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APPLICATION OF GRAPH THEORY MATRIX APPROACH TO SELECT OPTIMAL COMBINATION OF OPERATING PARAMETERS ON DIESEL ENGINE TO REDUCE EMISSIONS

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ABSTRACT

The objective of this paper is to adopt graph theory matrix approach to find the optimal combination of operating parameters on a diesel engine. Based on the engine performance parameters like brake power, brake specific fuel consumption and brake thermal efficiency, the combination of 18A load, 27^0bTDC injection timing and 200 bar Injection pressure gives the best result. Based on the emission parameters like nitric oxide, hydro carbon, carbon monoxide, carbon dioxide and oxygen, the combination of 9A load, 19^0bTDC injection timing and 240 bar injection pressure gives the best result out of the engine.

Key words: Digraph, Matrix, Permanent function, Permanent index, MADM, GTMA.

INTRODUCTION

Graph theory is the study of graphs. A graph is a collection of nodes or vertices and a collection of edges that connect pairs of vertices and is a branch of graph theory that applies topology and geometry to derive two dimensional representations of graphs. A drawing of a graph is basically a pictorial representation usually aimed at visualization of certain properties of process or a system modeled by the graph. A straight-line embedding of a graph G = (V, E) suggested by Geetha[1] is an injective function $\pi: V \rightarrow R2$ such that for any two distinct edges ab and cd the straight line segments π(a) π(b) and π(c) π(d) are internally disjoint (i.e., they may only intersect at their endpoints).

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Graph theory matrix approach (GTMA) is used in modeling and solving a decisionmaking problem with multiple and interrelated attributes. Gandhi and Agrawal[2] proposed graph theory matrix approach to find the reliability of a mechanical and hydraulic system. Rao[3] employed GTMA for developing a performance evaluation system for technical education institutions, which is used for ranking of the technical institutions. Grover and Agrawal[4] developed a TQM index to quantify degree of TQM concepts implementation in an industry. Kaur et al.[5] proposed a supply chain coordination index to evaluate several coordination mechanisms. Upadhyay[6] proposed a systematic procedure for analyses of object oriented software systems, which is useful to avoid pitfalls in the quality of software development life cycle. Graph theory has served an important purpose in the modeling of systems, network analysis, functional representation, conceptual modeling, and diagnosis etc. Graph theory has proved its mettle in various fields of science and technology as suggested by Rao[7]. In the present study, Graph theory matrix approach is adopted to find the optimal combination of operating parameters i.e., load, injection pressure and Injection timing on a diesel engine. This is attained by considering the performance parameters like Brake power (BP), Brake specific fuel consumption (BSFC), Brake thermal efficiency (BTE) and emission parameters like oxides of nitrogen (NO_x), carbon monoxide (CO), hydrocarbon (HC), carbon dioxide (CO_2), and oxygen (O_2) as worked by Bridjesh[8-10]. To meticulously analyze the above parameters and their effect, a mathematical model is needed to correlate different factors, which give accurate results.

General Methodology of Graph Theory Matrix Approach

Graph theory helps to analyze and understand the system as a whole by identifying the system and subsystem up to the component level. So, it is a versatile tool that has been used in various applications compiled by Singh et al.[11] The mathematical model developed by graph theoretic approach considers both the contribution of attributes itself and the extent of dependence among the attributes. Digraph representation is useful for modeling and visual analysis. Matrix representation is useful in analyzing the digraph model. Permanent function characterizes the system. Permanent function index represents the unique number useful for comparison, ranking and optimum combination selection. The graph theory matrix approach is divided into three parts:

(i) Digraph representation

(ii) Matrix representation

(iii) Permanent function representation

Digraph representation

Digraph is a finite set of objects called vertices together with a finite set of directed edges or arcs, which are ordered pairs of vertices[12]. In the present work, digraph represents the attributes and their inter dependencies in terms of nodes and edges. This digraph consists of a set of nodes V = {v_i}, with I = 1,2,3,…..M and a set of edges D = {d_{ij}}. If a node *i* has a relative importance over another node *j*, a directed edge or arrow is drawn from node *i* to *j* (d_{ij}). If node *j* has a relative importance over *i*, then a directed edge is drawn from node *j* to *i* (d_{ji}) proposed by Rao and Gandhi[13]. Fig. 1 shows the digraph of performance and emission parameters. It consists of 8 nodes, which represent BP, BSFC, BTE, NO_x, CO, HC CO_2 and O_2 and their interdependence. As the number of nodes and their relative importance increases, the digraph becomes complex.

Matrix representation

Matrix representation of a digraph is developed to reduce its complexity if the number of nodes is more and also matrix representation presents a one-to-one relationship between attributes and their relative importance. If the digraph contains M nodes, the attribute's matrix is of size NxN. The attributes are represented as diagonal elements R_i and the relative importance between attributes is represented as off diagonal elements a_{ij}. The attributes matrix for the digraph is given in Eq. (1).

$$A = \begin{bmatrix} R_1 & a_{12} & a_{13} & \dots & \dots & a_{1m} \\ a_{21} & R_2 & a_{23} & \dots & \dots & a_{2m} \\ a_{31} & a_{32} & R_3 & \dots & \dots & a_{3m} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & \dots & R_m \end{bmatrix} \quad \dots(1)$$

Permanent function representation and permanent index

The permanent of a matrix is a polynomial in the entries of matrix. It is interpreted as the sum of weights of cycle covers of a directed graph and as the sum of weights of perfect matching in a bipartite graph suggested by Muir and Thomas[14]. Permanents are used in combinatorial mathematics as no information is lost due to the positive signature of the permutations. The permanent function for a matrix shown in Eq. (1) is written as:

$$\begin{aligned}\mathrm{Per}(A) = {} & \prod_{i=1}^{M} S_i + \sum_i\sum_j\sum_k \cdots \sum_m (a_{ij}a_{ji}) R_k R_l \ldots R_m + \sum_i\sum_j\sum_k \cdots \sum_m (a_{ij}a_{ji}a_{ki} + a_{ik}a_{kj}a_{ji}) R_l R_m \ldots R_m \\ & + \left(\sum\sum\sum \cdots \sum (a_{ij}a_{ji})(a_{kl}a_{lk}) R_m R_n \ldots R_m + \sum_i\sum_j\sum_k \cdots \sum_m (a_{ij}a_{jk}a_{kl}a_{li} + a_{il}a_{lk}a_{kj}a_{ji}) R_m R_n \ldots R_m \right) \\ & + \left(\sum_i\sum_j\sum_k \cdots \sum_m (a_{ij}a_{ji})(a_{kl}a_{lm}a_{mk} + a_{km}a_{ml}a_{lk}) R_n R_o \ldots R_m + \sum_i\sum_j\sum_k \cdots \sum_m (a_{ij}a_{jk}a_{kl}a_{lm}a_{mi} + a_{im}a_{ml}a_{lk}a_{kj}a_{ji}) R_n R_o \ldots R_m + \ldots\ldots \right) \quad \ldots(2)\end{aligned}$$

The values of R_i are obtained from experimental results, which are objective. These objective values will have different units. So, they are to be normalized on the same scale as the subjective values i.e., 0 to 1. The relative importance (a_{ij}) between attributes can also be assigned a value between 0 and 1 on scale shown in Table 1 proposed by Adil Baykasoglu[15]. The values of a_{ji} are calculated using the following Eq.

$$a_{ji} = 1 - a_{ij} \text{ or } a_{ji} = 1/a_{ij} \quad \ldots(3)$$

Table 1: Relative importance of attributes

Description	Relative importance		
	a_{ij}	**$a_{ji} = 1/a_{ij}$**	**$a_{ji} = 1-a_{ij}$**
Two attributes are equally important	0.5	2.000	0.5
One attribute is slightly more important over the other	0.6	1.666	0.4
One attribute is strongly more important over the other	0.7	1.428	0.3
One attribute is very strongly important over the other	0.8	1.250	0.2
One attribute is extremely important over the other	0.9	1.111	0.1
One attribute is exceptionally more important over the other	1.0	1.000	0.0

The parameter index for each experiment is evaluated using Eq. (2) and tabulated in the descending or ascending order to rank them. The experiment, for which the permanent index is highest, is the best or optimal combination of operating parameters.

Application of graph theory matrix approach

Considering performance parameters

Graph theory matrix approach is adopted to find the optimal combination of operating parameters i.e., load, injection timing and Injection pressure based on the performance parameters i.e., brake power, brake specific fuel consumption and brake thermal efficiency.

Digraph representation

The attributes for the representation of digraph are BP, BSFC and BTE, which are represented as nodes 1, 2 and 3, respectively. The attribute BP is influenced by other attributes BSFC and BTE. Hence, directed edges are drawn from node 1 to the other nodes 2 and 3. The attribute BSFC is also dependent on BP and BTE. Hence, directed edges are drawn from node 2 to other nodes 3 and 1. The attribute BTE is also influenced by BP and BSFC. Hence, directed edges are drawn from node 3 to nodes 1 and 2. The performance attributes digraph is shown in Fig. 1.

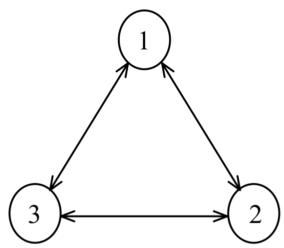

Fig. 1: Performance parameter digraph

Matrix representation

The performance parameter attributes matrix, B, for the digraph is shown in Eq. (4). It is a 3 x 3 matrix with diagonal elements R_i, which represents the attributes and off diagonal elements a_{ij} represents the relative importance.

$$B = \begin{bmatrix} R_1 & a_{12} & a_{13} \\ a_{21} & R_2 & a_{23} \\ a_{31} & a_{32} & R_3 \end{bmatrix} \quad \ldots(4)$$

The values of R_1, R_2 and R_3 are taken from Table 2, which are normalized between 0 and 1. The values of a_{12}, a_{13}, a_{21}, a_{23}, a_{31} and a_{32} are taken from Table 1.

Table 2: Normalized values of attributes

Exp. No.	Operating parameter			Engine performance values			Normalized performance values		
	Load	IT	IP	BP (kW)	BSFC (kg/h kW)	BTE (%)	BP (kW)	BSFC (kg/h kW)	BTE (%)
1	9	19	200	2.322	0.298	27	0.039	0.907	0.000
2	9	19	220	2.264	0.304	28	0.000	0.954	0.082
3	9	19	240	2.301	0.299	28	0.025	0.915	0.082
4	9	23	200	2.322	0.296	29	0.039	0.891	0.162
5	9	23	220	2.273	0.303	28	0.006	0.946	0.082
6	9	23	240	2.301	0.299	28	0.025	0.915	0.082
7	9	27	200	2.304	0.299	28	0.027	0.915	0.082
8	9	27	220	2.264	0.304	28	0.000	0.954	0.082
9	9	27	240	2.301	0.299	28	0.025	0.915	0.082
10	13	19	200	3.392	0.254	33	0.624	0.529	0.454
11	13	19	220	3.269	0.249	34	0.567	0.482	0.522
12	13	19	240	3.330	0.232	36	0.595	0.315	0.651
13	13	23	200	3.377	0.248	34	0.617	0.473	0.522
14	13	23	220	3.295	0.247	34	0.579	0.463	0.522
15	13	23	240	3.330	0.232	36	0.595	0.315	0.651
16	13	27	200	3.326	0.252	34	0.594	0.511	0.522
17	13	27	220	3.269	0.249	34	0.567	0.482	0.522
18	13	27	240	3.330	0.232	36	0.595	0.315	0.651
19	18	19	200	4.328	0.238	36	1.000	0.376	0.651
20	18	19	220	4.240	0.221	38	0.968	0.201	0.773
21	18	19	240	4.287	0.219	39	0.985	0.179	0.832
22	18	23	200	4.328	0.247	34	1.000	0.463	0.522
23	18	23	220	4.264	0.220	38	0.977	0.190	0.773
24	18	23	240	4.287	0.219	40	0.985	0.179	0.890
25	18	27	200	4.284	0.233	36	0.984	0.326	0.651
26	18	27	220	4.240	0.203	42	0.968	0.000	1.000
27	18	27	240	4.287	0.206	41	0.985	0.035	0.945

Permanent function representation and permanent index

The permanent function for the matrix shown in Eq. (4) is given in Eq. (5).

$$\text{Per (B)} = \prod_{i=1}^{3} R_i + \sum_{ijk} (a_{ij} * a_{ji}) R_k + \sum_{ijk} (a_{ij} * a_{jk} * a_{ki} + a_{ik} * a_{ij} \; * a_{ji}) \quad \ldots(5)$$

$$= R_1 * R_2 * R_3 + a_{12} * a_{23} * a_{31} + a_{13} * a_{21} * a_{32} + a_{13} * R_2 * a_{31} + a_{12} * a_{21} * R_3 + R_1 * a_{23} * a_{32}$$

Substituting the values of attributes R_i and their relative importance a_{ij} in Eq. (5), the permanent index is evaluated.

Example: For Experiment No. 1,

$$B = \begin{bmatrix} 0.0390 & 0.5 & 0.5 \\ 2.0 & 0.9068 & 0.8 \\ 2.0 & 1.25 & 0.000 \end{bmatrix}$$

$$\text{Per(B)} = 0.0390*0.9068*0.000 + 0.5*0.8*2.0 + 0.5*2.0*1.25 + 0.5*0.9068*2.0 + 0.5*2.0*0.000 + 0.0390*0.8*1.25$$

$$= 3.0401$$

The permanent index values for 27 experiments are given in descending order in Table 3.

Table 3: Permanent index values and Rank

Exp. No.	Operating parameter			Permanent index	Rank
	Load	IT	IP		
22	9	19	200	4.6144	1
19	9	19	220	4.5465	2
25	9	19	240	4.4115	3
24	9	23	200	4.3248	4
21	9	23	220	4.2675	5
20	9	23	240	4.2354	6

Cont…

Exp. No.	Operating parameter			Permanent index	Rank
	Load	IT	IP		
23	9	27	200	4.2225	7
10	9	27	220	4.0704	8
27	9	27	240	4.0581	9
16	13	19	200	4.0550	10
13	13	19	220	4.0266	11
26	13	19	240	4.0183	12
11	13	23	200	3.9630	13
17	13	23	220	3.9630	14
14	13	23	240	3.9498	15
12	13	27	200	3.8467	16
15	13	27	220	3.8467	17
18	13	27	240	3.8467	18
4	18	19	200	3.1851	19
7	18	19	220	3.1049	20
3	18	19	240	3.1006	21
6	18	23	200	3.1006	22
9	18	23	220	3.1006	23
5	18	23	240	3.0917	24
2	18	27	200	3.0862	25
8	18	27	220	3.0862	26
1	18	27	240	3.0401	27

From Table 3, it can be seen that the Exp. No. 22 has the highest value (4.614) of Permanent index. Hence, the engine operating parameter combination of 18A Load, 27^0bTDC Injection timing and 200 bar Injection pressure gives the best result.

Considering emission parameters

The emission parameters like NO_x, CO, HC, CO_2 and O_2 are considered to find the optimal combination of operating parameters i.e., Load, Injection timing and Injection pressure.

Digraph representation

The attributes chosen for representation of emission parameters digraph are NO_x, CO, HC, CO_2 and O_2. These are represented as nodes 1, 2, 3, 4 and 5 respectively. Based on the interdependence among the attributes, directed edges are drawn from various nodes. The attribute's digraph is shown in Fig. 2.

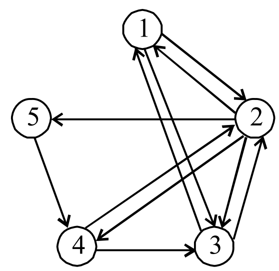

Fig. 2: Emission parameters digraph

Matrix representation

As stated earlier, matrix representation of digraph gives one-to-one representation. The matrix called emission parameter matrix, C, is shown in Eq. (6).

$$C = \begin{bmatrix} R_1 & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & R_2 & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & R_3 & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & R_4 & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & R_5 \end{bmatrix} = \begin{bmatrix} R_1 & a_{12} & a_{13} & 0 & 0 \\ a_{21} & R_2 & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & R_3 & 0 & 0 \\ 0 & a_{42} & a_{43} & R_4 & 0 \\ 0 & 0 & 0 & a_{54} & R_5 \end{bmatrix} \quad \ldots(6)$$

The values of R_1, R_2, R_3, R_4 and R_5 are taken from Table 4, which are normalized between 0 and 1. The values of a_{12}, a_{13}, a_{21}, a_{23}, a_{24}, a_{25}, a_{31}, a_{32}, a_{42}, a_{43} and a_{54} are taken from Table 1.

Permanent function and permanent index

The permanent function for the attribute's matrix shown in Eq. 6 is evaluated using the Eq. (2). A computer program is developed in C++ to compute the values of Permanent index for all experiments and tabulated in descending order in Table 5.

From Table 5, it can be seen that the experiment number 3 has the highest value (6.3750) of permanent index. Hence, based on the engine emission parameters, combination of 9A Load, 19^0 bTDC Injection timing and 240 bar Injection pressure gives the best result.

Table 4: Normalized values of attributes

Exp. No.	Operating parameters			Emission characteristics values					Normalized characteristics values				
	Load	IT	IP	Nox (ppm)	HC (ppm)	CO (%)	CO_2 (%)	O_2 (%)	Nox (ppm)	HC (ppm)	CO (%)	CO_2 (%)	O_2 (%)
1	9	19	200	242	36	0.03	3.3	15.98	0.982	0.444	0.585	0.310	0.654
2	9	19	220	381	35	0.02	3.4	15.92	0.598	0.506	0.000	0.290	0.640
3	9	19	240	375	44	0.02	1.2	17.62	0.611	0.000	0.000	1.000	1.000
4	9	23	200	237	31	0.03	3.5	15.89	1.000	0.775	0.585	0.270	0.634
5	9	23	220	313	35	0.02	3.4	15.92	0.764	0.506	0.000	0.290	0.640
6	9	23	240	518	44	0.02	2.1	17.62	0.337	0.000	0.000	0.618	1.000
7	9	27	200	475	31	0.03	3.5	15.89	0.411	0.775	0.585	0.270	0.634
8	9	27	220	481	35	0.02	3.4	15.92	0.400	0.506	0.000	0.290	0.640
9	9	27	240	518	44	0.02	1.3	17.62	0.337	0.000	0.000	0.945	1.000
10	13	19	200	320	34	0.04	4.0	15.23	0.745	0.570	1.000	0.179	0.483
11	13	19	220	472	32	0.02	4.1	15.11	0.416	0.705	0.000	0.162	0.455
12	13	19	240	423	38	0.02	4.0	15.19	0.509	0.324	0.000	0.179	0.474
13	13	23	200	281	28	0.03	4.1	14.97	0.856	1.000	0.585	0.162	0.422
14	13	23	220	347	32	0.02	4.1	15.11	0.677	0.705	0.000	0.162	0.455
15	13	23	240	598	38	0.02	4.0	15.19	0.215	0.324	0.000	0.179	0.474
16	13	27	200	618	28	0.03	4.1	14.97	0.188	1.000	0.585	0.162	0.422
17	13	27	220	633	32	0.02	4.1	15.11	0.167	0.705	0.000	0.162	0.455
18	13	27	240	598	38	0.02	4.0	15.19	0.215	0.324	0.000	0.179	0.474
19	18	19	200	422	34	0.04	4.6	14.08	0.511	0.570	1.000	0.084	0.205
20	18	19	220	526	34	0.02	4.8	14.04	0.324	0.570	0.000	0.055	0.195
21	18	19	240	485	32	0.02	4.8	14.000	0.393	0.705	0.000	0.055	0.185
22	18	23	200	313	33	0.03	5.2	13.29	0.764	0.636	0.585	0.000	0.000
23	18	23	220	385	34	0.02	4.8	14.04	0.589	0.570	0.000	0.055	0.195
24	18	23	240	613	32	0.02	4.8	14.00	0.194	0.705	0.000	0.055	0.185
25	18	27	200	771	30	0.03	4.9	13.3	0.000	0.847	0.585	0.041	0.003
26	18	27	220	756	33	0.02	4.4	14.9	0.017	0.636	0.000	0.114	0.405
27	18	27	240	669	34	0.02	4.1	14.23	0.120	0.570	0.000	0.162	0.242

Table 5: Permanent index and rank

Exp. No.	Operating parameters			Permanent index	Rank
	Load	IT	IP		
3	9	19	200	6.3750	1
9	9	19	220	5.5426	2
6	9	19	240	4.5296	3
1	9	23	200	4.4256	4
4	9	23	220	4.3335	5
10	9	23	240	3.3389	6
5	9	27	200	3.2095	7
7	9	27	220	3.1504	8
2	9	27	240	2.9975	9
13	13	19	200	2.9456	10
8	13	19	220	2.7460	11
14	13	19	240	2.2960	12
12	13	23	200	2.1766	13
11	13	23	220	2.0383	14
16	13	23	240	1.9126	15
15	13	27	200	1.8782	16
18	13	27	220	1.8782	17
19	13	27	240	1.8296	18
17	18	19	200	1.7924	19
26	18	19	220	1.4743	20
23	18	19	240	1.4441	21
22	18	23	200	1.3437	22
27	18	23	220	1.2955	23
21	18	23	240	1.2919	24
20	18	27	200	1.2636	25
24	18	27	220	1.1585	26
25	18	27	240	0.7389	27

CONCLUSION

In this research, Graph theory matrix approach was used to find the optimal combination of operating parameters on a diesel engine. Based on the engine performance parameters like Brake power, Brake specific fuel consumption and Brake thermal efficiency, the combination of 18A Load, 27^0 bTDC Injection timing and 200 bar Injection pressure forms the optimal combination of operating parameters. Based on the emission parameters like nitric oxide, hydrocarbon, carbon monoxide, carbon dioxide and oxygen the combination of 9A Load, 19^0 bTDC Injection timing and 240 bar Injection pressure forms the optimal combination of operating parameters.

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