 1)\rho_m, \qquad 1 \le \gamma \le 2$$

We get energy density of matter, energy density of radiation and total energy density as

$$\rho_m = -\frac{6a^2}{(4 - 3\gamma)(at + k)^2} \qquad \ldots(14)$$

$$\rho_r = \frac{3a^2(3\gamma - 2)}{(4 - 3\gamma)(at + k)^2} \qquad \ldots(15)$$

$$\rho = \rho_m + \rho_r$$

$$\rho = \frac{3a^2(3\gamma - 4)}{(4 - 3\gamma)(at + k)^2} \qquad \ldots(16)$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{4}(H_1 + H_2 + H_3) \ ,$$

where , $H_1 = \frac{\dot{A}}{A}$, $\quad H_2 = \frac{\dot{B}}{B}$, $\quad H_3 = \frac{\dot{C}}{C}$

are the directional HubbleParameter in the direction of *x*, *y* and *z* axes respectively

Case I: Dust model. In order to investigate the physical behaviour of the fluid parameters we consider the particular case of dust i.e. when $\gamma = 1$.

The scalar of expansion, shear scalar and declaration parameter are given by $\theta = 3H = \frac{3a}{at + k}$

$$\sigma^2 = \frac{3a^2}{2(at + k)^2} \qquad \ldots(17)$$

$$q = 0$$

The density parameters for matter and radiation are iven by

$$\Omega_m = -2$$

$\Omega_r = 1$

$\Omega_o = -1$,

where Ω_o is the total density parameter.

Case II: Radiation universe $\left(when\ \gamma = \frac{4}{3} \right)$. On substituting $\gamma = \frac{4}{3}$, we get, the scalar of expansion, shear scalar and declaration parameter as

$$\theta = 3H = \frac{3a}{at+k}$$

$$\sigma^2 = \frac{3a^2}{2(at+k)^2} \qquad \ldots(18)$$

$$q = 0$$

And the density parameters using $\gamma = \frac{4}{3}$ in (14) and (15) are given by

$\Omega_m = \infty$

$\Omega_r = \infty$

Case III: Hard universe $\left(\gamma \in \left(\frac{4}{3}, 2 \right) Let\ \gamma = \frac{5}{3} \right)$ the scalar of expansion, shear scalar and declaration parameter in Hard universe are

$$\theta = 3H = \frac{3a}{at+k}$$

$$\sigma^2 = \frac{3a^2}{2(at+k)^2} \qquad \ldots(19)$$

$$q = 0$$

For $\gamma = \frac{5}{3}$, (14) and (15) imply density parameters as

$\Omega_m = 2$

$\Omega_r = -3$

$\Omega = \Omega_m + \Omega_r = -1$

Here ρ_m is positive and total density ρ whereas ρ_r is negative.

Case IV: Zeldovich Universe $(\gamma = 2)$. In this case, we get, the scalar of expansion, shear scalar and declaration parameter as

$$\theta = 3H = \frac{3a}{at+k}$$

$$\sigma^2 = \frac{3a^2}{2(at+k)^2} \qquad \ldots(21)$$

$q = 0$

We get, the energy density of matter, energy density of radiation and total energy density as

$\Omega_m = 1$

$\Omega_r = -2$

$\Omega = -1$

Here total density ρ and ρ_r is negative and ρ_m is positive.

CONCLUSION

The sign of deceleration, parameters q indicates whether the model accelerates or not. The positive sign of q (>1) corresponds to decelerating model whereas the negative sign ($-1 < q < 0$) indicates acceleration and $q = 0$ corresponds to expansion with constant velocity. Here in all cases, we get $q = 0$. This implies that these two fluid models are expanding with constant velocity.

In all cases, we get, the ratio $\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{6} \neq 0$.

Therefore, these models do not approach isotropy for large value of t. These models come out to be rotating as well as expanding ones, the rate of expansion decrease with time, which can be thought of as realistic models.

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