Int. J. Chem. Sci.: 12(2), 2014, 618-624
ISSN 0972-768X
www.sadgurupublications.com

AN EVALUATION OF CRITICAL FREQUENCY OF VORTEX IN BOSE-EINSTEIN CONDENSATES AS A FUNCTION OF NUMBER OF ATOMS N

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ABSTRACT

In this paper, we have presented a method of evaluation of critical frequent Ω_c of vortex whose circulation number $\kappa = 1$ confined in a spherical trap with N atoms of ^{87}Rb. We have taken the values of $a_{h0} = 0.791$ μm. Our theoretically evaluated results indicate that Ω_c decreases with N and decrease is more pronounced as one goes to higher N. Our theoretical results are consistent with those of the others theoretical workers.

Key words: Super fluidity, Bose-Einstein condensation (BEC), Critical frequency, Circulation number κ, Phase coherence, Atom laser, Quantized vertex.

INTRODUCTION

Super fluidity is one of the most spectacular consequences of Bose-Einstein condensation. However, the explicit connection between super fluidity and BEC is not trivial and has been the object of a long-standing and deep investigation in the last decades, mainly forties importance in understanding the physics of liquid helium. In macroscopic bodies super fluidity shows up with many peculiar features: absence of viscosity, reduction of the moment of inertia, occurrence of persistent currents, new collective phenomena (second sound, third sound, etc.), quantized vortices and others. Several properties are usually interpreted as coherence effects associated with the phases of the super fluid through $\upsilon_s = (\hbar/m)\ \nabla s$. A major question is to understand whether some of these effects can be observed also in trapped gases. Of course, in a microscopic system, one expects the manifestations of super fluidity to be different from the ones exhibited by macroscopic bodies. In particular, traditional experiments based on the study of transport phenomena are not easily feasible in trapped gases. On the other hand, interference patterns, associated with

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phase coherence, have been already observed[1] and successfully compared with theory. This opens a promising field of research based on the investigation of coherence phenomena, including the realization of the so-called "atom laser".

Mathematical formula used in the evaluation

Among the several properties exhibited by super fluids, the occurrence of quantized vortices and the strong reduction of the moment of inertia represent effects of primary importance. In a dilute Bose gas, the structure of quantized vortices can be investigated starting from the Gross-Pitaevskii equation. Indeed one of the primary motivations of the theory was the study of vortex stated in weakly interacting bosons[2-4]. These studies were further developed by Fetter[5], including higher-order effects in the interaction.

A quantized vortex along the z axis can be described by writing the order parameter in the form -

$$\phi(r) = \phi_v(r_\perp, z)\exp(i\kappa\varphi) \qquad \ldots(1)$$

where φ is the angle around the z axis, κ is an integer, and $\phi_\upsilon(r_\perp, z) = \sqrt{n(r_\perp, z)}$ This vortex state has tangential velocity.

$$v = \frac{\hbar}{mr_\perp}\kappa \qquad \ldots(2)$$

The number κ is the quantum of circulation and the angular momentum along z is Nκħ. The equation for the modulus of the order parameter is obtained from the GP equation. The Kinetic energy brings a new centrifugal term arising from the velocity flow that pushes the atoms away from the z axis. The GP equation then takes the form –

$$[-\frac{\hbar^2\nabla^2}{2m} + \frac{\hbar^2\kappa^2}{2mr^2_\perp} + \frac{m}{2}(\omega^2{}_\perp r^2{}_\perp) + g\phi^2{}_v(r_\perp, z)] = \mu\phi_v(r_\perp, z) \qquad \ldots(3)$$

Due to the presence of the centrifugal term, the solution of this equation for κ has to vanish on the z axis.

For non interacting systems the solution of Eq. (3) is analytic and for κ = 1, has the form -

$$\phi_\upsilon(r_\perp, z) \propto r_\perp \exp\left[-\frac{m}{2\hbar}\left(\omega_\perp r_\perp^2 + \omega_z z^2\right)\right] \qquad \ldots(4)$$

In this case, the vortex state corresponds to putting all the atoms in the m = 1 single-partial state. Its energy is then $N\hbar\omega_{\perp}$ plus the ground state energy. Similarly to what happens for the ground state without vortices, the presence of repulsive forces; dramatically reduces the density with respect to the non interacting gas, the condensate wave function becoming much broader.

The structure of the core of the vortex is fixed by the balance between the kinetic energy and the two-body interaction term. For a uniform Bose gas, the size of the core is of the order of the healing length $\xi = (8\pi na)^{-1/2}$, where n is the density of the system. For the trapped gas, an estimate of the core size can be obtained using the central value of the density in the absence of vortices for n. If the trap is spherical, the ratio between and the radius R of the condensate takes the form[6].

$$\frac{\xi}{R} = \left(\frac{a_{ho}}{R}\right)^2 \qquad \ldots(5)$$

Where, we have used the Thomas-Fermi approximation for the central density and the radius R. For the condensate, the radius is about 4.1 in units of a_{ho} and the ratio ξ/R is then ~ 0.06. The actual core size depends, obviously, also on the position on the position on the z axis and becomes larger when the vortex line reaches the outer part of the condensate, where the density decreases.

The energy of the vortex can be evaluated through the energy functional (5). The difference between the energy of the vortex state and the one of the ground state allows one to calculate the critical frequency needed to create a vortex. In fact in a frame rotating with angular frequency Ω the energy of a system carrying angular momentum L_z is given by $(E-\Omega L_z)$, where E and L_z are defined in the laboratory frame. At low rotational frequencies this energy is minimal without the vortex. If Ω is large enough the creation of a vortex can become favorable due to the term $-\Omega L_z$. This happens at the critical frequency.

$$\Omega_c = (\hbar\kappa)^{-1}[(\frac{E}{N})_\kappa - (\frac{E}{N})_c] \qquad \ldots(6)$$

where E_κ is the energy of the system in the presence of a vortex with angular momentum $N\hbar\kappa$. When one plots the critical frequency for the creation of a vortex with $\kappa = 1$ as a function of the number of atoms, for rubidium in a spherical trap. The predicted critical frequency is a fraction of the oscillator frequency and, in typical experimental conditions, corresponds to a few Hz. It decreases when N increases, because for large systems the energy cost associated with the occurrence of vortex increases as In N, while the gain ΩL_z is always linear in N. This behavior is similar to the one exhibited by uniform systems where, approximating the vortex core as a cylindrical hole of radius ξ, one can explicitly calculate the critical frequency and finds $\Omega_c = (N/mR^2)$ in (R/ξ), where R is the radius of the region occupied by the vortex flow. Analogous expressions can also be derived in the presence of harmonic trapping for large. Baym and Pethick[6] and Sinha[7] have shown that the critical frequency, in units of ω_{ho} goes ~ $(a_{ho}/R)^2$ in $(R/\xi)^2$. Using the asymptotic solution of the GP equation in the large N limit, Lundh et al.[8] have found a useful analytic expression for the critical velocity in the case of axially symmetric traps :

$$\Omega_c = \frac{5\hbar}{2mR^2_{\perp}} \ln \frac{0.671 R_{\perp}}{\xi} \quad \ldots(7)$$

Where $R_{\perp}$ is the Thomas-Fermi radius of the cloud in the xy plane orthogonal to the vortex line, while the healing length is defined by $\xi = (8\pi na)^{-1/2}$ with n equal to the central density of the gas without vortex. This formula gives a critical frequency that significantly differs from the numerical result N smaller that about 2000, while for larger N it becomes more and more accurate.

RESULTS AND DISCUSSION

In this paper, we have presented a method of evaluation of critical frequency Ω_c of vortex whose circulation number $\kappa = 1$ confined in a spherical trap with N atoms of we have taken the value of $a_{ho} = 0.791$ µm. In Table 1, we have given the evaluated results of critical frequency Ω_c in the units of $\omega_{\perp}$ as a function of atoms N. We have also compared our theoretically evaluated results with those of the other worker[9-14].

Table 1: Evaluated results of critical angular velocity Ω_c in units of $\omega_{\perp}$ for the vortex $\kappa = 1$ in a spherical trap as a function of atoms N of 87_{Rb}. The value of $a_{ho} = 0.791$ µm

N	Ω_c in units of $\omega_{\perp}$ Ours	Other[6]
0	1.05	1.16
1000	0.85	1.10
2000	0.82	0.96
3000	0.78	0.82

Cont…

N	Ω_c in units of $\omega_\perp$ Ours	Other[6]
4000	0.68	0.76
5000	0.46	0.70
6000	0.42	0.62
7000	0.40	0.48
8000	0.38	0.42
9000	0.36	0.40
10000	0.35	0.40

Ω_c Decreases with N and decrease is more pronounced as one goes to higher N. Our theoretical results are consistent with those of the other[6] as far as the trend is concerned. In other calculation, we have evaluated the ratio of (Θ/Θ_{rig}) as a function of (T/T_c^0). The results are shown in Table 2.

Table 2: Evaluated results of (Θ/Θ_{rig}) as a function of (T/T_c^0)

(T/T_c^0)	$(\Theta/\Theta_{rig})^1$	$(\Theta/\Theta_{ris})^2$	$(\Theta/\Theta_{rig})^3$
0.1	0.052	0.087	0.142
0.2	0.087	0.112	0.262
0.3	0.104	0.147	0.305
0.4	0.118	0.236	0.386
0.5	0.287	0.305	0.408
0.6	0.326	0.368	0.435
0.7	0.453	0.488	0.552
0.8	0.567	0.605	0.786
0.9	0.669	0.708	0.835
1.0	0.728	0.814	0.926

$(\Theta/\Theta_{rig})^1$ = Interacting gas with $\eta = 0.4$
$(\Theta/\Theta_{ris})^2$ = Non-interacting gas with $N = 5 \times 10^7$ atoms
$(\Theta/\Theta_{rig})^3$ = Non-interacting gas with $N = 5 \times 10^4$ atoms

Here, we have evaluated the ratio in three different cases. In first case, we have taken the interacting gas with $\eta = 0.4$ and in other two cases, we have taken the non-interacting gas having no. of atoms 5×10^7 and 5×10^4, respectively. In all the three evaluated results (Θ/Θ_{rig}) increases with (T/T_c^0). The increases in more faster in the non-interacting gas case and less faster in interacting case[15]. Some recent calculations[16-26] also revel the same behavior.

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Revised : 03.03.2014 *Accepted : 05.03.2014*