



# **SIPHON AND TRAP ANALYSIS OF BUFFER OF A FLEXIBLE MANUFACTURING SYSTEM WITH THREE MACHINES AND TWO JOBS USING SIGN INCIDENCE MATRIX AND ITS DIGRAPHS**

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## **ABSTRACT**

There is an algorithm, which finds the set of places, which are both siphon and trap. In this paper we used that algorithm to find the set of places, which are both siphon and trap. There are two methods exists to convert a Petri net in to a digraph. In the first method the transitions of the Petri nets are changed into places and the places of the Petri nets are changed in to edges. In the first method we change both the places and transition and places as vertices and the arcs between them as edges. We used both the methods and found two digraphs which are not Euler/Hamiltonian digraphs. Therefore further analysis is not possible.

**Key words:** Petri nets, Marked graphs, Flexible manufacturing systems, Sign incidence matrix.

## **INTRODUCTION**

Introduction of Petri Nets are given in<sup>1,2</sup>. Generating Siphons and traps of Petri Nets using the Sign Incidence Matrix is discussed in<sup>3</sup> The analysis of marked graphs using sign Incidence matrix is given<sup>4,5</sup> we take the marked graph model of buffer of FMS given in<sup>6</sup> and analyze them using the method suggested in<sup>3-5</sup>. After that we oconvert the marked graph in to digraph by methods given in<sup>7</sup>.

This paper is organized as follows section I contains some basic definitions .section II Contains the algorithm contained in<sup>3-5</sup>. Section III contain examples of marked graph of Flexible manufacturing systems Section IV contains conclusion and references.

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## I-Basic definitions

**Definition 1.1: Petri net:** A Petri net is a 5-tuple,  $PN = \{P, T, F, W, M_0\}$ , where  $P$  is the finite set of places,  $T$  is the finite set of places,  $F$  is a set of arcs,  $w$  is a weight function,  $M_0$  is the initial marking.

**Definition 1.2: Marked graph:** A Marked graph is a Petri net in which each place as exactly one input transition and one output transition.

**Definition 1.3: Digraph:** A digraph  $D$  consists of a non-empty finite set  $V(D)$  of elements called vertices, and finite family  $A(D)$  of ordered pairs of elements of  $V(D)$  called arcs. We call  $V(D)$  the vertex set and  $A(D)$  the arc family of  $D$ .<sup>12</sup>

**Definition 1.4: Euler Digraph:** In a digraph  $G$  a closed directed walk which traverses every edge of  $G$  exactly once is called a directed Euler line. A digraph containing a directed Euler line is called directed Euler digraph<sup>12</sup>.

**Theorem 1.5:** A digraph  $G$  is an Euler digraph if and only if  $G$  is connected and is balanced i.e.  $d^-(v) = d^+(v)$  for every vertex  $v$  in  $G$ .<sup>12</sup>

**Definition 1.6:** For a Petri net  $N$  with  $n$ -transitions and  $m$ -places, the sign incidence matrix  $A = [a_{ij}]$  is an  $n \times m$  matrix whose entry is given as follows:

$a_{ij} = +$  If place  $j$  is an output place of transition  $i$ .

$a_{ij} = -$  If it is an input place of transition  $i$ .

$a_{ij} = \pm$  If it is both input and output places of transition  $i$  (i.e. transition  $i$  and place  $j$  form a self loop)

$a_{ij} = 0$  Otherwise.

**Definition 1.7:** The addition denoted by  $\oplus$  is a commutative binary operation on the set of four elements  $B = \{+, -, 0, \pm\}$  defined as follows:

$$+ \oplus - = \pm$$

$$X \oplus x = x, \quad \text{For every } x \in B$$

$$\pm \oplus x = \pm, \quad \text{For every } x \in B$$

$$0 \oplus x = x, \quad \text{For every } x \in B$$

**Definition 1.8:** A subset of places denoted as  $Z$  is both siphon and trap if  $Z^* = {}^*Z$

## II-Enumeration of siphon and trap as subsets of places of marked graphs

Here we present an algorithm given in<sup>4</sup> for marked graphs to find all subsets of places which are both siphon and trap. We define a siphon-trap matrix for marked graphs. A relation between sign incidence matrix and siphon-trap matrix for marked graphs is obtained.

We present the following theorem without given in<sup>4</sup>.

**Theorem 2.1:** A subset of  $k$ -places  $Z = \{p_1, p_2, \dots, p_k\}$  in a marked graph  $N$  is both siphon and trap if and only if the addition of  $k$ -column vectors of the sign incidence matrix of  $N$ ,  $A_1 \oplus A_2 \oplus \dots \oplus A_k$  contains either zero entry or  $\pm$  entry where  $A_j$  denote the column vector corresponding to place  $P_j$ ,  $j = 1, 2, \dots, k$ .

**Definition 2.2:** A  $+$  entry is said to be neutralized by adding a  $-$  entry to get a  $\pm$  entry.

**Algorithm 2.3: Input:** Sign incidence matrix  $A$  of order  $m \times n$ .

**Step 1:** Select  $A_j$ , the first column in the sign incidence matrix  $A$ , whose corresponding place is denoted as  $PLACE_j$ .

Set recursion level  $r$  to 1

Set  $V_{jr} = A_j$

Set  $PLACE_{jr} = PLACE_j$

**Step 2:** If  $V_{jr}$  has a  $\pm$  entry at  $i^{\text{th}}$  row then  $PLACE_j$  is a self loop with transition  $t_j$ . Go to step 5.

**Step 3:** If  $V_{jr}$  has a  $+$  entry in the  $k^{\text{th}}$  row find a column in  $A$ , which contains a  $-$  entry at the  $k^{\text{th}}$  row.

- (a) If no such column in  $A$ , exists, Go to Step 5.
- (b) If such  $A_s$ , exists add it to  $V_{jr}$  to obtain  $V_{j(r+1)} = V_{jr} \oplus A_s$  containing a  $\pm$  entry at  $k^{\text{th}}$  row. Then  $PLACE_{j(r+1)} = PLACE_{jr} \cup PLACE_s$ .
- (c) Repeat this step for all possible neutralizing columns  $A_s$ . This gives a new set of  $V_{j(r+1)}$ 's and  $PLACE_{j(r+1)}$ 's.

**Step 4:** Increment  $r$  by 1. Repeat step 3 until there are no more  $+$  entries in each  $V_{jr} = A_1 \oplus A_2 \oplus \dots \oplus A_{jr}$  or no neutralizing column can be defined.

**Step 5:** Any  $V_{jr}$  without + entries and without-entries (i.e., all the entries are either zero or  $\pm$ ) represents siphon and trap (By theorem). i.e., the places in  $PLACE_{jr}$  form both siphon and trap.

**Step 6:** Delete  $A_j$

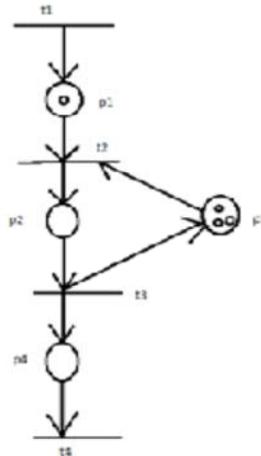
$j = j + 1$

Go to Sep 1.

**Output:** All sets which are both siphon and trap.

### III-Enumeration of siphon and trap subsets of places of marked graph of Buffer of a FMS with three machines and two jobs

In this section we take the buffer of a FMS with three machines and two jobs from [6] and analyze it using Sign Incidence Matrix



**Fig. 1: Buffer of an FMS**

The sign incidence matrix of the above marked graph is given below:

$$A = \begin{matrix} & \begin{matrix} p1 & p2 & p3 & p4 \end{matrix} \\ \begin{matrix} t1 \\ t2 \\ t3 \\ t4 \end{matrix} & \begin{bmatrix} + & 0 & 0 & 0 \\ - & + & - & 0 \\ 0 & - & + & + \\ 0 & 0 & 0 & - \end{bmatrix} \end{matrix}$$

$$V_{11} = A_1 = \begin{bmatrix} + \\ - \\ 0 \\ 0 \end{bmatrix} \text{ PLACE}_{21} = \{p_1\} \text{ } V_{11} \text{ has a + in I st Row. But there is no (-) in first}$$

Row.

$$V_{21} = A_2 = \begin{bmatrix} 0 \\ + \\ - \\ 0 \end{bmatrix} \text{ PLACE}_{21} = \{p_2\}. A_2 \text{ has a + in IIInd Row. In the given matrix there}$$

are two (-) in second Row viz first column, third column. Therefore the neutralizing column are  $A_1, A_3$ . According to the Algorithm.

$$V_{21}^{(1)} = V_{21} + A_1 = \begin{bmatrix} 0 \\ + \\ - \\ 0 \end{bmatrix} + \begin{bmatrix} + \\ - \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} + \\ \pm \\ - \\ 0 \end{bmatrix} \text{ PLACE}_{21}^{(1)} = \{p_2, p_1\}$$

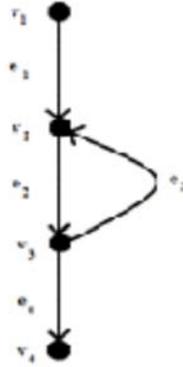
$$V_{21}^{(2)} = V_{21} + A_3 = \begin{bmatrix} 0 \\ + \\ - \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ - \\ + \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \pm \\ \pm \\ 0 \end{bmatrix} \text{ PLACE}_{21}^{(2)} = \{p_2, p_1, p_3\}$$

Since all entries in  $V_{21}^{(2)}$  are either zero or  $\pm$  we have the set of places  $\{p_2, p_1, p_3\}$  is both siphon and trap.

We continue this manner we get the following sets of places form both siphon and trap.

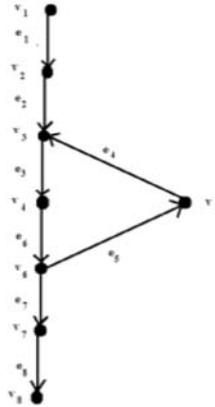
$$\text{PLACE}_{21}^{(2)} = \{p_2, p_1, p_3\}, \text{ PLACE}_{31}^{(1)} = \{p_3, p_2\}$$

Now we convert the Petri net in Fig. 1 into a digraph as per the methods given in [7]. We get the following directed graph in Fig. 2. Here transitions are replaced by vertices and places input output arcs as edges<sup>7</sup>



**Fig. 2: Digraph**

Fig. 3 is obtained by changing places and transition into vertices arcs as edges. Since the two digraphs are not Euler digraphs as in degree is not equal to out degree in each case<sup>12</sup>, We cannot proceed further according to [4] and [5].



**Fig. 3: Digraph**

## CONCLUSION

From the above analysis we conclude that the following sets of places are both siphon and trap  $(p_2, p_1, p_3), \{p_3, p_2\}$

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