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SELF-FOCUSING OF ELECTROMAGNETIC WAVES IN AN INHOMOGENEOUS PLASMAS WITH ARBITRARY LARGE MAGNITUDE OF PONDEROMOTIVE NON-LINEARITY

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ABSTRACT

In this paper, we have investigated the self-focusing behaviour of a radially symmetrical Gaussian Electromagnetic beam propagating in axially inhomogeneous plasma. Considering the non-linearity to arise from ponderomotive phenomena and following the extended version of Sodha et al theory based on the WKB approximation and paraxial ray approximation, the self-focusing behaviour has been investigated in some detail. The effects of different type of axial inhomogeneities in plasma, on the self-focusing of electromagnetic beam have been studied for arbitrary large magnitude of nonlinearity. Results indicate that the plasma behaves as an oscillatory wave–guide. The self-focusing is found to depend on type of axial inhomogeneity as well as characteristics scale length of axial inhomogeneity.

Key words: Self-focusing behaviour, Electromagnetic waves, Inhomogeneous plasmas, Pondermotive phenomena.

INTRODUCTION

The supply of inexpensive energy is one of the key problems of modern civilization. It can be accepted only as a compromise that such valuable resources as fossil oil or coal are used for power generation instead of as raw materials for organic chemistry. This presumably temporary compromise is based on comparison of the cost of the present type of nuclear fission reactor with the high cost of fossil fuel resources. Nuclear fission at present hardly reaches the level of competitivity because the basic material – uranium and thorium ore is extremely expensive. Economically available ore can supply energy only for 50 years, when the minimum annual consumption of 10 power 22 joules predicted for the year 2050 is taken into account. The problems of immense production of fissionable materials that are radioactive, the possibility of a catastrophic explosion of a fission power station, which cannot be completely ruled out, and another questions of security basically categorize the present fission reactors only as a short range energy source. The problems of the interaction of a high intensity laser radiation with plasma represent a very new, fascinating, and fundamental field of research, which could be an important key for solving the energy crisis. Whatever other results may come from this field, the technique of inertial confinement of compressed pure hot deuterium by laser radiation may be the only clean, inexpensive, and inexhaustible nuclear energy source of the future. For the short-term, perhaps within 10 years under the most advantageous conditions, laser fusion may lead to a practicable reactor if compression of plasma by lasers to densities 10,000 times the

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solid-state density can be achieved within the near future. The interaction of intense radiation beams with plasmas plays an important role in such areas as laser driven fusion, ionospheric processes and pulsar radiation. Recently, the possibility of employing intense laser beams to accelerate electrons to ultrahigh energy has also stimulated interest in radiation self-focusing in plasmas[1-12]. For example, a laser energy density of the order of 10^{16}-10^{18} w/cm^2 is required to reach fusion conditions. The self- focusing of laser beam in non-linear medium has been a subject of many theoretical, numerical as well as experimental investigations[1-12]. However, most of these studies are limited to various approximations such as a homogeneous medium, non-linear part of the dielectric constant much smaller than the linear part etc. These approximations are rather restrictive and limit the applicability of the theory to many real life situations. Results of these studies, in much case, are far off from the observed experimental observations. Transmission of an intense light beam through an inhomogeneous non-linear waveguide has a wide variety of potential applications such as an integrated optics (four-wave mixing), medical procedure (surgery, cauterization) industrial processes (cutting, welding etc) and power transmission.

There are number of mechanisms that can degrade the uniformity of a electromagnetic beam. At very high intensities, the relativistic mass variation of the electrons oscillating in the laser electric field can increase the index of refraction in the center of a beam or in a hot spot in the beam[4-5]. This leads to focusing that increases the mass variation further, causing the system to go unstable. At high intensities, the ponderomotive force of the laser can drive plasma from the interior of a beam thus raising the index of refraction there, leading to focusing and instability[6]. Finally, at lower intensities, if the beam or hot spot width is large compared with an electron mean free path λ_m, inverse bremsstrahlung heating can raise the pressure in the interior of the beam. The increased pressure drives plasma out of the beam, once again raising the index of refraction there, leading to instability[7]. The first mechanism, relativistic self-focusing is not of interest to laser fusion. With the intensities, wavelengths, and plasma scale lengths envisioned for reactor targets, little self-focusing is expected from this mechanism. Ponderomotive self-focusing could be important for small-scale hot spots. The third mechanism, thermal self-focusing, is important in the focusing of whole beams and large-scale hot spots.

Sodha and coworkers had developed a steady state paraxial theory of self-focusing of laser beam in a non-linear, non-absorbing homogeneous medium[8]. One of the important feature of their theory is that non-linearity is arbitrarily large as observed in many of the real life situations. The extended version of this theory has been used in the present study of self-focusing of electromagnetic beam in inhomogeneous plasma in the paraxial ray approximation, taking into account the ponderomotive mechanism. The ponderomotive force of a laser creates density depletion in the background plasma, which modifies the index of refraction and focuses laser light into density channel, thus overcoming the effect of light diffraction. This study is restricted to the Gaussian beam in linearly increasing and decreasing, exponentially varying inhomogeneous plasma. To compute the results, nonlinear mechanism considered in the present study is collisional heating.

Inhomogeneous plasma medium

Inhomogeneous plasma means that charge density is not uniform throughout the space where laser plasma interactions are considered. For the study of self-focusing of electromagnetic beam in plasma, some simple models for variations of charge density are devised and considered here in the present analysis. The relation can represent the inhomogeneity in charge density of the plasma at any time in space

$$N(x, y, z, t) = N_o W(x, y, z, t), \qquad \ldots(1)$$

where N_o is the density of the plasma at $x=0$, $y=0$, $z=0$ and $t=0$. Here, $W(x, y, z, t)$ is the density profile function and may have different shapes for different types of inhomogeneities. Let the electromagnetic beam, whose effect is to be studied is propagating in z-direction in plasma. In axially inhomogeneous plasma, the electron density varies along the z-direction only i.e. the non-uniformity in charge density is present in the propagation direction only and system is supposed to be under steady state i.e. time-independent. For such type of inhomogeneity (axial only), the Eq. (1) can be rewritten as -

$$N(z) = N_o\, W(z), \qquad \ldots(2)$$

where N_o is a constant (density of plasma medium at the boundary where wave is incident on it i.e. at z=0) and density profile function W(z) is only z-dependent. This function W (z) can have different shapes corresponding to different types of axially inhomogeneous plasma. In the present study, few shapes are considered which are founded to be of practical importance.

(a) Linearly increasing axial inhomogeneity

The charge density is supposed to increase linearly with the propagation distance. For such type of axially inhomogeneous plasma, density profile function, which is of practical importance, can be written as -

$$W(z) = 1 + z/L. \qquad \ldots(3)$$

Here, z is the propagation distance in the plasma medium and L is the characteristics scale length of axial inhomogeneity.

(b) Exponentially varying axial inhomogeneities

The electron charge density functions for such type of axially inhomogeneous plasma which are considered in the present study, can be written as -

$$W(z) = 1 + \frac{z^2}{L^2}\ exp\left(-\frac{z^2}{L^2}\right), \qquad \ldots(4)$$

and

$$W(z) = 1 + exp\left(1-\frac{z}{L}\right)^2 \qquad \ldots(5)$$

The value of plasma frequency depends on the plasma charge density. Therefore, it is noticed that the plasma frequency is not a constant (as in case of homogeneous plasma) but varies in inhomogeneous plasma as -

$$\omega_p^2 = \frac{4\pi N e^2}{m} \qquad \ldots(6)$$

Substitution of N from Eq. (2) for axially inhomogeneous plasma, gives –

$$\omega_p^2 = \frac{4\pi N_o e^2 W(z)}{m} \qquad \ldots(7)$$

$$= \omega_{po}^2\ \mathrm{W}(z),$$

where $\omega_{po}^2 = \dfrac{4\pi N_o e^2}{m}$ is the homogeneous plasma frequency or the plasma frequency at the boundary of inhomogeneous plasma. Eq. (7) gives the z-dependence of plasma frequency in case of axially inhomogeneous plasma. For different shapes of *W (z)* i.e. for different type of inhomogeneities, this dependence is going to be different.

Self-focusing Equation with arbitrary large non-linearity

The intensity distribution of a linearly polarized Gaussian electromagnetic beam can be written as -

$$E\,E^{*} = E_o^2\ \exp\left(-r^2/r_o^2\right), \qquad \ldots(8)$$

where r is the radial coordinate of the cylindrical coordinate system and r_o is the initial beam width. E_o represents the amplitude of the electric field due to propagating laser beam. For the study of self-focusing phenomena, the nonlinear dielectric constant of the medium can be written as –

$$\in(\langle E\,E^{*}\rangle) = \in_o + \phi\,(\langle EE^{*}\rangle), \qquad \ldots(9)$$

in the paraxial-ray approximation, one generally expands ϕ around $\phi \cong o$. However with such an expansion one can study only those cases where $\phi << \in_o$. To study self-focusing for arbitrary large non-linearity, one should expand ϕ around an arbitrary large value at $r = o$. In order to do this, the non-linear dielectric constant of the medium may be rewritten as -

$$\in(\langle EE^{*}\rangle) = \in_o + \phi\left[\left\langle \frac{k(o)E_o^2}{2k(f)f^2}\right\rangle\right] + \phi\,[\langle EE^{*}\rangle] - \phi\left[\left\langle \frac{k(o)E_o^2}{2k(f)f^2}\right\rangle\right] \qquad \ldots(10)$$

or,
$$\in(\langle EE\rangle) = \in_o'(f) + \in_1(f) \qquad \ldots(11)$$

where
$$\in_o'(f) = \in_o + \phi\left(\left\langle \frac{k(o)E_o^2}{2k(f)f^2}\right\rangle\right), \qquad \ldots(12)$$

$$\psi(r,f) = \in_1(f) \quad = \phi\,[\langle EE\rangle] - \phi\left(\left\langle \frac{k(o)E_o^2}{2k(f)f^2}\right\rangle\right) << \in_o'(f) \qquad \ldots(13)$$

Here, f is the dimensionless beam-width parameter, defined below in Eq. (19) and k is the propagation constant defined below in Eq. (15). Here, in Eq. (13), < > represents time average of many cycles. Using the WKB approximation and following the procedure used by Sodha et al.[8] and Akhmanov et al.[9], one can write.

$$E\,(r,z) = A\,(r,z)\left[\frac{k(o)}{k(f)}\right]^{1/2} \exp\,[-ik\,(f)\,z], \qquad \ldots(14)$$

where,
$$k(f) = \frac{\omega}{c}\left[\in_o'(f)\right]^{1/2} \text{ and } k(o) = \frac{\omega}{c}\left[\in_o'(f=1)\right]^{1/2} \qquad \ldots(15)$$

In wave equation $\nabla^2 E + \frac{\omega^2}{c^2}\in E = o$. Values of $\in$ and E can be substituted from Eqs. (11) and (14), which leads to parabolic equation as -

$$-2ik\,(f)\,\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega^2}{c^2}\,\psi\,(r,f)\,A = o. \qquad \ldots(16)$$

Putting

$A\ (r, z) = A_o\ (r, z)\ exp\ [-i \int k\ (f)\ dS]$

and separating real and imaginary parts, one gets.

$$2\frac{\partial S}{\partial z}+\left[\frac{\partial S}{\partial r}\right]^2=\frac{1}{k^2(f)A_o}\left[\frac{\partial^2 A_o}{\partial r^2}+\frac{1}{r}\frac{\partial A_o}{\partial r}\right]+\frac{\omega^2}{k^2(f)c^2}\,\psi(r,f) \qquad \ldots(17)$$

$$\frac{\partial A_o^2}{\partial z}+\frac{\partial S}{\partial r}\frac{\partial A_o^2}{\partial r}+A_o^2\left[\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}\right)=\mathrm{o} \qquad \ldots(18)$$

The solution of Eqs. (17) and (18) can be written as -

$$A_o^2=\frac{E_o^2}{f^2}\ exp\left[-\frac{r^2}{r_o^2 f^2}\right], \qquad \ldots(19)$$

$$S=\frac{r^2}{z}\beta(z)+\eta(z),$$

$$\beta=\frac{1}{f}\frac{\partial f}{\partial z}$$

where β corresponds to the inverse radius of curvature of the wave front and $r_o f$ is the width of the main beam in the medium. Substituting S from Eq. (19) in Eq. (18), using the paraxial approximation, equating the coefficients of r^2 in both sides of the resulting equation and substituting for β on obtains.

$$\frac{d^2 f}{dz^2}=\frac{1}{k^2(f)r_o^4 f^3}-\frac{\omega^2 \in_1(f)f}{c^2k^2(f)} \qquad \ldots(19)$$

Collisionless plasma: Ponderomotive nonlinearity

Following Sodha, Ghatak and Tripathi the dielectric constant of the medium can be written as –

$$\in(\langle E\,E^*\rangle)=\in_o+\phi\,(\langle EE^*\rangle), \qquad \ldots(20)$$

and

$$\phi\,(\langle EE^*\rangle)=\frac{\omega_p^2}{\omega^2}\left[1-exp\left(-\frac{3}{2}\frac{m}{M}\right)\alpha\,(E\,E)\right] \qquad \ldots(21)$$

Where $\in_o = 1-\frac{\omega_p^2}{\omega^2}$, $\omega_p^2=\frac{4\pi N e^2}{m}$ is the plasma frequency in the absence of the beam, N and e are the density and charge of electrons. The characteristic parameter α is given by $\alpha=\frac{e^2M}{6k_BT_0m^2\omega^2}$, where k_B, T_0, are the Boltzmann constant and equilibrium plasma temperature respectively. Substituting for E from Eq. (14) and S, A_0 and β from (19) in Eq. (21), and using the paraxial approximation, ϕ can be written as -

$$\phi\,(\langle EE^*\rangle)=\phi\left(\frac{k(0)}{k(f)}\frac{E_0^2}{2f^2}\right)-r^2\left[\frac{3}{4}\frac{m}{M}\alpha\frac{k(0)}{k(f)}\frac{E_0^2}{r_0^2f^4}\frac{\omega_p^2}{\omega^2}\exp\left(\frac{3}{4}\frac{m}{M}\alpha\frac{k_0}{k_f}\frac{E_0^2}{f^2}\right)\right] \qquad \ldots(22)$$

correct to term in r^2,

where

$$\phi\left(\frac{k(0)}{k(f)}\frac{E_0^2}{2f^2}\right)=\frac{\omega_p^2}{\omega^2}\left[1-\exp\left(-\frac{3}{4}\frac{m}{M}\alpha\frac{k_0}{k_f}\frac{E_0^2}{f^2}\right)\right] \qquad \ldots(23)$$

Eq. (22) can be put in the convenient form of Eq. (11), thus

$$\in_o^{/}(f)=\in_0+\frac{\omega_p^2}{\omega^2}\left[1-\exp\left(-\frac{3}{4}\frac{m}{M}\alpha\frac{k_0}{k_f}\frac{E_0^2}{f^2}\right)\right] \qquad \ldots(24)$$

and

$$\in_1(f)=\frac{3}{4}\frac{m}{M}\alpha\frac{k_0}{k_f}\frac{E_0^2}{r_0^2 f^4}\frac{\omega_p^2}{\omega^2}\exp\left(-\frac{3}{4}\frac{m}{M}\alpha\frac{k_0}{k_f}\frac{E_0^2}{f^2}\right) \qquad \ldots(25)$$

Substituting for $\in_o^{/}$ and $\in_1$ from Eqs. (24) and (25) in Eq. (19a), the equation governing the beam width parameter is seen to be -

$$\frac{d^2 f}{dz^2}=\frac{1}{k^2(f)r_o^4 f^3}-\frac{3}{4}\frac{m}{M}\alpha\frac{k(0)}{k(f)}\frac{E_0^2}{r_0^2 f^3}\frac{\omega_p^2}{c^2}\exp\left(-\frac{3}{4}\frac{m}{M}\alpha\frac{k_0}{k_f}\frac{E_0^2}{f^2}\right) \qquad \ldots(26)$$

For an initial plane wavefront of the beam the initial conditions on *f* are *f (z = 0)* = 1 and $\frac{df}{dz}1_{z=0}=0$. When the two terms on RHS of equation cancel each other at *z=0,* $\frac{d^2(f)}{d^2(z)}$ *= 0, f =1* at *z=0, f* =1 for all value of *z*; in other words the beam propagates without convergence or divergence. The condition for self-trapping is therefore :

$$\frac{3}{4}\frac{m}{M}\alpha E_{ocr}^2\exp\left(-\frac{3}{4}\frac{m}{M}\alpha E_{ocr}^2\right)=\frac{1}{\left(\frac{r_0\omega_p}{c}\right)^2} \qquad \ldots(27)$$

The critical power of the beam for self-focusing is thus,

$$P_{cr}=\frac{c}{8}r_o^2 E_{ocr}^2\left[\in_o(f=1)\right]^{\frac{1}{2}} \qquad \ldots(28)$$

The initial condition on *f* are *f (z=0)=1* and *df/dz |z=0* corresponding to an initially plane wave front. The first term on the right hand side (RHS) of Eq. (29) corresponds to diffraction divergence and second term corresponds to convergence due to nonlinearity.

Above self-focusing equation has been modified and rewritten for different types of axial inhomogeneities by putting the different functions of density profile W(z) as discussed in section (2). All above mentioned modified self-focusing equations are difficult to solve analytically and thus have been solved numerically by the computer for the typical sample plasma with the following parameters *ω= 1* ×

10^{14} *rad* s^{-1}, $\omega_p = 5.5 \times 10^{13}$ *rad* s^{-1}, $T_o = 10^5$ K, $r_0 = 30$ µm, $N_0 = 9.5 \times 10^7$ cm^{-3} and $\alpha E_0^2 = 1$. Fig. 1 shows the variation of focusing parameter *(f)* versus *(z)* for different types of axially inhomogeneous plasma medium, namely, axially decreasing, increasing, exponential varying, parabolic varying etc., for the nonlinearity introduced due to ponderomotive effect. Examination of Figure 1 indicates that the value of Z_{min} (1) is almost the same for all types of axial inhomogeneities, except for exponential type, where it is very small.

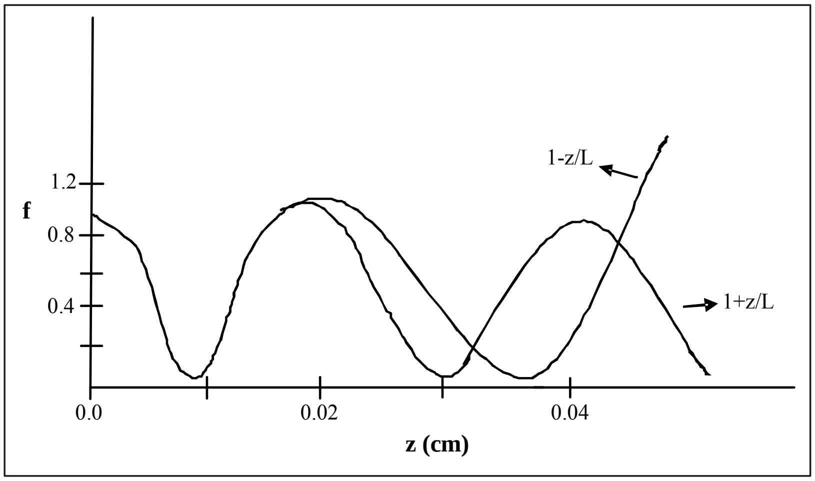

Fig. 1: Graphs between focusing parameter (f) versus propagation distance (z) in different types of axially inhomogeneous plasma. Here L = 0.2 cm

RESULTS AND DISCUSSION

In this paper, results of self focusing study of linearly polarized wave with a Gaussian initial intensity profile, based on steady state non-linear refraction theory in an inhomogeneous transparent plasma in the aberration-less paraxial approximation, are presented. Intense electromagnetic wave with nonuniform intensity distribution along its wave front causes various non–linear phenomenons while propagating through plasma. A nonlinear variation in the dielectric constant of plasma along the wavefront is setup because redistribution of charges due to ponderomotive effect. The combined effect both the non-linearity induced in the transverse direction due to propagating intense electromagnetic beam and plasma inhomogeneity in axial direction has been considered. Equation of self-focusing for different situations (i.e. for different types of axial inhomogeneities) have been obtained and solved numerically with the help of computer. Plot of beam width parameter *(f)* as a function of propagation distance has been drawn. In the present study, the plasma is considered axially inhomogeneous and is closer to more realistic situation. In all types of axially inhomogeneous plasma considered in the present study, initially up to some degree of penetration, self-focusing behavior shows oscillatory nature. Just on entering the plasma, laser beam aperture starts decreasing and after reaching minimum value (different for different types of plasma and for different types non-linearties responsible for self-focusing) at focusing (i.e. for f_{min}), starts increasing. Due to small value of beam aperture, diffraction effects start dominating over focusing effect and beam starts defocusing i.e. value of f starts increasing after reaching a minimum value. When beam spread is large, contribution due to diffraction effect is countered- balanced or in fact, dominated by self-focusing effect due to various types of nonlinearities. Beam again start starts focusing i.e. value of f starts decreasing after reaching f_{max} value. This process repeats repeatedly giving oscillatory nature to beam aperture. Because of inhomogeneous nature of plasma, repetitive nature is not exactly the same as observed in case of homogeneous plasma. In case of linearly increasing inhomogeneous plasma, value of f_{min} (1) is slightly larger than f_{min} (2) where as f_{max} (2) is slightly less than f_{min} (1). Careful analysis of Figure 1 indicates that value of f_{min} decreases with increasing order of minima i.e. focusing effect is much stronger at higher penetration in the medium. In case of linearly decreasing axially inhomogeneous plasma, beam initially focuses, then defocuses and again focuses, this pattern continues up to some extent beyond, which defocusing of the beam prevails. This behaviour is not common to other type of inhomogeneous mediums considered in the present study. In this type of inhomogeneous medium, as the beam propagates, charge density decreases causing ω_p to decrease. Beyond some distance, due to decrease in ω_p, minimum value of beam power for self-focusing increases i.e. here the critical power is a function of propagation distance. Now at a certain value of z, incident beam power levels becomes less than critical power level and hence beam does not focus. Apart from that for $z > L$, charge density function for linearly decreasing axially inhomogeneous plasma becomes negative, leading to purely imaginary plasma frequency. This may also contribute to the observed non-oscillatory behaviour of a beam at a deeper penetration in medium. The present analysis of self-focusing in inhomogeneous plasma leads to some very interesting information. It is observed that the self-focusing behaviour of the propagating beam is a complex function of inhomogeneity and very strongly depends on the density function. Axial inhomogeneity plays an important role and focusing length depends upon the nature of axial inhomogeneity.

In case of laser induced fusion process, the nature and characteristics of the plasma near the pellet is going to play a very important role in surface heating of the fusion pellet. The knowledge of the plasma density profile, which will be created across the pellet is essential to assess the laser energy density function at pellet.

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