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AN EVALUATION OF SUPER FLUID DENSITY (ρ_S/ρ), QUASI PARTICLE CONTRIBUTION ($\rho^F{}_N/\rho$) AND FLUCTUATION CONTRIBUTION ($\rho^B{}_N/\rho$) AS A FUNCTION OF (T/T_C) FOR BCS-BEC CROSSOVER REGIME

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ABSTRACT

We have evaluated ratio of super fluid density to the total density (ρ_s/ρ), Quasi Particle Contribution ($\rho^F{}_n/\rho$) and Fluctuation Contribution ($\rho^B{}_n/\rho$) as a Function of (T/T_c) for three limits BCS, Pseudo gap and BEC. These results are quite useful in the study of dynamical properties in the BCS- BEC crossover region at finite temperature.

Key words: Super fluid density, Quasi particle contribution, Fluctuation contribution, Pseudo gap, Bogoliubov collective excitations.

INTRODUCTION

In earlier paper[1], we have studied the BCS-BEC crossover physics and evaluated energy gap parameter Δ/E_F and chemical potential μ/E_F from BCS theory. In this paper, we have evaluated the super fluid density (ρ_s/ρ), quasi particle contribution ($\rho^F{}_n/\rho$) and fluctuation contribution ($\rho^B{}_n/\rho$) as a function of (T/T_c) for BCS limit [$(K_F\ a_s)^{-1}$ = - 2.0], pseudo gap limit [$(K_F\ a_s)^{-1}$ = 0.0] and BEC limit [$(K_F\ a_s)^{-1}$ = 2.0]. Here, K_F is Fermi wave vector and a_s is the s-wave scattering length.

As we know that in a Fermi gas with a Feshbach resonance (FR), one can tune the strength of the paring interaction by adjusting the threshold energy of FR[2]. The BCS-BEC crossover has been realized by using this unique property[3,4]. Here, if one increases the strength of the paring interaction, the character of super fluidity continuously changes from weak coupling BCS type to strong coupling BEC type of tightly bound cooper pairs[5,6]. In the

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super fluid phase, the super fluid density ρ_s is the most fundamental quantities. The value of ρ_s is always equal to the total carrier density ρ at T = 0, while it vanishes at the super fluid phase transition T_c. These properties are satisfied in both; Fermi and Bose super fluids, irrespective of the strength of the pairing interactions[7]. But there is a crucial difference between ρ_s in a Fermi super fluid and that in a Bose super fluid. In a mean field BCS theory, ρ_s (T > 0) originates from the thermal dissociation of Cooper pairs. The resulting normal fluid density $\rho_n = \rho - \rho_s$ is determined by quasi particle excitations. On the other hand, ρ_n in the Bose super fluid is dominated by Bogoliubov collective excitation[7]. Therefore, it is a very interesting problem to see as to how ρ_s in a Fermi super fluid changes into ρ_s in a Bose super fluid in BCS-BEC crossover.

In this paper, taking theoretical formalism of Ohashi[8] and Ohashi & Griffin[9,10], We have theoretically evaluated the super fluid density in the BCS-BEC crossover. These authors have taken an uniform super fluid Fermi gas at finite temperature and extended the strong coupling Gaussian fluctuation theory for T_c developed by Nozieres and Schmitt-Rink (N.S.R.)[4,5] to super fluid phase below T_c. They self-consistently determined Δ and μ. We have used their formalism to calculate (ρ_s/ρ), $(\rho^F{}_n/\rho)$ and $(\rho^B{}_n/\rho)$ for BCS-BEC crossover.

Mathematical formulae used in the evaluation:

One starts with the BCS Hamiltonian in Nambau representation -

$$H = \frac{\Delta^2}{U}\sum_{\mathbf{p}}\xi_{\mathbf{p}} + \sum_{\mathbf{p}}\psi^+_{\mathbf{p}}[\xi_{\mathbf{p}}\tau_3 - \Delta\tau_1]\psi_{\mathbf{p}} - \frac{U}{4}\sum_{\mathbf{q}}[\rho_{1,\mathbf{q}}\rho_{1,-\mathbf{q}} + \rho_{2,\mathbf{q}}\rho_{2,-\mathbf{q}}] \qquad \ldots(1)$$

Here, one assumes two atomic hyperfine states described by pseudo-spin $\sigma = \uparrow, \downarrow$.

$\Psi_p^+ = (C_{p\uparrow}^+, C_{-p\downarrow})$ is a Nambau field operator. $C_{p\sigma}^+$ is the creation operator of a Fermi atom and τ_j are the Pauli matrices (j = 1,2,3), which act on the particle-hole space.

$\xi_p = \varepsilon_p - \mu = (p^2/2m - \mu)$ is the atomic kinetic energy measured from the chemical potential μ. U is the tunable pairing interaction associated with FR.

$$\rho_{j,\mathbf{q}} = \sum_{\mathbf{p}}\psi^+_{\mathbf{p}+\frac{\mathbf{q}}{2}} \times \tau_j \psi_{\mathbf{p}-\frac{\mathbf{q}}{2}} \, (j = 1,2) \qquad \ldots(2)$$

ρ_{jq} represents the generalized density operator. $\rho_{1\mathbf{q}}$ and $\rho_{2\mathbf{q}}$ describe the amplitude and phase fluctuations of the order parameter Δ, respectively. In equation (1) the interaction is described by the sum of the interaction between amplitude fluctuations ($\rho_{1\mathbf{q}}\ \rho_{1\text{-}\mathbf{q}}$) and the phase fluctuations ($\rho_{2\mathbf{q}}\ \rho_{2\text{-}\mathbf{q}}$).

In the NSR theory[6], T_c is described by the Thouless criterion in the t-matrix approximation. The resulting equation for T_c has the same form as the mean-field BCS gap equation with $\Delta \rightarrow 0$. However, in contrast to the weak- coupling BCS theory (where μ is equal to the Fermi energy E_F), μ remarkably deviates from E_F in the BCS –BEC crossover regime due to strong pairing fluctuations. The NSR theory includes the strong-coupling effect by solving the equation of state within the Gaussian fluctuation approximation[6,7] in terms of pairing fluctuation -

Now one extends the NSR theory to the super-fluid phase below T_c. To calculate Δ, one uses the BCS gap equation.

$$1 = \frac{-4\pi a_s}{m} \sum_{\mathbf{p}} [\frac{\tanh(\beta \frac{E_F}{2})}{2E_{\mathbf{p}}} - \frac{1}{2\xi_{\mathbf{p}}}] \qquad \ldots(3)$$

Where $E_{\mathbf{p}} = [(\varepsilon_{\mathbf{p}} - \mu)^2 + \Delta^2]^{1/2}$ is the single particle excitation spectrum. In equation (3), one eliminates the well known ultraviolet divergence by employing a two-body scattering length a_s.

$$\frac{4\pi a_s}{m} = \frac{-U}{1 - U \sum_{\mathbf{p}}^{p_c} \frac{1}{2\varepsilon_{\mathbf{p}}}} \qquad \ldots(4)$$

Where, a_s is two body scattering length.

To calculate μ, one considers the thermodynamic potential Ω density is given by –

$$\rho = \frac{-\partial \Omega}{\partial \mu} \qquad \ldots(5)$$

Fluctuation contribution to Ω ($= \partial\, \Omega$) is calculated from relevant Feynman diagrams.

Now summing up these diagrams, one obtains[9] total densities

$$\rho = \rho_F{}^0 - \frac{1}{2\beta} \sum_{\mathbf{q},\upsilon_n} \frac{\partial}{\partial \mu} \ln \det[1 - \frac{4\pi a_s}{m} \{\Xi(\mathbf{q}, i\upsilon_n) + \frac{1}{2\varepsilon_p}\}] \qquad \ldots(6)$$

$$\rho_F{}^0 = \sum_p (1 - \frac{\xi_p}{E_p}) \tanh(\frac{\beta E_p}{2}) \qquad \ldots(7)$$

$\rho_F{}^0$ is the number of Fermi atoms in the mean field approximation. In equation (6), the second term describes the fluctuation contribution. $\Xi(q, i\,\omega_m)$ is the matrix correlation function. Π_{ij} is the generalized density correlation function. ν_n is the boson Matsubara frequency. ω_m is the fermion-Matsubara frequency. Super fluid density in the BCS-BEC cross-over is determined as -

$$\rho_s = \rho - \rho_n \qquad \ldots(8)$$

ρ is the total carrier density and it is given by –

$$\rho = \sum_p 1 + \frac{1}{\beta} \sum_{p,\omega_m} Tr[\tau_3 G(p, i\omega_m)] \qquad \ldots(9)$$

$$G(\mathbf{p}, i\omega_m) = G_0(\mathbf{p}, i\omega_m) + G_0(\mathbf{p}, i\omega_m) \sum_s (\mathbf{p}, i\omega_m) G_0(\mathbf{p}, i\omega_m) \qquad \ldots(10)$$

where

$$G(\mathrm{p}, i\omega_m)^{-1} = i\omega_m - \delta\varepsilon_\mathrm{p}\tau_3 + \Delta\tau_1 \qquad \ldots(11)$$

G_0 (p, iω_m) is the matrix single particle thermal Green's function. Σ is the self-energy, which involves corrections to G_o. ρ_n is the well known BCS normal fluid density. ρ_n is calculated for both; boson and fermion.

$$\rho_n^F = \frac{-2}{\beta} \sum_{p,\omega_m} p_z \, \partial \big/ \partial Q \, Tr[Gso(G(\mathbf{p}, i\omega_m)] Q \to 0 \qquad \ldots(12)$$

Here G_0 is replaced by G_{s0}. Super current state is described[11] by order parameter -

$$\Delta(z) = \Delta exp(iQz)$$

Super fluid velocity V_s = Q/2 m

G_s is the matrix single particle Green's function in the super current state.

$$Gso^{-1} = (i\omega_m - Q_{pz} \big/ 2m - \varepsilon_\mathbf{p}\tau_3 + \Delta\tau_1) \qquad \ldots(13)$$

$$\rho_n^F = \frac{-2}{3m}\sum_{\mathbf{p}} p^2 \frac{\partial f(E)_{\mathbf{p}}}{\partial E_{\mathbf{p}}} \quad \ldots(14)$$

Where f(E) is the Fermi-Dirac distribution function. Boson normal density (fluctuation correction) is given by –

$$\rho_n^B = \frac{-2}{\beta}\sum_{\mathbf{p},\omega_m} p_z \, {}^{\partial}\!/_{\partial Q} Tr[Gso(\mathbf{p}, i\omega_m)\sum_s(\mathbf{p}, i\omega_m)Gso(\mathbf{p}, i\omega_m)]Q \mapsto 0 \quad \ldots(15)$$

In the weak-coupling BCS regime $(K_F\, a_s)^{-1} << -1$, pairing fluctuation are weak and one finds that $\rho_n \approx \rho_n^F$ or $\rho_s = \rho - \rho_n^F$. In this regime, equation (12) shows that ρ_n is dominated by the quasi-particle excitations with excitation gap Δ. In the BCS – BEC crossover regime, the chemical potential deviates from the Fermi energy E_F and becomes negative in the strong coupling BEC regime[4-6]. One can calculate the chemical potential μ in the BEC limit, where $(K_F a_s)^{-1} >> 1$. Using equation (3), μ is calculated as –

$$\mu = \frac{-1}{2ma_s^{\,2}} \quad \ldots(16)$$

In BEC regime, the chemical potential μ works as a large expectation gap and therefore, quasi particle excitation as well as ρ_n^F are suppressed. This shows that Cooper pair do not dissociate in the Fermi atoms due to large binding energy.

From equation (15), one can calculate the fluctuation contribution ρ_n^B. This is the dominant term in the strong coupling regime BEC. From (15), one obtains ρ_n^B as –

$$\rho_n^B = \frac{-2}{3M}\sum_{\mathbf{q}} q^2 \frac{\partial n_B(E_q^B)}{\partial E_q^B} \quad \ldots(17)$$

where n_B (E) is the Bose distribution function. M = 2 m is the molecular mass

$$E_q^B = [\frac{q^2}{2m}(\frac{q^2}{2m} + 2\nu_B\,\varphi^2)]^{\frac{1}{2}} \quad \ldots(18)$$

Equation (18) is the Bogoliubov phonon spectrum in a dilute molecular Bose gas with a repulsive interaction $\nu_B = 4\pi\,(2a_s)/M$ and the BCS order parameter $\varphi = (a_s/8\,\pi\, m\Delta)^{1/2}$. In the BEC regime the normal fluid density is dominated by Bogoliubov collective excitations in a molecular Bose super-fluid.

RESULTS AND DISCUSSION

In this paper, we have presented the method of evaluation of ratio of super fluid density and total density $(\frac{\rho_s}{\rho})$ as a function of $(\frac{T}{T_c})$ for BCS limit $[(K_F a_s)^{-1} = -2]$, pseudo gap limit $[(K_F a_s)^{-1} = 0]$ and BEC limit $[(K_F a_s)^{-1} = 2]$. Our theoretical results indicate that $(\frac{\rho_s}{\rho})$ is larger in BEC limit and smaller in BCS limit as a function of $(\frac{T}{T_c})$. $(\frac{\rho_s}{\rho})$ decreases with $(\frac{T}{T_c})$ for all three cases. The results are shown in Table 1. In Table 2; We have presented the method of evaluation of quasi particle contribution $(\frac{\rho_n^F}{\rho})$ as a function of $(\frac{T}{T_c})$ for all the three limits. Our theoretical results indicate that $(\frac{\rho_n^F}{\rho})$ is larger for BCS limit and smaller in pseudo gap limit. In Table 3; we have shown the evaluated results of fluctuation contribution $(\frac{\rho_n^B}{\rho})$ as a function of $(\frac{T}{T_c})$ for the above three limits. Our evaluated results show that fluctuation contribution $(\frac{\rho_n^B}{\rho})$ is smaller for BCS limit $[(K_F a_s)^{-1} = -2]$ and larger for BEC limit $[(K_F a_s)^{-1} = 2]$.

Table 1: An evaluated result of $(\frac{\rho_s}{\rho})$ as a function of $(\frac{T}{T_c})$ for BCS limit $[(K_F a_s)^{-1} = -2]$, pseudo gap $[(K_F a_s)^{-1} = 0]$ and BEC limit $[(K_F a_s)^{-1} = 2]$

$(\frac{T}{T_c})$	$(\frac{\rho_s}{\rho})$		
	BCS $[(K_F a_s)^{-1} = -2]$	**Pseudo gap $[(K_F a_s)^{-1} = 0]$**	**BEC $[(K_F a_s)^{-1} = 2]$**
0.0	1.0	1.0	1.0
0.1	0.975	0.982	0.995
0.2	0.956	0.967	0.977
0.3	0.932	0.955	0.964
0.4	0.897	0.902	0.912
0.5	0.824	0.855	0.866
0.6	0.746	0.797	0.805

Cont…

$(\frac{T}{T_c})$	$(\frac{\rho_s}{\rho})$		
	BCS [$(K_F a_s)^{-1} = -2$]	**Pseudo gap [$(K_F a_s)^{-1} = 0$]**	**BEC [$(K_F a_s)^{-1} = 2$]**
0.7	0.618	0.639	0.652
0.8	0.546	0.568	0.574
0.9	0.348	0.382	0.403
0.95	0.226	0.267	0.288
1.00	0.059	0.122	0.147
1.05	0.002	0.097	0.106

Table 2: An evaluated results of quasi particle contribution $(\frac{\rho_n^F}{\rho})$ as a function of $(\frac{T}{T_c})$ for BCS limit [$(K_F a_s)^{-1} = -2$], pseudo gap [$(K_F a_s)^{-1} = 0$] and BEC limit [$(K_F a_s)^{-1} = 2$]

$(\frac{T}{T_c})$	$(\frac{\rho_n^F}{\rho})$		
	BCS [$(K_F a_s)^{-1} = -2$]	**Pseudo gap [$(K_F a_s)^{-1} = 0$]**	**BEC [$(K_F a_s)^{-1} = 2$]**
0.2	0.004	0.002	0.0
0.4	0.098	0.008	0.0
0.6	0.185	0.086	0.0
0.8	0.274	0.105	0.0
1.0	0.456	0.126	0.0
1.2	0.684	0.149	0.0
1.4	0.756	0.185	0.0
1.5	0.889	0.225	0.0
1.6	0.954	0.246	0.0
1.7	1.038	0.278	0.0
1.8	1.176	0.304	0.0

Table 3: An evaluated results of quasi particle contribution $(\frac{\rho_n^B}{\rho})$ as a function of for $(\frac{T}{T_c})$ BCS limit $[(K_F a_s)^{-1} = -2]$, pseudo gape $[(K_F a_s)^{-1} = 0]$ and BEC limit $[(K_F a_s)^{-1} = 2]$

$(\frac{T}{T_c})$	$(\frac{\rho_n^B}{\rho})$		
	BCS $[(K_F a_s)^{-1} = -2]$	**Pseudo gap $[(K_F a_s)^{-1} = 0]$**	**BEC $[(K_F a_s)^{-1} = 2]$**
0.2	0.002	0.004	0.056
0.4	0.006	0.016	0.089
0.6	0.009	0.108	0.126
0.8	0.012	0.148	0.248
1.0	0.122	0.288	0.336
1.2	0.142	0.326	0.409
1.4	0.167	0.449	0.526
1.5	0.198	0.567	0.678
1.6	0.207	0.659	0.776
1.7	0.226	0.787	0.892
1.8	0.245	0.896	0.967
2.0	0.268	1.052	1.122

From the above calculation, one observes that if one increases the strength of the pairing interaction, BCS –type normal fluid density dominated by quasi – particle excitation changes into BEC type normal fluid density dominated by Bogoliubov collective excitations. As ρ_s plays an important role in two fluid hydrodynamics; these results would be useful in study of dynamical properties in the BCS-BEC crossover region at finite temperature[12-15].

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