



# NUMERICAL SOLUTION TO MASS TRANSFER ON MHD FLOW OF CASSON FLUID WITH SUCTION AND CHEMICAL REACTION

**T. HYMAVATHI<sup>a</sup> and W. SRIDHAR<sup>\*</sup>**

Department of Mathematics, K. L. University, Vaddeswaram, GUNTUR (A.P.) INDIA

<sup>a</sup>Department of Mathematics, Adikavi Nannaya University, RAJAHMUNDRY (A.P.) INDIA

## ABSTRACT

In the present work, the effect of mass transfer of a MHD Casson fluid over a porous stretching sheet in presence of chemical reaction is investigated using Keller box method. The resulting nonlinear flow is solved to get a series solution. The variations in velocity and concentration fields are presented for various flow parameters. We further analyzed that the concentration profile decreases rapidly compared with the velocity of the fluid with increase in suction parameter.

**Key words:** Casson fluid, MHD, Suction, Mass transfer, Chemical reaction.

## INTRODUCTION

Generally, non Newtonian fluids are more stable when compared with Newtonian fluids. Hence the study of such fluids is more important. Some examples of non Newtonian fluids are Maxwell fluid, Jeffery fluid, casson fluid, visco-elastic fluid etc. In recent years, casson fluids become more popular in the study of non Newtonian fluids. Tomato soup, jelly, honey, blood of human etc., are some examples of casson fluids.

Crane<sup>1</sup> investigated the study of flow past a stretching surface. The recent investigations of Hayat and Qasim<sup>2</sup> about influence of thermal radiation and joule heating on MHD flow of a Maxwell fluid in presence of thermophoresis, Fang et al.<sup>3</sup> about unsteady boundary layers over a stretching surface, Khan and Pop<sup>4</sup> about boundary flow of a nanofluid past a stretching sheet, Hayat et al.<sup>5</sup> about mixed convection flow of a micropolar fluid with radiation and chemical reaction and Ibrahim et al.<sup>6</sup> about chemically reacting MHD boundary layer flow of heat and mass transfer past a moving vertical plate

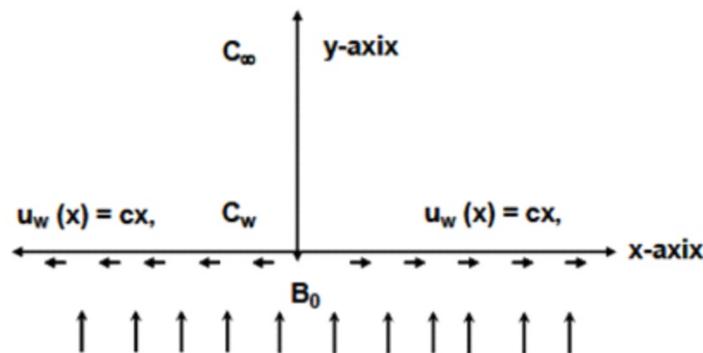
---

<sup>\*</sup> Author for correspondence; E-mail: sridharwuriti@kluniversity.in

with suction motivated us to do the present work. Shehzad et al.<sup>7</sup> studied the effect of mass transfer on MHD flow of casson fluid with chemical reaction and suction analytically using Homotopy analysis method. In the present work, numerical solution to mass transfer on MHD flow of a casson fluid with suction and chemical reaction using keller box method<sup>8</sup>. The constitutive equations of casson fluid model are considered from Nakamura and Sawada<sup>9</sup>, Eldabe and Salwa<sup>10</sup>, Dash<sup>11</sup> and Boyd et al.<sup>12</sup> The constitutive governing equations are converting to ordinary equations by using similarity transformations and solved using kellerbox method. The velocity and concentration profiles are presented for various values of casson parameter, reaction ate parameter, magnetic parameter, suction parameter and Schmidh number. The results are found good in agreement.

### Governing problems

Consider a MHD incompressible flow of a Casson fluid over a porous stretching surface at  $y = 0$ , as shown in Fig. 1. We select the Cartesian coordinate system such that the x-axis be taken parallel to the surface and y is perpendicular to the surface. The fluid occupies a half space  $y > 0$ . The mass transfer phenomenon with chemical reaction is also retained. The flow is subjected to a constant applied magnetic field  $B_0$  in the y direction. The flow is taken to be steady and the magnetic Reynolds number is considered to be very small so that the induced magnetic field is negligible in comparison to the applied magnetic field. The fluid properties are constant.



**Fig. 1: Physical sketch of the present problem**

The rheological equation of state for an isotropic flow of a Casson fluid can be expressed as<sup>10</sup> –

$$\tau_j = \begin{cases} 2(\mu_B + P_y/\sqrt{2\pi})e_{ij}, & \pi > \pi_c \\ 2(\mu_B + P_y/\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c \end{cases} \quad \dots(1)$$

In the above equation  $\pi = e_{ij}e_{ij}$  and  $e_{ij}$  denotes the  $(i, j)^{\text{th}}$  component of the deformation rate,  $\pi$  the product of the component of deformation rate with itself,  $\pi_c$  a critical value of this product based on the non-Newtonian model,  $\mu_B$  the plastic dynamic viscosity of the non-Newtonian fluid and  $p_y$  the yield stress of the fluid. The equations governing the steady boundary layer flow of the Casson fluid are<sup>13</sup> –

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots(2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad \dots(3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \quad \dots(4)$$

where  $u$  and  $v$  are the components of velocity respectively in the  $x$  and  $y$  directions,  $\nu$  is the kinematic viscosity,  $\rho$  is the fluid density (assumed constant),  $\beta = \mu_B \sqrt{2\pi_c} / P_y$  is the non-Newtonian parameter of the Casson fluid,  $D$  is the diffusion coefficient,  $k_1$  reaction rate and  $C$  is the concentration rate.

#### A. Boundary Conditions

The appropriate boundary conditions for the problem are given by –

$$u = u_w(x) = cx, v = v_0, C = C_w \text{ at } y = 0 \quad \dots(5)$$

$$\text{as } y \rightarrow \infty, u \rightarrow 0, C \rightarrow C_\infty \quad \dots(6)$$

Equations (2)-(6) can be made dimensionless by introducing the following change of variables –

$$u = cx f'(\eta), v = -\sqrt{c\nu} \psi(\eta), \eta = y \sqrt{\frac{c}{\nu}}, \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad \dots(7)$$

The dimensionless problem satisfies –

$$\left( 1 + \frac{1}{\beta} \right) f''' + f f'' - f'^2 - M f' = 0 \quad \dots(8)$$

$$\phi'' - Sc f \phi' - Sc \gamma \phi = 0 \quad \dots(9)$$

and the boundary conditions take the following form –

$$\text{at } \eta = 0, f = S, \phi = 1, f' = 1 \quad \dots(10)$$

$$\text{as } \eta \rightarrow \infty, f' \rightarrow 0, \phi \rightarrow 0 \quad \dots(11)$$

where Eq. (2) is satisfied identically,  $M = \frac{\sigma B_0^2}{\rho c}$ , the Hartman number,  $Sc = \nu / D$  the Schmidt number,  $\gamma = k_1 / c$  the chemical reaction parameter and  $S = v_0 / \sqrt{\vartheta c}$  the suction parameter.

### Numerical procedure

Equation subject to boundary conditions is solved numerically using an implicit-finite difference scheme known as Keller box method, as described by Cebeci and Bradshaw<sup>8</sup>. The steps followed are –

1. Reduce (8)-(9) to a first order equation
2. Write the difference equations using central differences
3. Linearize the resulting algebraic equation by Newton's method and write in matrix vector form
4. Use the block tridiagonal elimination technique to solve the linear system.

Consider the flow equation and concentration equations

$$\left(1 + \frac{1}{\beta}\right) f''' + f f'' - 2f'^2 = 0 \quad \dots(12)$$

$$\phi'' - Sc f \phi' - Sc \gamma \phi = 0 \quad \dots(13)$$

and the boundary conditions

$$f(\eta) = 0, f'(\eta) = 1, \phi(\eta) = 1 \text{ at } \eta = 0 \quad \dots(14)$$

$$f'(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ at } \eta \rightarrow \infty \quad \dots(15)$$

$$\text{Introduce } f' = p, \quad \dots(16)$$

$$p' = q, \quad \dots(17)$$

$$g' = n \quad \dots(18)$$

Eqn (12) and (13) reduces to –

$$\left(1 + \frac{1}{\beta}\right) q' + fq - 2p^2 = 0 \quad \dots(19)$$

$$n' - Scfn - Sc\gamma g = 0 \quad \dots(20)$$

Consider the segment  $\eta_{j-1}, \eta_j$  with  $\eta_{j-1/2}$  as the mid point  $\eta_0=0, \eta_j=\eta_{j-1}+h_j, \eta_j=\eta_\infty \dots(21)$

where  $h_j$  is the  $\Delta\eta$  spaces and  $j = 1, 2, \dots, J$  is a sequence number that indicates the coordinate locations.

$$\frac{f_j - f_{j-1}}{h_j} = \frac{p_j + p_{j-1}}{2} = p_{j-1/2} \quad \dots(22)$$

$$\frac{p_j - p_{j-1}}{h_j} = \frac{q_j + q_{j-1}}{2} = q_{j-1/2} \quad \dots(23)$$

$$\frac{g_j - g_{j-1}}{h_j} = \frac{n_j + n_{j-1}}{2} = n_{j-1/2} \quad \dots(24)$$

$$\left(1 + \frac{1}{\beta}\right) \frac{q_j - q_{j-1}}{h_j} + \left(\frac{f_j + f_{j-1}}{2}\right) \left(\frac{q_j + q_{j-1}}{2}\right) - \left(\frac{p_j + p_{j-1}}{2}\right)^2 - M \cdot \frac{p_j + p_{j-1}}{2} = 0 \quad \dots(25)$$

$$\frac{n_j - n_{j-1}}{h_j} - Sc \left(\frac{f_j + f_{j-1}}{2}\right) \left(\frac{n_j + n_{j-1}}{2}\right) - Sc \cdot \gamma \cdot \frac{g_j + g_{j-1}}{2} = 0 \quad \dots(26)$$

Equations (22) to (26) are imposed for  $j = 1, 2, 3, \dots, J$  and the transformed boundary layer thickness  $\eta_j$  taken to the sufficiently large so that it is beyond the edge of the boundary layer .

$$\begin{aligned} \text{The bc's are } f_0 &= 0, & p_j &= 0 \\ p_0 &= 1, & g_0 &= 1 & g_j &= 0 \end{aligned} \quad \dots(27)$$

### Newton's method

Linearizing the non linear system of equations (22) to (26)

Introduce

$$\begin{aligned}
 f_j^{(k+1)} &= f_j^{(k)} + \delta f_j^{(k)} \\
 p_j^{(k+1)} &= p_j^{(k)} + \delta p_j^{(k)} \\
 q_j^{(k+1)} &= q_j^{(k)} + \delta q_j^{(k)} \\
 g_j^{(k+1)} &= g_j^{(k)} + \delta g_j^{(k)} \\
 n_j^{(k+1)} &= n_j^{(k)} + \delta n_j^{(k)}
 \end{aligned} \dots (28)$$

Substitute in equations (12) to (15)

Write  $\delta f_j - \delta f_{j-1} - \frac{h_j}{2} (\delta p_j + \delta p_{j-1}) = (r_1)_{j-\frac{1}{2}}$  ... (29)

$$\delta p_j - \delta p_{j-1} - \frac{h_j}{2} (\delta q_j + \delta q_{j-1}) = (r_2)_{j-\frac{1}{2}} \dots (30)$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2} (\delta n_j + \delta n_{j-1}) = (r_3)_{j-\frac{1}{2}} \dots (31)$$

$$(a_1)_j \delta q_j + (a_2)_j \delta q_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta p_j + \delta p_{j-1} = (r_4)_{j-\frac{1}{2}} \dots (32)$$

$$(b_1)_j \delta n_j + (b_2)_j \delta n_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} + (b_5)_j \delta g_j + (b_6)_j \delta g_{j-1} = (r_5)_{j-\frac{1}{2}} \dots (33)$$

Where  $(a_1)_j = 1 + \frac{\beta h_j}{4(\beta + 1)} (f_j + f_{j-1})$

$$(a_2)_j = (a_1)_j - 2.0$$

$$(a_3)_j = \frac{\beta h_j}{4(\beta + 1)} (q_j + q_{j-1})$$

$$(a_4)_j = (a_3)_j$$

$$(a_5)_j = \frac{\beta h_j}{2(\beta + 1)}(p_j + p_{j-1}) - \frac{\beta h_j M}{2(\beta + 1)}$$

$$(a_6)_j = (a_5)_j$$

$$(b_1)_j = 1 + \frac{Sch_j}{4}(f_j + f_{j-1})$$

$$(b_2)_j = (b_1)_j - 2.0$$

$$(b_3)_j = \frac{Sch_j}{2}(n_j + n_{j-1})$$

$$(b_4)_j = (b_3)_j$$

$$(b_5)_j = \frac{-Sc\gamma h_j}{2}$$

$$(b_6)_j = (b_5)_j$$

and  $(r_1)_j = f_{j-1} - f_j + \frac{h_j}{2}(p_j + p_{j-1})$

$$(r_2)_j = p_{j-1} - p_j + \frac{h_j}{2}(q_j + q_{j-1})$$

$$(r_3)_j = g_{j-1} - g_j + \frac{h_j}{2}(n_j + n_{j-1}) \quad \dots(35)$$

$$(r_4)_j = q_{j-1} - q_j - \frac{\beta h_j}{4(\beta + 1)}(f_j + f_{j-1})(q_j + q_{j-1}) + \frac{\beta h_j}{4(\beta + 1)}(p_j + p_{j-1})^2$$

$$(r_5)_j = n_{j-1} - n_j - \frac{Sch_j}{4}(f_j + f_{j-1})(n_j + n_{j-1}) + \frac{Sc\gamma h_j}{2}(g_j + g_{j-1})$$

Taking  $j = 1, 2, 3, \dots$

The system of equations becomes –

$$[A_1][\delta_1]+[C_1][\delta_2]=[r_1] \quad \dots(36)$$

$$[B_2][\delta_1]+[A_2][\delta_2]+[C_2][\delta_3] = [r_2] \quad \dots (37)$$

$$\dots[B_{j-1}][\delta_1]+[A_{j-1}][\delta_2]+[C_{j-1}][\delta_3] = [r_{j-1}]$$

$$[B_j][\delta_{j-1}]+[A_j][\delta_j]=[r_j]$$

Where

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ d & 0 & 0 & d & 0 \\ 0 & d & 0 & 0 & d \\ (a_2)_1 & 0 & (a_3)_1 & (a_1)_1 & 0 \\ 0 & (b_2)_1 & (b_3)_1 & 0 & (b_1)_1 \end{bmatrix} \quad A_j = \begin{bmatrix} d & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ (a_6)_j & 0 & (a_3)_j & (a_1)_j & 0 \\ 0 & (b_6)_j & (b_3)_j & 0 & (b_1)_j \end{bmatrix}$$

$$B_j = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & d \\ 0 & 0 & (a_4)_j & (a_2)_j & 0 \\ 0 & 0 & (b_4)_j & 0 & (b_2)_j \end{bmatrix} \quad C_j = \begin{bmatrix} d & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ (a_5)_j & 0 & 0 & 0 & 0 \\ 0 & (b_5)_j & 0 & 0 & 0 \end{bmatrix} \quad \dots(38)$$

**The block elimination method**

The linearized differential equations of the system has a block diagonal structure. This can be written in matrix form as –

$$\begin{bmatrix} [A_1] & [C_1] \\ [B_2] & [A_2] & [C_2] \\ \vdots \\ [B_{j-1}] & [A_{j-1}] & [C_{j-1}] \\ [B_j] & [A_j] \end{bmatrix} \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{j-1}] \\ [\delta_j] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{j-1}] \\ [r_j] \end{bmatrix} \quad \dots(39)$$



Now  $[\alpha_1] [w_1] = [r_1]$

$[\alpha_j] [w_j] = [r_j] - [B_j][W_{j-1}]$  for  $2 \leq j \leq J$

Once the elements of W are found, substitute in  $L\delta = W$  and solve for  $\delta$

$[\delta_j] = [W_j]$

$[\delta_j] = [W_j] - [\Gamma_j][\delta_{j+1}]$ ,  $1 \leq j \leq J-1$

These calculations are repeated until some convergence criterion is satisfied and we stop the calculations when  $|\delta g_0^{(i)}| \leq \epsilon$ , where  $\epsilon$  is very small prescribed value taken to be  $\epsilon = 0.0000001$ .

### RESULTS AND DISCUSSION

The velocity and concentration profiles are plotted graphically using MATLAB for various value of casson parameter, suction parameter, magnetic parameter, reaction rate parameter, Schmidt number. It is observed that the velocity is found to be decreasing with increase in casson parameter as shown in Fig. 2a. where the values of  $M, S = 0.5, Sc = 10$  and  $\gamma = 0.3$ . Velocity is found to be decreasing with increase in magnetic parameter as shown in Fig. 2b. where the values of  $\gamma = 0.3, \beta = 0.5, Sc = 10$  and  $S = 0.5$  and also velocity is found to be decreasing with increase in suction parameter as shown in Fig. 2c. where the values of  $\gamma = 0.3, \beta = 0.6, Sc = 10$  and  $M = 0.5$ , velocity is found to be decreasing with increase in reaction rate parameter shown in Fig. 2d. where the values of  $S = 0.5, \beta = 0.6, Sc = 0.8$  and  $M = 0.5$ .

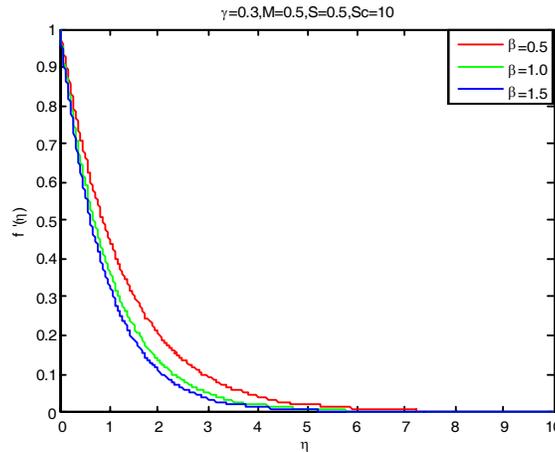
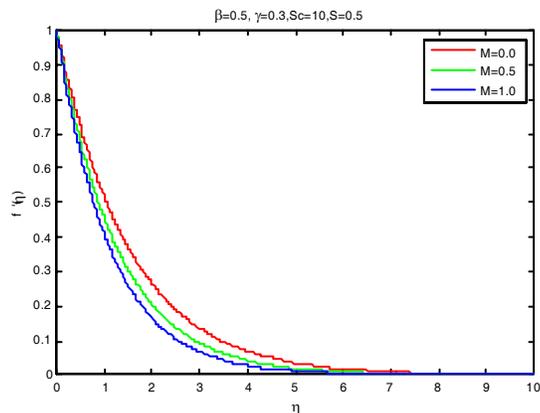
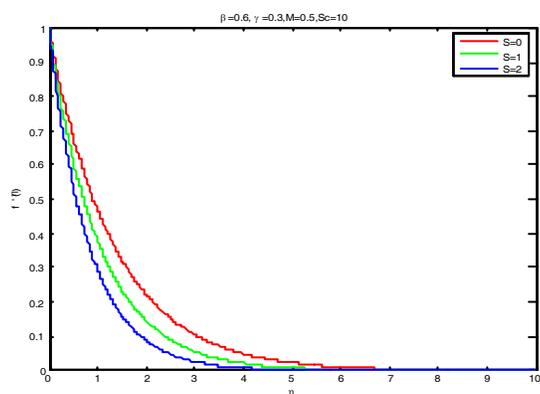


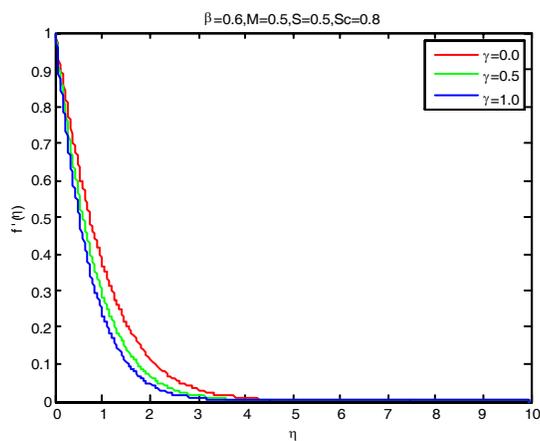
Fig. 2a: Velocity profiles for variation in  $\beta$



**Fig. 2b: Velocity profiles for variation in M**

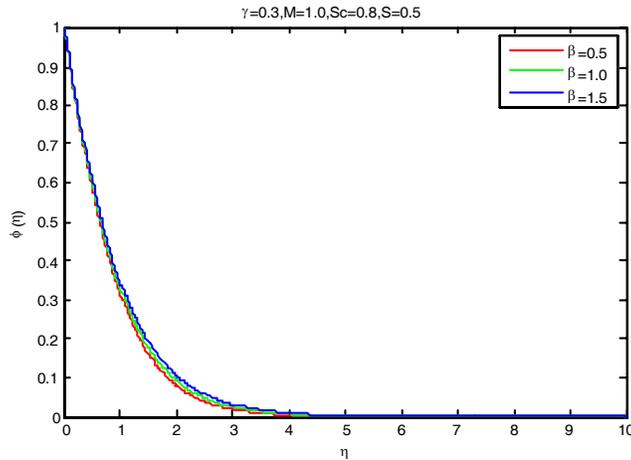


**Fig. 2c: Velocity profiles for variation in S**

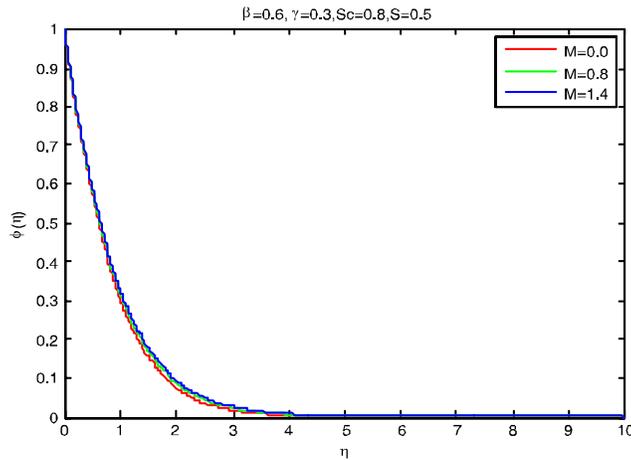


**Fig. 2d: Velocity profiles for various values of  $\gamma$**

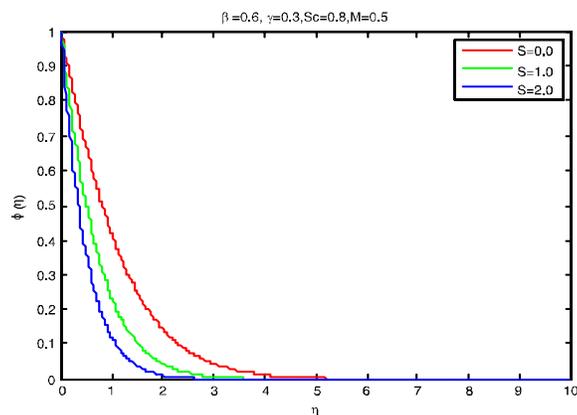
Concentration is found to be increasing with increase in casson parameter as shown in Fig. 3a. where the values of  $M = 1.0$ ,  $S = 0.5$ ,  $Sc = 0.8$  and  $\gamma = 0.3$ . Concentration is found to be decreasing with increase in magnetic parameter as shown in Fig. 3b. where the values of  $\gamma = 0.3$ ,  $\beta = 0.6$ ,  $Sc = 0.8$  and  $S = 0.5$  and also concentration is found to be decreasing with increase in suction parameter as shown in Fig. 3c. where the values of  $\gamma = 0.3$ ,  $\beta = 0.6$ ,  $Sc = 0.8$  and  $M = 0.5$ , concentration is found to be decreasing with increase in reaction rate parameter shown in Fig. 2d. where the values of  $S = 0.5$ ,  $\beta = 0.6$ ,  $Sc = 0.8$  and  $M = 0.5$ . Concentration is found to be decreasing with increase in Schmidt number shown in Fig. 3e. where the values of  $S = 0.5$ ,  $\beta = 0.6$ ,  $Sc = 0.8$  and  $M = 0.5$ .



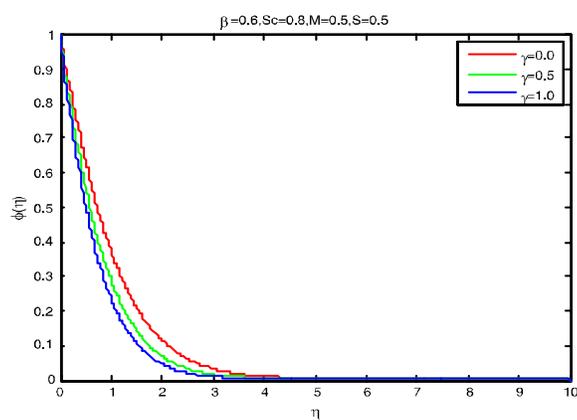
**Fig. 3a: Concentration profiles with change in  $\beta$**



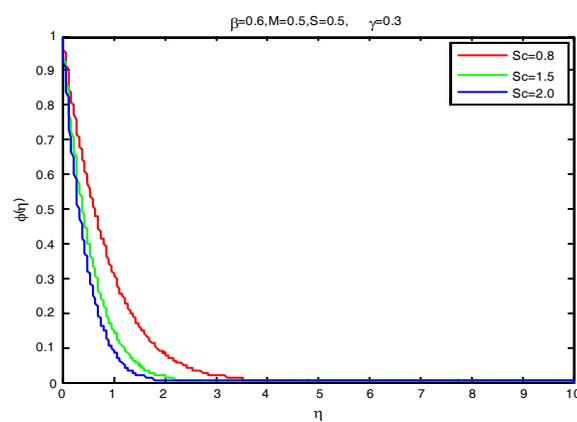
**Fig. 3b: Concentration profiles with change in  $M$**



**Fig. 3c: Concentration profiles with change in S**



**Fig. 3d: concentration profiles with change in  $\gamma$**



**Fig. 3e: Concentration profiles with change in Sc**

## CONCLUSION

Effects of mass transfer on the MHD boundary layer flow of a Casson fluid model with chemical reaction are presented. The following observations are found.

- (i) The Casson parameter  $\beta$ ,  $S$ ,  $\gamma$  and  $M$  have similar effects on the velocity profile  $f'(\eta)$ .
- (ii) Casson parameter  $\beta$  has opposite effects on the velocity and concentration profiles.
- (iii) An increase in the Schmidt number  $Sc$  causes a decrease in the concentration profile and the boundary layer thickness.
- (iv) When  $\gamma = 0$ , there is no chemical reaction. An increase in  $\gamma$  decreases concentration  $\phi(\eta)$ .

## REFERENCES

1. L. J. Crane, Flow Past a Stretching Plate, *Z. Angew. Math. Mech.*, **21**, 645 (1970).
2. T. Hayat and M. Qasim, Influence of Thermal Radiation and Joule Heating on MHD Flow of a Maxwell Fluid in the Presence of Thermophoresis, *Int. J. Heat Mass Transfer*, **53**, 4780 (2010).
3. T. Fang, J. Zhang and S. Yao, A New Family of Unsteady Boundary Layers Over a Stretching Surface, *Appl. Math. Comput.*, **217**, 3747 (2010).
4. W. A. Khan and I. Pop, Boundary Flow of a Nanofluid Past a Stretching Sheet, *Int. J. Heat Mass Transfer*, **53**, 2477 (2010).
5. T. Hayat, S. A. Shehzad and M. Qasim, Mixed Convection Flow of a Micropolar Fluid with Radiation and Chemical Reaction, *Int. J. Num. Methods Fluids*, **67**, 1418 (2011).
6. S. Y. Ibrahim and O. D. Makinde, Chemically Reacting MHD Boundary Layer Flow of Heat and Mass Transfer Past a Moving Vertical Plate with Suction, *Scientific Research Essays*, **5**, 2875 (2010).
7. S. A. Shehzad, T. Hayat, Qasim and S. Asghar, Effects of Mass Transfer on MHD Flow of Casson Fluid with Chemical Reaction and Suction, *Brazilian J. Chem. Engg.*, **30(1)**, 187-195 (2013).

8. T. Cebeci and P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer*, Springer-Verlag, Newyork (1988).
9. M. Nakamura and T. Sawada, Numerical Study on the Flow of a Non-Newtonian Fluid through an Axisymmetric Stenosis, *ASME J. Biomech. Eng.*, **110**, 137 (1988).
10. N. T. M. Eldabe and M. G. E. Salwa, Heat Transfer of MHD Non-Newtonian Casson Fluid Flow Between Two Rotating Cylinders, *J. Phys. Soc. Japan*, **64**, 41 (1995).
11. R. K. Dash, R. K. Mehta and G. Jayarama, Casson Fluid Flow in a Pipe Filled with a Homogenous Porous Medium, *Int. J. Eng. Sci.*, **34**, 1145 (1996).
12. J. Boyd, J. M. Buick and S. Green, Analysis of the Casson and Carreau-Yasuda Non-Newtonian Blood Models in Steady and Oscillatory Flow using the Lattice Boltzmann Method, *Phys. Fluids*, **19**, 93 (2007).
13. M. Mustafa, T. Hayat, I. Pop and A. A. Hendi, Stagnation-Point Flow and Heat Transfer of a Casson Fluid Towards a Stretching Sheet, *Z. Naturforsch.*, **67a**, 70 (2012).

*Revised : 01.08.2016*

*Accepted : 03.08.2016*