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NUMERICAL SOLUTION OF MHD FLOW OVER A MOVING VERTICAL POROUS PLATE WITH HEAT AND MASS TRANSFER

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ABSTRACT

An analysis is performed to study numerical solution of MHD flow over a vertical porous plate with heat and mass transfer. The coupled nonlinear partial differential equations governing the flow, heat and mass transfer have been reduced to a set of coupled nonlinear ordinary differential equations by using similarity transformation. The reduced equations are solved numerically using Runge-Kutta fourth-order integration scheme together with shooting method. The effect of various physical parameters on the velocity, temperature, and concentration fields has been studied.

Key words: MHD, Chemical reaction, Heat and mass transfer, Vertical plate, Porous medium.

INTRODUCTION

The study of convective flow with heat and mass transfer under the influence of magnetic field and chemical reaction with heat source has practical applications in many areas of science and engineering. This phenomenon plays an important role in chemical industry, petroleum industry, cooling of nuclear reactors, and packed-bed catalytic reactors. Natural convection flows occur frequently in nature due to temperature differences, concentration differences, and also due to combined effects. The concentration difference may sometimes produce qualitative changes to the rate of heat transfer.

Bestman and Adjepong[1] studied the unsteady hydro-magnetic free convection flow with radiative heat transfer in a rotating fluid. Jha[2] studied MHD free convection and mass

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transfer flow through a porous medium but did not consider the effect of radiation, which is of great relevance to astrophysical and cosmic studies. The effects of Hall current on hydro-magnetic free convection with mass transfer in a rotating fluid was studied by Agarwal et al.[3] Singh and Sacheti[4] presented a study on the finite difference analysis of unsteady hydromagnetic free convection flow with constant heat flux, while Ram and Jain[5] presented the result of a study on hydro-magnetic Ekman layer on convective heat generating fluid in slip flow regime. Helmy[6] focused on MHD flow in a micro-polar fluid. Recently, Chamkha[7] investigated unsteady convective heat and mass transfer past a semi-infinite permeable moving plate with heat absorption where it was found that increase in solutal Grashoff number enhanced the concentration buoyancy effects leading to an increase in the velocity. In another recent study, Ibrahim et al.[8] investigated unsteady magneto-hydrodynamic micro-polar fluid flow and heat transfer over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. Chamkha[7] and Cookey et al.[9] gave a good review on MHD flows through a porous medium. Das et al.[10] considered the effects of first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumaraswamy and Meenakshisundaram[11] studied chemical reaction effects on vertical oscillating plate with variable temperature. Thermal radiation effect on flow past a vertical plate with mass transfer is examined by Muralidharan and Muthucumaraswamy[12] and Rajput and Kumar[13]. Natural convective flow past an oscillating plate with constant mass flux in the presence of radiation was studied by Chaudhary and Jain[14].

The coupled nonlinear partial differential equations governing the flow, heat and mass transfer have been reduced to a set of coupled nonlinear ordinary differential equations by using similarity transformation. Following[15] the similarity solutions exist, if the convective heat transfer associated with the hot fluid on lower surface of the plate is proportional to the inverse square root of the axial distance. The reduced equations are solved numerically using Runge-Kutta fourth-order integration scheme together with shooting method. The effect of various physical parameters on the velocity, temperature, and concentration fields has been studied.

Formulation of the problem

We consider a steady two-dimensional boundary layer flow of a stream of cold incompressible electrically conducting fluid over a moving vertical porous flat plate at temperature T_∞ in presence of heat source and chemical reaction. The left surface of the plate is being heated by convection from a hot fluid at temperature T_f that gives a heat transfer coefficient h_f and T_∞ is the temperature of the fluid away from the plate. The cold fluid in contact with the upper surface of the plate generates heat internally at the volumetric rate Q_0.

Here, the *x*-axis is taken along the direction of plate and *y*-axis is normal to it. A magnetic field of uniform field strength B_0 is applied in the negative direction of *y*-axis.

The continuity, momentum, energy, and concentration equations describing the flow under the Boussinesq approximation can be written as –

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho}\mathrm{u} - \frac{\nu}{K'}u + \mathrm{g}\,\beta(\mathrm{T} - \mathrm{T}_\infty) + \mathrm{g}\,\beta^*(\mathrm{C} - \mathrm{C}_\infty) \qquad \ldots(2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_P}(T - T_\infty) \qquad \ldots(3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - Kr'C \qquad \ldots(4)$$

The symbols u and v denote the fluid velocity in the *x*- and *y*-direction. Here *T* and *C* are the temperature and concentration variables, γ is the kinematic viscosity, α is the thermal diffusivity, *D* is the mass diffusivity, β is the thermal expansion coefficient, *i* β^* s the solutal expansion coefficient, ρ is the fluid density, *g* is the gravitational acceleration, σ is the electrical conductivity, Q_0 is the heat source, C_p is the specific heat at constant pressure, and Kr^1 is the chemical reaction rate on the species concentration. In the above equations, several assumptions have been made. First, the plate is non-conducting, and the effects of radiant heating, viscous dissipation, Hall effects, and induced fields are neglected. Second, the physical properties, that is, viscosity, heat capacity, thermal diffusivity, and the mass diffusivity of the fluid remain invariant throughout the fluid. The appropriate boundary conditions at the plate surface and far into the cold fluid are –

$$\begin{aligned} &u(x,0) = U_0, v(x,0) = 0, -k\frac{\partial T}{\partial y}(x,0) = h_f\left[T_f - T(x,0)\right], C_w(x,0) = Ax^\lambda + C_\infty, \\ &u(x,\infty) = 0, T(x,\infty) = T_\infty, C(x,\infty) = C_\infty, \end{aligned} \qquad \ldots(5)$$

where C_∞ is the species concentration at the plate surface, A is the constant, λ is the power index of the concentration, U_0 is the plate velocity, *k* is the thermal conductivity coefficient, and C_∞ is the concentration of the fluid away from the plate. The boundary layer equations presented are nonlinear partial differential equations and, are in general, difficult to solve. However, the equations admit of a self-similar solution. Therefore, transformation allows them to be reduced to a system of ordinary differential equations that are relatively easy to solve numerically. We look for solution compatible with (1) of the form –

$$u = U_0 f'(\eta), \mathrm{v} = -\frac{1}{2}\sqrt{\frac{\nu U_0}{x}} f(\eta) + \frac{U_0 y}{2x} f'(\eta) \qquad \ldots(6)$$

where $\eta = y\sqrt{U_0/(\nu x)}$, and prime denotes the differentiation with respect to η. Let us introduce the dimensionless quantities, that is –

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ Gr_x = \frac{g\beta(\mathrm{T}_f - T_\infty)x}{U_0{}^2},\ K = \frac{K'\nu_0^2}{U_0^2}$$
$$Gc_x = \frac{g\beta^*(C_w - C_\infty)x}{U_0{}^2}, BI_x = \frac{h_f}{k}\sqrt{\frac{\nu x}{U_0}}, \mathrm{Pr} = \frac{\nu}{\alpha}, Sc = \frac{\nu}{D},$$
$$S_x = \frac{Q_0 x}{U_0 \rho C_P}, Kr_x = \frac{Kr'x}{U_0}, N_c = \frac{C_\infty}{C_w - C_\infty}, Ha_x = \frac{\sigma B_0{}^2 x}{\rho U_0}, \qquad \ldots(7)$$

Here Ha_x is the local magnetic field parameter, Gr_x is the local thermal Grash of number, Gc_x is the modified Grashof number, BI_x is the local convective heat transfer parameter, Pr is the Prandtl number, Sc is the Schmidt number, S_x is the local heat source parameter, Kr_x is the local chemical reaction parameter, and Nc is the concentration difference parameter. Using (6) and (7) in (2)–(4), we get the following equations:

$$f''' + \frac{1}{2} ff'' - Ha_x f' + Gr_x\theta + GC_x\phi = 0 \qquad \ldots(8)$$

$$\theta'' + \frac{1}{2}\mathrm{Pr}\, f\theta' + \mathrm{Pr}\, S_x\theta = 0 \qquad \ldots(9)$$

$$\phi'' + \frac{1}{2} Scf\phi' - ScKr_x(\phi + Nc) = 0 \qquad \ldots(10)$$

The corresponding boundary conditions for equation (5) for velocity, temperature, and concentration fields in terms of non-dimensional variables are –

$$f(0) = 0, f'(0) = 1, \theta'(0) = BI_x\left[\theta(0) - 1\right], \phi(0) = 1,$$
$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \qquad \ldots(11)$$

Solution of the problem

It is observed that in the absence of local source parameter and chemical reaction parameter, that is, for $S_x = 0$, and $Kr_x = 0$; (8), (9), and (10) together with boundary condition

(11) are the same as those obtained by Makinde[15]. It is noticed that the concentration equation (10) in presence of the chemical reaction parameter (Kr_x) in the fluid yields nonhomogeneous differential equation, which is coupled with momentum equation (8), and in general, difficult to solve analytically. In order to overcome this difficulty, we solve these equations numerically by fourth-order Runge-Kutta method in association with shooting technique. Firstly, these equations together with associated boundary conditions are reduced to first-order differential equations. Since equations to be solved are the third order for the velocity and second order for the temperature and concentration, the values of f', θ', and ϕ' are needed at $\eta = 0$. Therefore, the shooting method is used to solve this boundary value problem. The local skin friction coefficient, the local Nusselt number, the local Sherwood number, and the plate surface temperature are computed in terms of $f''(0)$, $-\theta'(0)$, $\phi'(0)$ and $\theta(0)$, respectively. It can be noted that the local parameters Ha_x, Gr_x, Gc_x, Bi_x, S_x and Kr_x in (8)–(10) are functions of x and generate local similarity solution. In order to have a true similarity solution, we assume the following relation:

$$h_f = \frac{a}{\sqrt{x}},\ \sigma = \frac{b}{x},\ \beta = \frac{c}{x},\ \beta^* = \frac{d}{x},\ Q_0 = \frac{e}{x},\ Kr' = \frac{m}{x}, \qquad \ldots(12)$$

where a, b, c, d, e, and m are the constants with appropriate dimensions. In view of relation (12), the parameters Ha_x, Gr_x, Gc_x, Bi_x, S_x and Kr_x are now independent of x and henceforth, we drop the index "x" for simplicity.

RESULTS AND DISCUSSION

In order to get physical insight into the problem, the numerical calculations are carried out to study the variations in velocity, temperature and concentration. The variation in skin-friction shear stress at the wall, rate of heat and mass transfer are computed.

Figs. 1-7 exhibit the velocity profiles obtained by the numerical simulations for various flow parameters involved in the problem. The simulated parameters are reported in the Fig caption. It is evident from Figs. 1 and 2 that greater cooling of surface, an increase in Gr, and an increase in Gc result in an increase in the velocity. It is due to the fact that the increase in the values of Grashof number and modified Grashof number has the tendency to increase the thermal and mass buoyancy effect. The increase is also evident due to the presence of source and chemical reaction parameters. Furthermore, the velocity increases rapidly and suddenly falls near the boundary and then approaches the far field. The effect of magnetic parameter on the velocity field is shown in Fig 3. It illustrates that the velocity profile decreases with the increase of magnetic parameter, because Lorentz force acts against the flow, if the magnetic field is applied in the normal direction. The velocity profiles for different values of the dimensionless permeability are shown in Fig. 4. The presence of porous media increases the resistance flow resulting in a decrease in the flow velocity. This behavior is depicted by the decrease in the velocity as permeability decreases and when $k \to \infty$ (i.e., the porous medium effect vanishes) the velocity is greater in the flow field. These behaviors are shown in Fig 4. A little increase in the velocity profile near the boundary layer is marked in Fig. 5 with the increase in the convective heat parameter because the fluid adjacent to the right surface of the plate becomes lighter by hot fluid and rises faster. The boundary layer flows develop adjacent to vertical surface and velocity reaches a maximum in the boundary layer. Fig. 6 illustrates the velocity profiles for different values of Prandtl number.

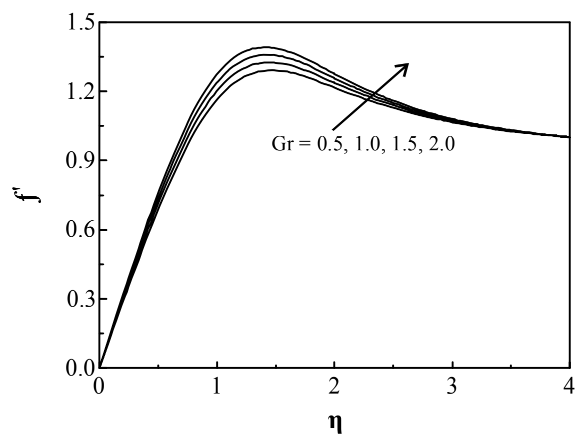

Fig. 1: Variation of the velocity component *f'* with *Gr*

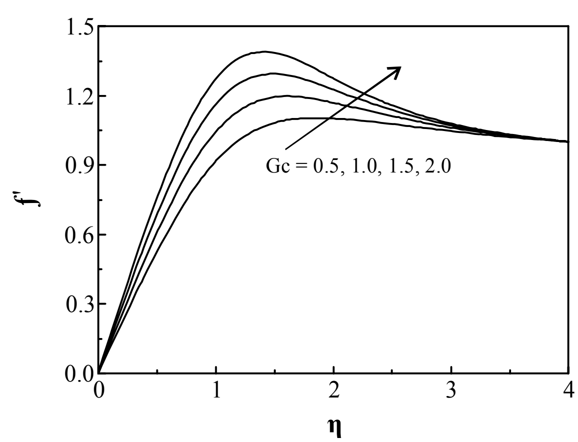

Fig. 2: Variation of the velocity component *f'* with *Gc*

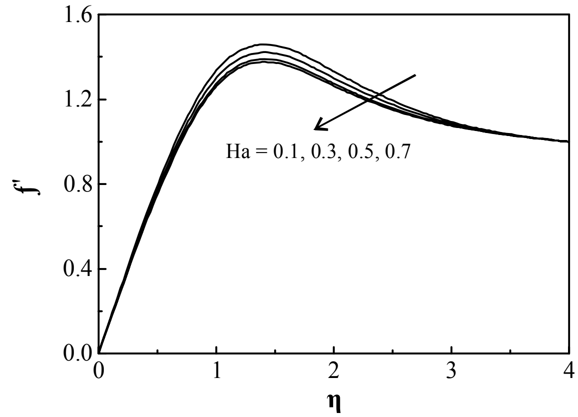

Fig 3: Variation of the velocity component *f′* with *Ha*

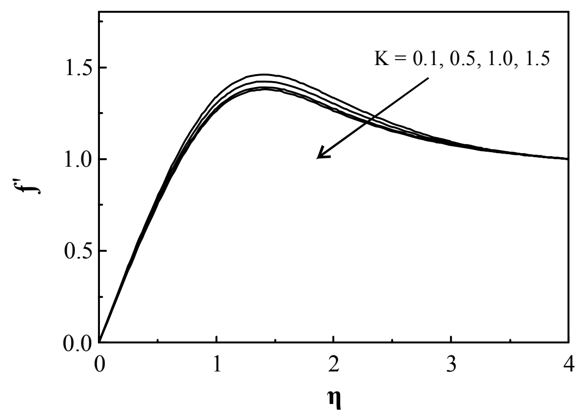

Fig. 4: Variation of the velocity component *f′* with *K*

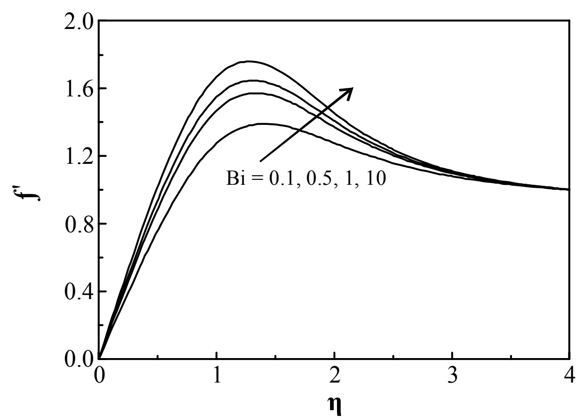

Fig. 5: Variation of the velocity component *f′* with *Bi*

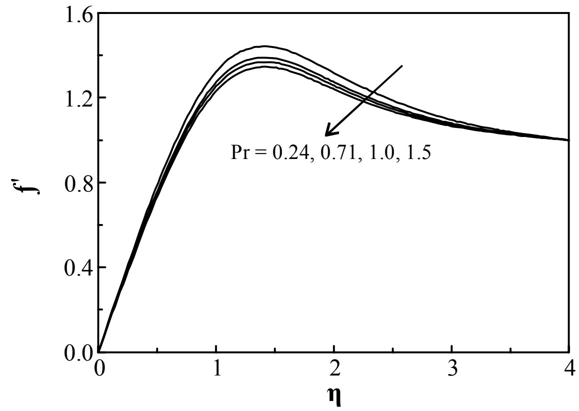

Fig. 6: Variation of the velocity component *f'* with *Pr*

The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity field. For different values of the radiation parameter and heat source parameters on the velocity profiles are shown in Figs. 7 and 8. It is noticed that an increase in the radiation parameter and heat source parameter results an increase in the velocity within the boundary layer, also it increases the thickness of the velocity boundary layers. Fig. 9 shows the velocity profiles for different values of chemical reaction parameter. As the chemical reaction parameter increases, the velocity profiles decreases.

Figs. 10-14 show the temperature profiles obtained by the numerical simulations for various values of flow parameters. Fig. 10 clearly demonstrates that the temperature profiles increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends to heat the fluid, and thus, reduces the heat transfer from the wall.

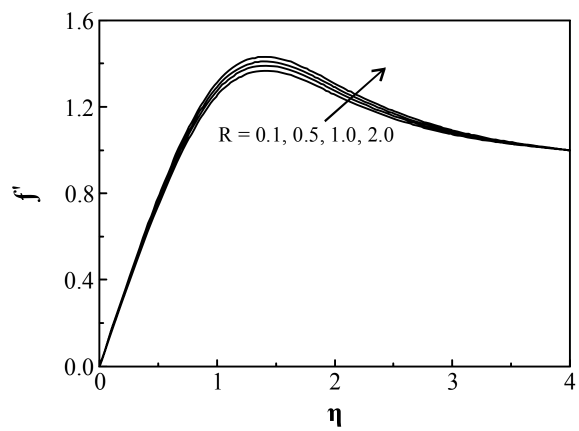

Fig. 7: Variation of the velocity component *f'* with *R*

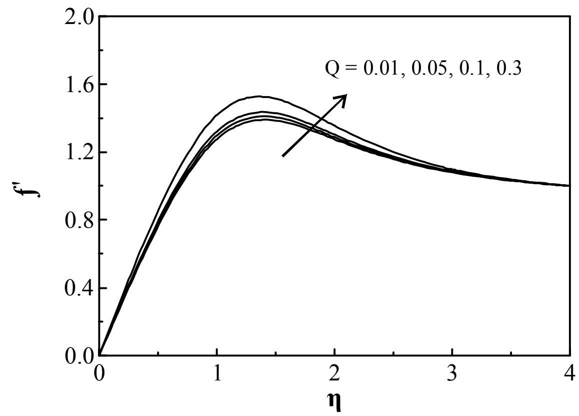

Fig. 8: Variation of the velocity component *f'* with *Q*

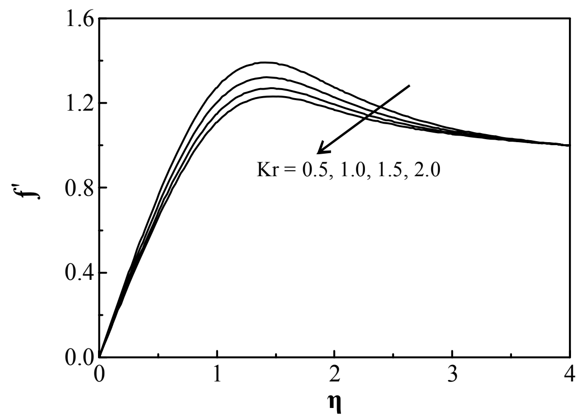

Fig. 9: Variation of the velocity component *f'* with *Kr*

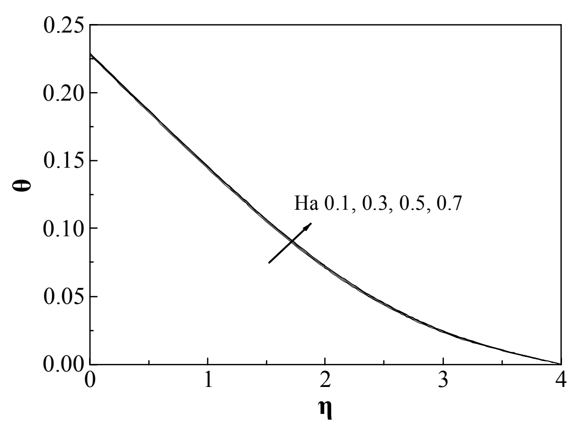

Fig. 10: Variation of the temperature θ with *Ha*

Fig. 11 illustrates the temperature profiles for different values of permeability parameter. The numerical results show that the effect of increasing values of permeability parameter, temperature profiles are increases. Fig. 12 illustrates the velocity profiles for different values of Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing temperature profiles. Also, it is shown that an increase in the Prandtl number results tends to a decreasing of the thermal boundary layer and in general, it lowers the average temperature through the boundary layer. The reason is that, the smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers, the thermal boundary layer is thicker and the rate of heat transfer is reduced. For different values of the radiation parameter *R,* temperature profiles are shown in Fig. 13.

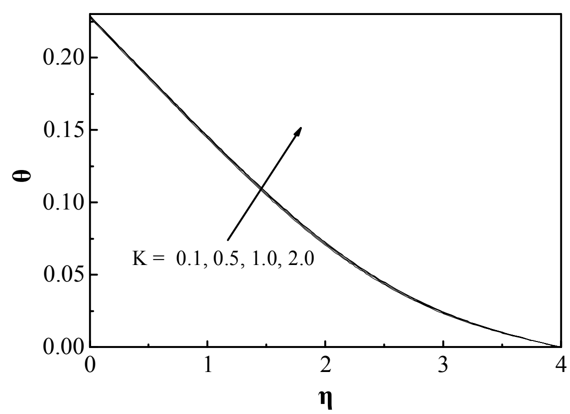

Fig. 11: Variation of the temperature θ with K

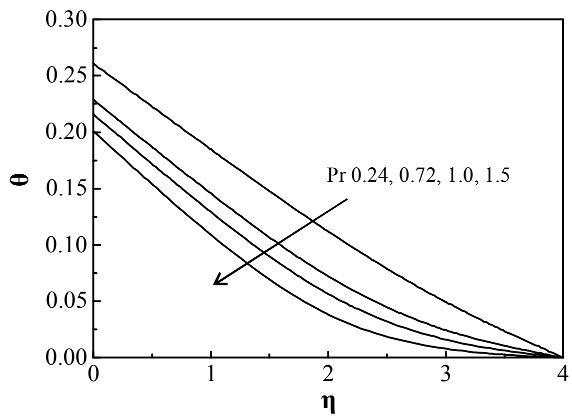

Fig. 12: Variation of the temperature θ with *Pr*

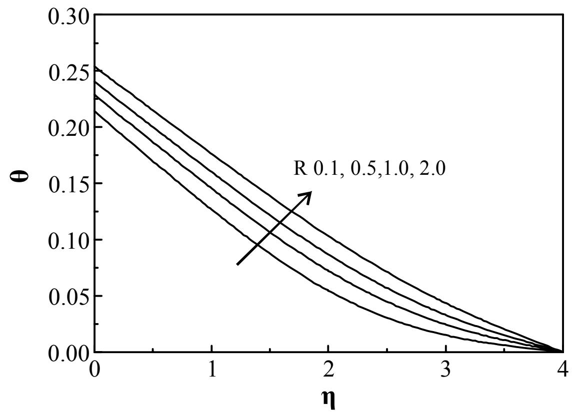

Fig. 13: Variation of the temperature θ with *R*

It is noticed that an increase in the radiation parameter results an increase in the temperature within the boundary layer, also it increases the thickness of the temperature boundary layers. Fig. 14 illustrates the temperature profiles for different values of convective heat parameter. Further, it can be seen that temperature profile increases due to increase of heat source parameter. The thermal boundary layer thickness increases with an increase in the plate surface convective heat parameter.

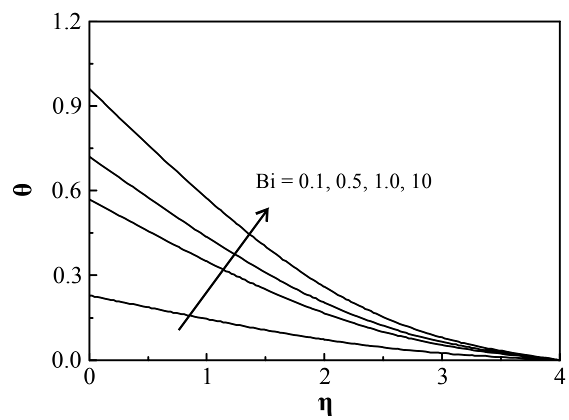

Fig. 14: Variation of the temperature θ with *Q*

CONCLUSION

The present numerical study has been carried out for heat and mass transfer of MHD flow over a moving vertical porous plate in presence of heat source and chemical reaction along with convective surface boundary condition. The shooting method with Runge-Kutta fourth-order iteration scheme has been implemented to solve the dimensionless velocity, thermal and mass boundary layer equations. It has been shown that the local Nusselt number increases whereas the plate surface temperature and Sherwood number decreases with an increase in source parameter. The increase in the strength of chemical reacting substances causes an increase in the plate surface temperature, and Sherwood number, but opposite behavior is seen for local Nusselt number. The velocity profile decreases by increasing the magnetic parameter and even the increase is more prominent with the increase in source and chemical reaction parameter. The thermal boundary layer thickness increases with the increase of source, chemical reaction parameter, plate surface convective heat parameter, and Schmidt number while the mass flux boundary layer thickness decreases. Moreover, the thermal boundary layer thickness, the mass boundary layer, and velocity decrease as the Prandtl number increases.

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