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COEFFICIENT DIAGRAM: A NOVEL TOOL IN POLYNOMIAL CONTROLLER DESIGN

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ABSTRACT

Design of controllers using the coefficient diagram method is a novel approach and can be implemented to all kind of processes, which can be approximated to FOPTD. In this paper, the various approaches used in time delay approximations are discussed. Coefficient diagram (CD) are drawn for FOPTD processes and second order process. From the CD, time response, stability indices and robustness of the system are analyzed. Polynomial controller has been designed and its control action on the FOPTD process for a servo problem is discussed in detail. The result indicates that polynomial based controller is most successful in the operation of closed loop system.

Key words: Polynomial controller, CD, FOPTD, Stability, Robustness.

INTRODUCTION

Due to the extensive use of control mechanism in various applications, it is essential to design a consistent control system. The conventional control design techniques are used for simple but not for complex systems. Modern control had been developed but there are many difficulties associated with it. The coefficient diagram overcomes these difficulties. Manabe introduced the Coefficient Diagram in 1991. CD is an algebraic approach that is applied to a polynomial loop, in which a coefficient diagram is used as a criterion for good design[1]. Plant transfer function G(S) = N(s)/D/(s) is specified before the design of the controller. The performance of the control system is determined by drawing the Coefficient Diagram and observing the time response, stability and robustness properties[2]. In CD (Fig. 1), the coefficient a_i is read on the left side scale, and the stability index γ_i, equivalent time constant τ, and the stability limit γ_i^* are read by the right hand scale. If the curvature of a_i

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becomes larger, the system becomes more stable corresponding to larger stability index. If the a_i curve is left-end down, the equivalent time constant is small and response is fast. The equivalent time constant (τ_{eq}) is calculated to the desired settling time (t_s) using $\tau = t_s/(2.5 \sim 3)$ and the stability indices are selected as γ_i = (2.5, 2, 2, …, 2) for i = 1 to n-1 according to standard Manabe form[4].

Coefficient diagram stability indices was studied in 1953 by Graham, who proposed the ITAE. This was followed by Kessler in 1960 to decrease the oscillations and overshoot. On comparing the ITAE and Kessler model, it was found that Kessler model was more stable and has 8% overshoot. In Kessler model, all stability indices were chosen as 2 whereas in the CD model, stability indices are selected as [2, 2, 2..., and 2.5]. In the case of CD (Manabe's standard form), the responses are obtained without overshoot and with smallest settling time compared to other methods[5].

Table 2.1: Stability indices of the standard

Forms	n	γ_4	γ_3	γ_2	γ_1
Binomial	2	3	3	3	-
	4	2.5	2	2	2.5
ITAE	2	1.42	2.64	-	-
	4	1.57	1.62	1.78	2.10
Kessler	2	2	2	-	-
	4	2	2	2	2
CD	2	2	2.2	-	-
	4	2	2	2	2.5

In the block diagram (Fig. 2), N(S) is the numerator polynomial of degree m and D(S) is the denominator polynomial of degree n ($m \leq n$)[6]. In case of lag in the system, $e^{-}\theta s$ is represented by first order Pade approximation[7]. $N(s)$ and $D(s)$ are numerator and denominator polynomials of the transfer function of the plant. $A(s)$ is the forward denominator polynomial while $F(s)$ and $B(s)$ are the reference numerator and the feedback numerator polynomials of the controller transfer function, respectively. Since, the transfer function of the controller has two numerators, it resembles to a two degree of freedom (2DOF) system structure. $A(s)$ and $B(s)$ are designed in such a way to satisfy the desired transient behavior. $F(s)$ is determined as zero order polynomial and used to provide the steady-state gain[8].

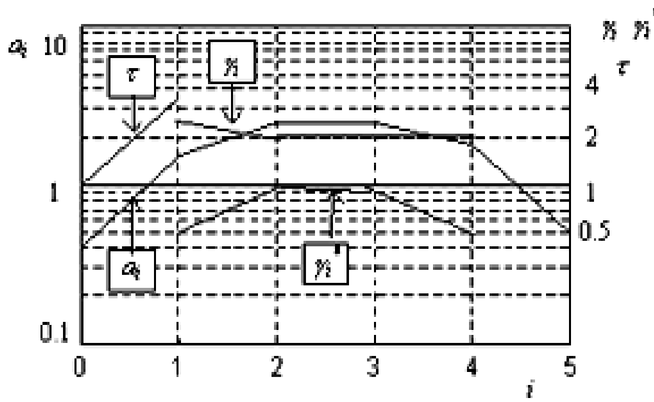

Fig. 1: Coefficient diagram[1]

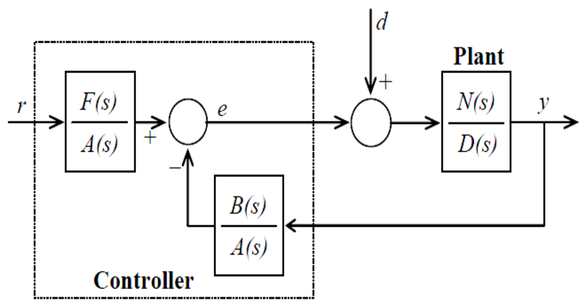

Fig. 2: Closed loop block diagram with polynomial Control System

Therefore, the output of the steady state system is given by –

$$y = \frac{N(s)\,F(s)}{P(s)} r + \frac{A(s)\,F(s)}{P(s)} d \qquad \ldots(1)$$

where P(s) is considered as the characteristic polynomial of the closed-loop system and is defined by –

$$P(s) = A(s)D(s) + B(s)N(s) = \sum_{i=0}^{n} a_i s^i \qquad \ldots(2)$$

At $a_i > 0$ in which $A(s) = \sum_{i=0}^{p} l_i s^i$ and $B(s) = \sum_{i=0}^{q} k_i s^i$

The polynomial CD controller design consists of equivalent time constant ($\tau_{eq} = t_s/2.5$) and stability indices (γi). According to Manabe's standard form, γ_i values are selected as {2.5, 2, 2...2}. Using the design parameters (τ_{eq}, γ_i^*), a target characteristic polynomial is determined as –

$$P_{target}(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\lambda_{i-j}^{j}} \right) \left(\tau_{eq} s\right)^i \right\} + \tau s + 1 \right] \quad \ldots(3)$$

Equating the polynomials represented in the above equations, a Diophantine equation of $A(s)D(s) + B(s)N(s) = P_{t\,arget}(s)$ is obtained. It is then transformed into Sylvester matrix. Solving the algebraic equations, the controller parameters (k_i and l_i) are computed. The CD controller polynomials A(s), B(s) and closed loop characteristic polynomial P(s) are determined using k_i, and l_i.[10] The reference numerator, F(s) is obtained from $F(s) = (P(s)/N(s))|_{s=0}$.

Development of coefficient diagram

Coefficient diagrams are drawn and illustrated with two first order systems with time delay and one second order system without time delay[7].

Example 1: Process transfer function,

$$G(s) = \frac{e^{-0.2\theta}}{0.5\,s + 1} \quad \ldots(4)$$

$$\text{Approximated } G(s) = \frac{2 - 0.2s}{0.1\,s^2 + 1.2\,s + 2} \quad \ldots(5)$$

From the transfer function, $N(s) = 2 - 0.2s$ and $D(s) = 0.1s^2 + 1.2s + 2$ are obtained. It is assumed as $A(s) = l_2 s^2 + l_1 s$, $B(s) = k_2 s^2 + k_1 s + k_0$. According to the standard Manabe form $\gamma = [2\ 2\ 2.5]$. $\tau = 1$ for $t_s = 2.5$ are selected. Target polynomial is found to be $P(s) = 0.008s^4 + 0.08s^3 + 0.4s^2 + s + 1$. Equating the target polynomial to the right side of the Diophantine equation and solving, $A(s) = 0.08s^2 + 0.079s$, $B(s) = 0.12s^2 + 0.47s + 0.5$ and $\gamma_i^* = [0.5\ 0.9\ 0.5]$ are obtained. From the coefficient diagram (Fig. 3) it is noticed that $a_2 > (A+B)$, and $\gamma_i > 1.5\gamma_i^*$; hence, it is reported as the given system is stable and robust.

Example 2: Process transfer function,

$$G(s) = \frac{0.1\,e^{-2s}}{18\,s + 1} \quad \ldots(6)$$

$$\text{Approximated } G(s) = \frac{0.1 - 0.1s}{18\,s^2 + 19\,s + 1} \qquad \ldots(7)$$

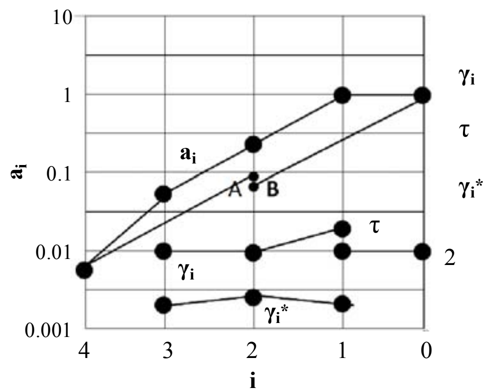

Fig. 3 (a): CD (Example 1)

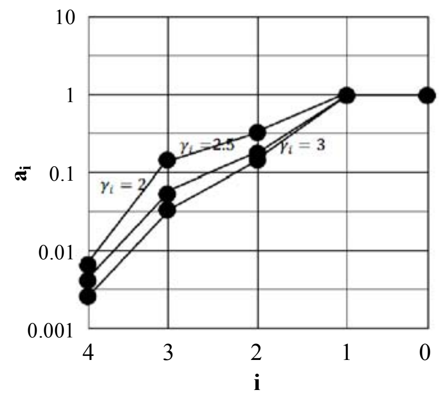

Fig. 3 (b): Effect of γ_i (Example 1)

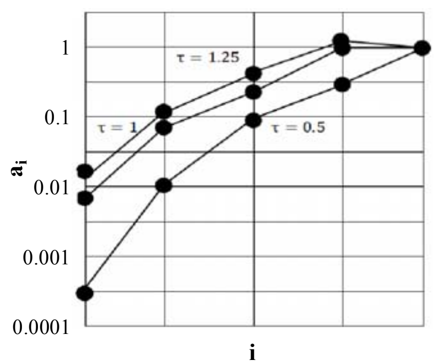

Fig. 3 (c): Effect of τ (Example 1)

From the transfer function, $N(s) = 0.1 - 0.1s$ and $D(s) = 18s^2 + 19s + 1$ are obtained. It is assumed as $A(s) = l_2s^2 + l_1s$ and $B(s) = k_2s^2 + k_1s + k_0$. According to the standard Manabe form $\gamma = [2\ 2\ 2.5]$ and $\tau = 1$ for $t_s = 2.5$ are selected. Target polynomial is found to be $P(s) = 0.008s^4 + 0.08s^3 + 0.4s^2 + s + 1$. Equating the target polynomial to the right side of the Diophantine equation and solving, $A(s) = 4.4*10^{-4}s^2 + 0.065s$, $B(s) = 10.99s^2 + 19.35s + 10$ and $\gamma_i^* = [0.5\ 0.9\ 0.5]$ are obtained. From the coefficient diagram (Fig. 4) it is noticed that $a_2 > (A+B)$, and $\gamma_i > 1.5\gamma_i^*$; hence, it is reported as the given system is stable and robust.

Example 3: Process transfer function,

$$G(s) = \frac{8s^2 + 18s + 32}{s^3 + 6s^2 + 14s + 24} \qquad \ldots(8)$$

From the transfer function, $N(s) = 8s^2 + 18s + 32$ and $D(s) = s^3 + 6\,s^2 + 14s + 24$ are obtained. It is assumed as $A(s) = l_3s^3 + l_2s^2 + l_1s$ and $B(s) = k_3s^3 + k_2s^2 + k_1s + k_0$. According to the standard Manabe form, $\gamma = [2\ 2\ 2\ 2\ 2.5]$ and $\tau = 1$ for $t_s = 2.5$ are selected. Target polynomial is found to be $P(s) = 1 \times 10^{-5}s^6 + 4 \times 10^{-4}s^5 + 0.008s^4 + 0.08s^3 + 0.4s^2 + s + 1$. Equating the target polynomial to the right side of the Diophantine equation and solving, $A(s) = 10^{-5}s^3 - 0.017s^2 - 9.0625 \times 10^{-3}s$, $B(s) = 2.166 \times 10^{-3}s^3 + 7.7185 \times 10^{-3}s^2 + 6.875 \times 10^{-3}s + 0.03125$ and $\gamma_i^* = [0.5\ 1\ 1\ 0.9\ 0.5]$ are obtained. From the coefficient diagram (Fig. 4), it is noticed that $a_2 > (A+B)$, and $\gamma_i > 1.5\gamma_i^*$; hence, it is reported as the given system is stable and robust.

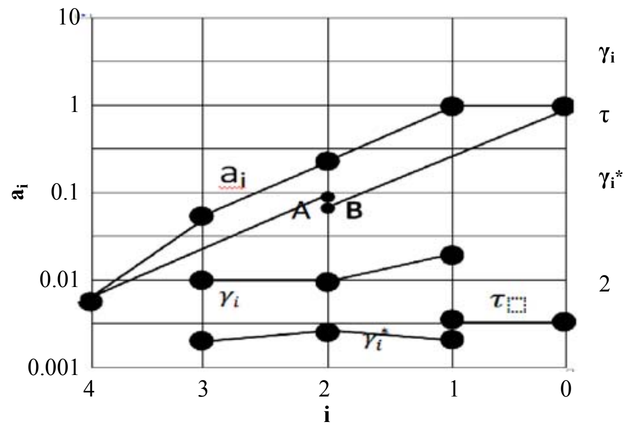

Fig. 4 Coefficient diagram of example: 2

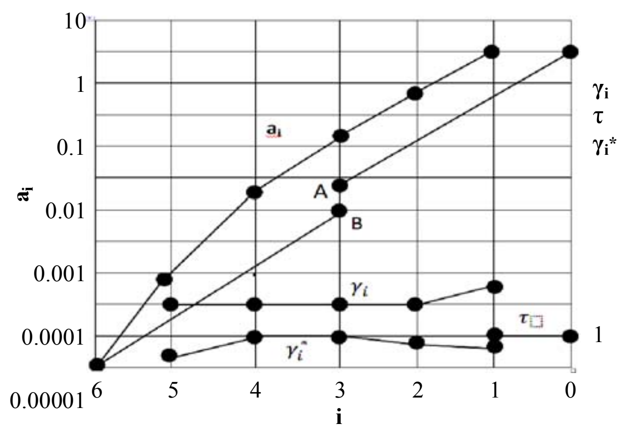

Fig. 5: Coefficient diagram of example

The effect γ and τ are studied in Fig. 3b and Fig. 3c respectively. In Fig. 3, the curvature of a_i is larger for $\gamma_i = 2.5$ than others and hence, the system is more stable. Even though, the a_i curvature for $\tau = 0.5$ and $\tau = 1$ are left end down, the curvature of a_i for $\tau = 1$ is larger than $\tau = 0.5$ comparatively. Hence, the system with $\tau = 1$ has given better response[8].

CD polynomial controller

Polynomial CD controller is designed (Fig. 2) for a FOPTD process (Example 1). Approximation of dead time in the given process (Table 3) is done using Pade approximation, numerator approximation and denominator approximation. Approximated transfer function models are simulated in closed loop with step input and the responses are recorded (Fig. 9). The step responses indicates that Pade approximation[9] is most suitable method for the given system. Performance of the PN CD controller is analyzed using step response analysis (Fig. 6). The effect of stability indices and equivalent time constant on the system performance (Y(t)) are investigated (Fig. 7, Fig. 8 and Table 2). It is observed that the stability index of 2.5 and equivalent time constant of 1 performed better than others[3].

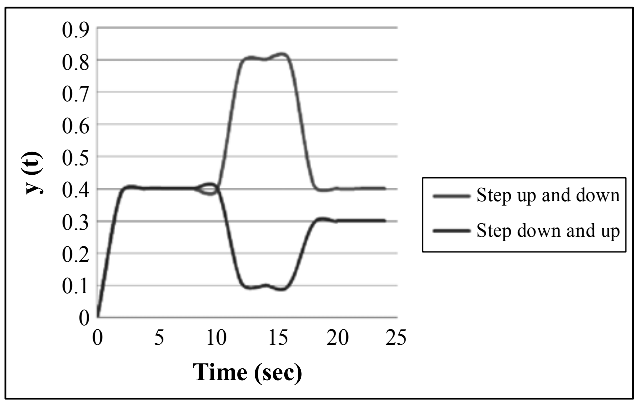

Fig. 6: Step up and step down response of example 1 with Pn controller

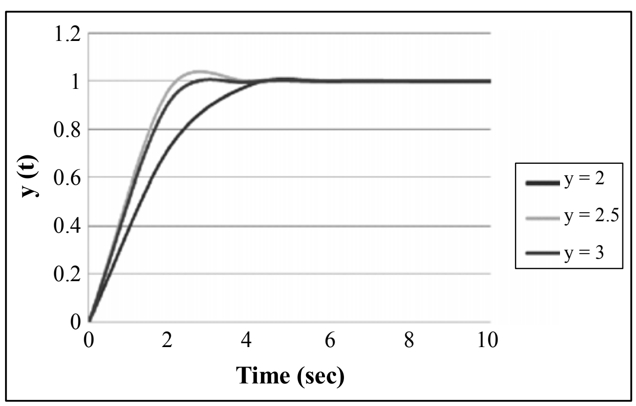

Fig. 7: Effect of stability indices of example 1 with Pn controller

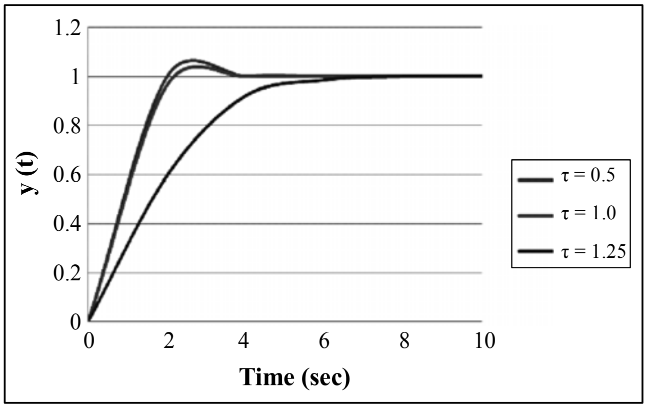

Fig. 8: Effect of equivalent time constant of example 1 with Pn controller

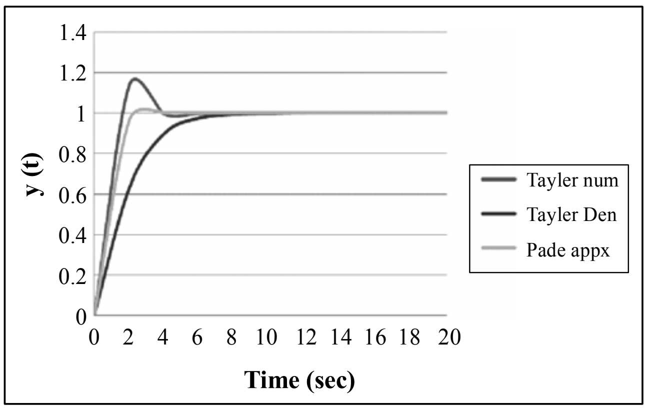

Fig. 9: Effect of dead time approximation of example 1 with Pn controller

Table 2: Effect of equivalent time constant and stability index of example 1

Equivalent time constant and stability index		**A(s)**	**B(s)**
Stability index (γ_1) = 2.5	τ = 1.25	$0.195s^2 + 0.171s$	$0.305s^2 + 0.85s + 0.5$
	τ = 1.0	$0.08s^2 + 0.079s$	$0.12s^2 + 0.47s + 0.5$
	τ = 0.5	$0.005s^2 + 0.079s$	$0.02s^2 + 0.22s + 0.5$
Time constant (τ) = 1	γ_1 = 2.0	$0.156s^2 – 0.135s$	$0.2435s^2 + 0.685s + 0.5$
	γ_1 = 2.5	$0.08s^2 + 0.079s$	$0.12s^2 + 0.47s + 0.5$
	γ_1 = 3.0	$0.046s^2 – 0.153s$	$0.052s^2 + 0.398s + 0.5$

Table 3: Time delay approximation of example 1

Transfer function	Pade approximation	Tayler's numerator approximation	Tayler's denominator approximation
G(s)	$2\text{-}0.2S/(0.1S^2 + 1.2S + 2)$	$1\text{-}0.2S/(0.5S + 1)$	$1/(0.5s^2 + 0.7s + 1)$

CONCLUSION

In this paper, coefficient diagram (a novel tool) is drawn and discussed with three examples. The approaches used in the approximations of the time delay process are discussed and observed that Pade approximation is most suitable. The most important characteristic properties of the system namely time response, stability indices and robustness are recorded in a single and simple coefficient diagram. The effect of stability indices and equivalent time constant on the system performance are analyzed using coefficient diagram as well as polynomial CD controller based closed loop response. It is noticed that the designed polynomial controller exhibits better performance for a servo problem.

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