



DETERMINATION OF ATTRIBUTES PERMANENT INDEX ON A COMPRESSION IGNITION ENGINE USING GRAPH THEORY MATRIX APPROACH

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ABSTRACT

In this paper, a systematic method of Graph theory matrix approach is adopted ingeniously to find the optimal combination of operating parameters on a compression ignition engine. For quick appraisal, the Performance attributes digraph is developed to represent the attributes and their relative importance. For one-to-one representation of the attributes, Matrix method is adopted. Permanent function is used to characterize the structure of the matrix. A computer program is developed to find the parameter index from the permanent function. The results of Graph theory matrix approach are compared with other multi attribute decision making methods like Simple additive weighting, Weighted products method and Analytic hierarchy process.

Key words: GTMA, Digraph, Matrix, Permanent function, SAW, WPM, AHP.

INTRODUCTION

Diesel engine enjoy importance among the internal combustion engines because of relatively better fuel economy, sturdy operation, reduced Hydro carbon (HC), Carbon monoxide (CO) emissions, higher Oxides of Nitrogen (NO_x) and Particulate matter (PM) when compared with gasoline engines. Kweonha and Byung-Hyun¹ stated that compression ignition engines employ a high pressure fuel injection to improve fuel efficiency and reduce harmful emissions. The higher nozzle opening pressure results in increase of maximum fuel pipe pressure and shorter combustion duration which increases the brake thermal efficiency of the engine as suggested by Shin et al.,² Ha et al.³ proposed that the combustion characteristics were greatly influenced for a complete open throttle ratio with early injection

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timing and for a partial throttle ratio with late injection timing. With the control of the fuel injection timing and varying the ratio of the throttle opening, the emissions are reduced. When compared to standard injection timing longer delay period, higher cylinder pressure, higher heat release rate and shorter combustion duration were observed at advanced injection timing as shown by Saravanan⁴. De and Panua⁵ showed that at higher compression ratios, the diesel engine gives the best performance in-terms of thermal efficiency, exhaust gas temperature. The experimental results of Santhosh and Padmanaban⁶, showed that, on a diesel engine, the brake thermal efficiency increases as the compression ratio increases either with diesel fuel or ethanol diesel blends.

Literature review

Many different state of the art methods were proposed in the literature to solve multiple attribute decision making(MADM) problems like Analytic Hierarchy Process(AHP), Analytic Network Process(ANP), The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Preference ranking Organization Method for Enrichment Evaluation (PROMETHEE), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR), ELimination Et Choix Traduisant la Realité (ELECTRE), Grey Relational Analysis (GRA), The Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP), etc.⁷ Venkatasamy and Agrawal⁸ proposed GTMA for structural analyses of automobile in-terms of their main components, subsystems, and elements. The optimal structural design of an automobile system was selected by evaluating the performance index. Wani and Gandhi⁹ adopted GTMA for evaluation of machinability index of mechanical systems. In modeling the digraph, they used accessibility, disassembly, standardization, simplicity, identification parameters as attributes and the maintainability index was quantified using permanent function. Grover et al.¹⁰ used human factors, behavioral factors, use of tools, non-behavioral factors and functional areas as attributes and their inter-relations were developed. In their work, GTMA was adopted to evaluate the Total Quality Management (TQM) index to quantify the degree of TQM concepts implementation in an industry. Rao and Padmanabhan¹¹ employed GTMA for selection of robot for manufacturing application. The permanent index was calculated considering attributes like purchase cost, load capacity, velocity, repeatability, number of degrees of freedom and man-machine interface. Upadhyay¹² proposed GTMA for analyses of object oriented software systems to avoid the pitfalls in the quality of software development cycle. Darvish et al.¹³ adopted GTMA in selecting a most suitable contractor for a given construction project. Attributes like work experience, technology and equipment, experience, financial stability, quality, reputation were considered in developing the selection index. Samantary et al.¹⁴ proposed GTMA based feeder routing in power distribution network the reliability index, the optimal radial networks and optimal radial path were assessed. Zhang et al.¹⁵ adopted

GTMA for stability of multi group models with dispersal. In their work, single graph based method was generalized into multi-digraph by constructing a Lyapunov function for Multi Group Models with Dispersal (MGMD). Znakis et al.¹⁶ compared the performance of eight MADM methods like ELECTRE, TOPSIS, Multiplicative Exponential Weighting (MEW), SAW and observed that all versions of MADM methods behaved similarly and closer values of ranks were obtained. Hajkowicz and Higgins¹⁷ employed various MADM methods to water management decision problems and showed that MADM methods were in strong agreement with high correlations amongst rankings.

In this paper, GTMA is used to find the optimal combination of operating parameters on a single cylinder diesel engine by varying Load, Fuel injection timing and Fuel injection pressure. The results of GTMA are compared and analyzed with other MADM methods like SAW, Weighted Products Method (WPM) and AHP.

EXPERIMENTAL

Materials and methods

Experimental setup

The engine used in this work was a four stroke single cylinder diesel engine of Kirloskar make. The eddy current dynamometer was coupled to the engine for loading. The set-up was provided with necessary instruments for measuring combustion pressure and crank angle. Various sensors are connected to the setup and they are interfaced to a computer. The experimental setup is shown in Fig. 1 and the specifications of the engine are shown in Table 1.



Fig. 1: Experimental setup

Experimental procedure

All the tests were conducted at a constant speed of 1500 rpm and varying the load as 4 Ampere (A), 13A and 18A. The fuel injection pressure was varied as 200 bar, 220 bar and 240 bar. The fuel injection timing was varied as 19⁰bTDC (before Top Dead Center), 23⁰bTDC and 27⁰bTDC. For every reading, the engine was run for five minutes to attain steady state. The performance and emission parameters were calculated and shown in Table 2.

Table 1: Specifications of the engine

S. No.	Component	Specification
1	Make	Kirloskar Engines Ltd, Pune
2	Type of engine	Four stroke single cylinder water cooled engine
3	Bore and Stroke	87.5 mm & 110 mm
4	Compression ratio	17.5 : 1
5	BHP and rpm	4.4kW & 1500 rpm
6	Fuel injection pressure	180 N/mm ²
7	Fuel injection timing	21 ⁰ bTDC
8	Dynamometer	Eddy current dynamometer

Table 2: Experimental results

Exp. No.	Factors			Engine performance			Emission characteristics	
	Load (A)	IT (⁰ bTDC)	IP (bar)	BP kW	BSFC kg/h kW	BTE (%)	NOx ppm	HC ppm
1	9	19	200	2.422	0.460	29	235	35
2	9	19	220	2.364	0.454	30	374	34
3	9	19	240	2.401	0.449	30	368	43
4	9	23	200	2.422	0.446	31	230	30
5	9	23	220	2.373	0.453	30	306	34
6	9	23	240	2.401	0.449	30	511	43
7	9	27	200	2.404	0.449	30	468	30
8	9	27	220	2.364	0.454	30	474	34

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Exp. No.	Factors		Engine performance				Emission characteristics	
	Load (A)	IT (⁰ bTDC)	IP (bar)	BP kW	BSFC kg/h kW	BTE (%)	NO _x ppm	HC ppm
9	9	27	240	2.401	0.449	30	511	43
10	13	19	200	3.492	0.404	35	313	33
11	13	19	220	3.369	0.399	36	465	31
12	13	19	240	3.430	0.382	38	416	37
13	13	23	200	3.477	0.398	36	274	27
14	13	23	220	3.395	0.397	36	340	31
15	13	23	240	3.430	0.382	38	591	37
16	13	27	200	3.426	0.402	36	611	27
17	13	27	220	3.369	0.399	36	626	31
18	13	27	240	3.430	0.382	38	591	37
19	18	19	200	4.428	0.388	38	415	33
20	18	19	220	4.340	0.371	40	519	33
21	18	19	240	4.387	0.369	41	478	31
22	18	23	200	4.428	0.397	36	306	32
23	18	23	220	4.364	0.370	40	378	33
24	18	23	240	4.387	0.369	42	606	31
25	18	27	200	4.384	0.383	38	764	29
26	18	27	220	4.340	0.353	44	749	32
27	18	27	240	4.387	0.356	43	662	33

IT: Injection Timing; IP: Injection Pressure

Graph theory matrix approach

Graph theory matrix approach is a systematic and logical approach¹⁸. It consists of the following steps:

- (i) Digraph representation
- (ii) Matrix representation
- (iii) Permanent function representation

Digraph representation

A directed graph is a graph with directed edges. The digraph gives graphical representation of the attributes and their relative importance for a quick visual appraisal¹⁹. In the present work, five attributes, Brake power(BP), Brake specific fuel consumption(BSFC), Brake thermal efficiency(BTE), Nitric oxide(NO_x) and Hydro carbon(HC) are taken as nodes and their inter-dependencies are represented as edges. The performance attributes digraph is shown in Fig. 2.

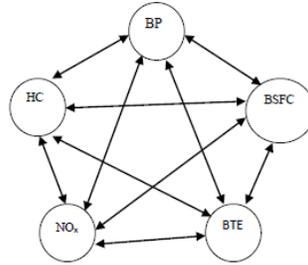


Fig. 2: Performance attributes digraph

In digraph model, the qualitative parameters can be given different numerical values and be made part of the model. To give better appreciation, the inter-dependencies are considered^{16,19}. As the number of nodes and their relative importance increases, the digraph becomes complex. To overcome this difficulty, the digraph is represented in matrix form.

Matrix representation

The one-to-one representation of the attributes in digraph is presented in attributes matrix. It is an M x M matrix which considers all attributes (D_i) and their relative importance (a_{ij}). The Attributes Matrix, P, is shown in Equation 1.

$$P = \begin{bmatrix} D_1 & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & D_2 & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & D_3 & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & D_4 & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & D_5 \end{bmatrix} \quad \dots(1)$$

Where D_i is the normalized value of i^{th} attribute represented by node V_i and a_{ij} is the relative importance of the i^{th} attribute over the j^{th} attribute of edge d_{ij} . Table 3 shows the normalized values of experimental results of Table 2.

Table 3: Normalized values of experimental results

Exp. No.	Factors			Engine performance						Emission Characteristics						Normalized values					
				Engine performance			Emission Characteristics			Engine performance			Emission characteristics								
	Load (A)	IT (°bTDC)	IP (bar)	BP (kW)	BSFC (kg/h kW)	BTE (%)	NOx (ppm)	HC (ppm)	BP (kW)	BSFC (kg/h kW)	BTE (%)	NOx (ppm)	HC (ppm)	BP (kW)	BSFC (kg/h kW)	BTE (%)	NOx (ppm)	HC (ppm)			
1	9	19	200	2.422	0.460	29	235	35	0.547	1.000	0.659	0.979	0.771								
2	9	19	220	2.364	0.454	30	374	34	0.534	0.987	0.682	0.615	0.794								
3	9	19	240	2.401	0.449	30	368	43	0.542	0.976	0.682	0.625	0.628								
4	9	23	200	2.422	0.446	31	230	30	0.547	0.970	0.705	1.000	0.900								
5	9	23	220	2.373	0.453	30	306	34	0.536	0.985	0.682	0.752	0.794								
6	9	23	240	2.401	0.449	30	511	43	0.542	0.976	0.682	0.450	0.628								
7	9	27	200	2.404	0.449	30	468	30	0.543	0.976	0.682	0.491	0.900								
8	9	27	220	2.364	0.454	30	474	34	0.534	0.987	0.682	0.485	0.794								
9	9	27	240	2.401	0.449	30	511	43	0.542	0.976	0.682	0.450	0.628								
10	13	19	200	3.492	0.404	35	313	33	0.789	0.878	0.795	0.735	0.818								
11	13	19	220	3.369	0.399	36	465	31	0.761	0.867	0.818	0.495	0.871								
12	13	19	240	3.430	0.382	38	416	37	0.775	0.830	0.864	0.553	0.730								
13	13	23	200	3.477	0.398	36	274	27	0.785	0.865	0.818	0.839	1.000								

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Exp. No.	Normalized values												
	Factors					Emission Characteristics					Emission characteristics		
	Load (A)	IT (°bTDC)	IP (bar)	BP (kW)	BSFC (kg/h kW)	BTE (%)	NOx (ppm)	HC (ppm)	BP (kW)	BSFC (kg/h kW)	BTE (%)	NOx (ppm)	HC (ppm)
14	13	23	220	3.395	0.397	36	340	31	0.767	0.863	0.818	0.676	0.871
15	13	23	240	3.430	0.382	38	591	37	0.775	0.830	0.864	0.389	0.730
16	13	27	200	3.426	0.402	36	611	27	0.774	0.874	0.818	0.376	1.000
17	13	27	220	3.369	0.399	36	626	31	0.761	0.867	0.818	0.367	0.871
18	13	27	240	3.430	0.382	38	591	37	0.775	0.830	0.864	0.389	0.730
19	18	19	200	4.428	0.388	38	415	33	1.000	0.843	0.864	0.554	0.818
20	18	19	220	4.340	0.371	40	519	33	0.980	0.807	0.909	0.443	0.818
21	18	19	240	4.387	0.369	41	478	31	0.991	0.802	0.932	0.481	0.871
22	18	23	200	4.428	0.397	36	306	32	1.000	0.863	0.818	0.752	0.844
23	18	23	220	4.364	0.370	40	378	33	0.986	0.804	0.909	0.608	0.818
24	18	23	240	4.387	0.369	42	606	31	0.991	0.802	0.955	0.380	0.871
25	18	27	200	4.384	0.383	38	764	29	0.990	0.833	0.864	0.301	0.931
26	18	27	220	4.340	0.353	44	749	32	0.980	0.767	1.000	0.307	0.844
27	18	27	240	4.387	0.356	43	662	33	0.991	0.774	0.977	0.347	0.818

The values of relative importance between two attributes (a_{ij}) are also assigned on the scale of 0 to 1²⁰. The relative importance between i,j and j,i is given in Equation 2 as –

$$a_{ji} = 1/ a_{ij} \quad \dots(2)$$

The relative importance values of attributes are shown in Table 4.

Table 4: Relative importance of attributes

S. No.	Description	Relative importance	
		a_{ij}	a_{ji}
1	Two attributes are equally important	0.5	2.000
2	One attribute is slightly more important over the other	0.6	1.666
3	One attribute is strongly more important over the other	0.7	1.428
4	One attribute is very strongly important over the other	0.8	1.250
5	One attribute is extremely important over the other	0.9	1.111
6	One attribute is exceptionally more important over the other	1.0	1.000

Permanent function

The permanent function of the parameter matrix is a standard matrix function used in Combinatorial mathematics¹⁹. The concept of permanent leads to a better appreciation as no negative sign will appear in the expression and hence no information will be lost¹⁷. The parameter index for each experiment is evaluated using the Equation 3.

$$\begin{aligned}
 Per(A) = & \prod_{i=1}^M D_i + \sum_{i=1}^{M-1} \sum_{j=i+1}^M \dots \sum_{M=t+1}^M (d_{ij}d_{ji})D_kD_lD_mD_nD_o\dots D_tD_m \\
 & + \sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^M \dots \sum_{M=t+1}^M (d_{ij}d_{jk}d_{ki} + d_{ik}d_{kj}d_{ji})D_lD_mD_nD_o\dots D_tD_M \\
 & + [\sum_{i=1}^{M-3} \sum_{j=i+1}^M \sum_{k=i+1}^{M-1} \sum_{l=i+2}^M \dots \sum_{M=t+1}^M (d_{ij}d_{ji})(d_{kl}d_{lk})D_mD_nD_o\dots D_tD_M + \\
 & \sum_{i=1}^{M-3} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^M \sum_{l=j+1}^M \dots \sum_{M=t+1}^M (d_{ij}d_{jk}d_{kl}d_{li} + d_{il}d_{lk}d_{kj}d_{ji})D_mD_nD_o\dots D_tD_m]
 \end{aligned}$$

Table 5: Comparison of GTMA with SAW, WPM and AHP

GTMA			SAW			WPM			AHP		
Rank	Exp No.	Parameter index									
1	22	6.099	1	22	5.2780	1	22	0.3603	1	22	146.360
2	13	6.084	2	23	5.2570	2	23	0.3491	2	13	144.643
3	23	5.707	3	19	5.2530	3	19	0.3464	3	4	142.890
4	19	5.581	4	21	5.2490	4	13	0.3427	4	23	142.219
5	21	5.523	5	24	5.2400	5	21	0.3418	5	19	140.920
6	4	5.466	6	13	5.2330	6	20	0.3336	6	21	140.722
7	10	5.428	7	20	5.2320	7	24	0.3324	7	10	139.991
8	14	5.362	8	27	5.2290	8	10	0.3307	8	14	139.482
9	25	5.353	9	26	5.2250	9	14	0.3261	9	1	138.609
10	24	5.305	10	25	5.2220	10	27	0.3253	10	24	138.466
11	20	5.261	11	10	5.2030	11	26	0.3203	11	20	137.638
12	1	5.118	12	14	5.1930	12	25	0.3194	12	25	136.265
13	27	5.098	13	4	5.1700	13	12	0.3144	13	27	136.055

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GTMA			SAW			WPM			AHP		
Rank	Exp No.	Parameter index									
14	26	5.045	14	12	5.1690	14	11	0.3117	14	26	135.718
15	11	4.929	15	11	5.1670	15	4	0.3079	15	16	135.059
16	16	4.903	16	16	5.1670	16	16	0.3056	16	11	134.685
17	12	4.861	17	17	5.1590	17	17	0.3023	17	5	133.600
18	5	4.723	18	1	5.1540	18	1	0.3011	18	12	133.277
19	17	4.627	19	15	5.1460	19	15	0.2993	19	17	131.322
20	15	4.494	20	18	5.1460	20	18	0.2993	20	2	130.247
21	18	4.494	21	5	5.1220	21	5	0.2900	21	7	129.601
22	2	4.459	22	2	5.1020	22	2	0.2817	22	15	129.074
23	7	4.384	23	7	5.0940	23	3	0.2776	23	18	129.074
24	3	4.210	24	3	5.0900	24	7	0.2765	24	3	126.508
25	8	4.210	25	8	5.0840	25	8	0.2725	25	8	126.209
26	6	3.900	26	6	5.0660	26	6	0.2652	26	6	122.375
27	9	3.900	27	9	5.0660	27	9	0.2652	27	9	122.375

CONCLUSION

The combinatorial mathematics procedure used in Graph theory matrix approach is relatively simple and enables more critical analysis among the attributes. In this approach any number of quantitative and qualitative attributes can be considered. The desirable properties of Graph theory matrix approach are ability to model criteria interactions and ability to generate hierarchical models for modeling and solving complex decision making problems. The decision-making capability of Graph theory matrix approach can be adopted for making decision in any field of science and technology.

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