



# **A THEORETICAL EVALUATION OF MAGNETIC FIELD DEPENDENCE OF VORTEX CORE SIZE AND COHERENCE LENGTH OF $V_3Si$ , $NbSe_2$ AND $LuNi_2B_2C$ SUPERCONDUCTORS**

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## **ABSTRACT**

In this paper, we have theoretically determined the vortex core size by taking Ginzburg-Landau parameter  $K = 1$  for superconductor  $V_3Si$ ,  $NbSe_2$  and  $LuNi_2B_2C$ . We have used Eilenberger equations in our calculation. We have shown that vortex core size  $r_0$  and coherence length  $\zeta_{ab}$  exhibit magnetic field dependence. The maximum value of the cut off parameter  $\zeta_{ab}$  measured by  $\mu$ SR corresponds to the GL coherence length calculated from  $HC_2$ . At low fields, where the vortices are weakly interacting, the fitted value of  $\zeta_{ab}$  agrees with expected from  $HC_2$ .

**Key words:** Vortex core,  $\mu$ SR, Type-II superconductor, Eilenberger equation, GL Parameter  $K$ .

## **INTRODUCTION**

Muon spin rotation ( $\mu$ SR) is an experimental technique primarily used to measure local magnetic fields inside samples. The discovery of high transition temperature (high-  $T_c$ ) superconductivity in 1986 brought about a rapid world-wide expansion in the use and application of  $\mu$ SR. Since the  $\mu$ SR has been routinely applied to investigations of these and other newly discovered type-II superconductors. The technique allows for studies in zero external magnetic field, which combined with its sensitivity as a local probe has provided distinctive information on the occurrence of internal magnetism as a coexisting or competing these, or as a consequence of time  $\mu$ SR reversal symmetry breaking superconductivity. From zero field  $\mu$ SR studies of cuprates, a generic temperature versus doping phase diagram has been constructed, showing the coexistence of high  $-T_c$  superconductivity with static

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magnetism in highly doped samples. Today there is still much debate on the origin of this magnetism and its importance to the high- $T_c$  'problems'.

The vortex state provides another avenue for investigation of type-II superconductors with  $\mu\text{SR}$ <sup>1</sup>. For many years such studies focused solely on obtaining experimental information to the magnetic penetration depth ( $\lambda$ ), through measurements of the muon spin depolarization rate ( $\sigma$ ) resulting from the broad internal magnetic field distribution  $n(B)$  of the flux-line lattice (FLL). The temperature and magnetic field dependences of  $\lambda$ , which in many systems can also be determined in the Meissner phase by other techniques, reflect the pairing state symmetry of the superconducting carriers. With further advances of the  $\mu\text{SR}$  method came the ability to focus attention on the properties of the vortex cores themselves.

The first-ever study to account for the finite size of the vortex cores in the analysis of  $\mu\text{SR}$  data was an investigation of  $n(B)$  in pure Nb single crystal<sup>2</sup>. The measured field distributions were shown to be consistent with numerical solutions of the macroscopic BCS-Gor'kov theory. Some years later, the magnetic field dependence of the vector core size was determined from  $\mu\text{SR}$  measurements on single crystal  $\text{NdSe}_2$ <sup>3</sup>. The results confirmed earlier scanning tunneling spectroscopy (STS) measurements on  $\text{NdSe}_2$  that showed a shrinking of the vortex cores with increasing magnetic field.<sup>4</sup> This behavior could be attributed to an increased overlap of the quasi-particle states around a vortex core with those coming from neighboring vortices. However, these  $\mu\text{SR}$  studies were more than just another means of accessing information obtainable by another experimental technique. Instead they marked the development of a more powerful method for investigating some of the intrinsic properties of vortex cores in type-II superconductors.

This STS technique, which is sensitive to the electronic structure of the vortex cores, is permitted to probing individual vortices near the sample surface. Near the surface the vortices spread out<sup>5,6</sup> and their properties are strongly influenced by surface inhomogeneities and/or defects. Today, one can study vortices immediately above or below the surface by  $\mu\text{SR}$  using low-energy (several KeV) positively charged muons ( $\mu^*$ )<sup>6,7</sup> or by  $\beta$  detected NMR using low-energy radioactive ions.<sup>8</sup> In contrast, the experiments of<sup>2,3</sup> used energetic ( $\sim 3$  MeV)  $\mu^+$  ions that stop at interstitial or bond sites in the bulk of the sample where they directly probe the local magnetic field. The term 'bulk' means that the stopping range of these faster muons is approximately  $150 \text{ mg cm}^{-2}$  which requires samples  $\sim 1$  mm thick. In further contrast to the STS method  $\mu\text{SR}$  studies yield average information on the vortex cores, using  $\sim 10^7 \mu^+$  to randomly probe the  $\sim 10^9$  vortices in a typical size sample.

Since the experiments<sup>3</sup> of, strong field and temperature dependences of the vortex core size have been found by  $\mu$ SR in a variety of superconductors. Through comparison of the results with theoretical models and experiments that are directly sensitive to quasi-particles properties, a good understanding of many of the  $\mu$ SR experiments has been achieved. In d-wave superconductors, it is now well established that the vortex core size depends on both the thermal occupancy of the bound quasi-particle core states and the overlap of the corresponding quasi-particle wave functions with those of nearest-neighbor vortices. However, in exotic systems such as high- $T_c$  superconductors, where localized core states may be absent, there is currently insufficient experimental information to make similar definitive statements. On the other hand, recent  $\mu$ SR studies of the vortex cores in under doped high- $T_c$  superconductors have shed new light on the ground state that emerges when superconductivity is suppressed. Combining information obtained from  $\mu$ SR experiments in zero and non-zero magnetic fields, the latest results support a picture of closely competing superconducting and magnetic ground states.

### **Muon spin rotation ( $\mu$ SR)**

The primary use and strength of  $\mu$ SR is its unmatched sensitivity to internal magnetism. Central to the  $\mu$ SR method is the use of nearly 100% spin-polarized muon beam, naturally generated from the weak interaction decay of pions. This is a great advantage over conventional NMR, which relies on thermal equilibrium nuclear spin polarization in a large magnetic field. Zero fields (ZF)  $\mu$ SR is routinely used to study small internal magnetic fields of natural origin. In contrast to neutron scattering, the information provided by  $\mu$ SR is integrated over reciprocal space, which makes it ideal for studies of short-range magnetic correlations or disordered magnetism. The magnetic moment of the muon is 3.18 times larger than that of proton, making it ever more sensitive to magnetism than NMR. Although generally a nuisance in experiments,  $\mu$ SR even detects the dipolar fields of nuclear moments. In fact, magnetic field is small as  $\sim 0.1$  G are detectable-although it is important to emphasize that it refers to the local field at the muon stopping site.

### **Transverse field $\mu$ SR**

The internal magnetic field distribution  $n(B)$  of a type-II superconductor in the vortex state is measured by the so-called 'transverse-field' muon spin rotation (TF-  $\mu$ SR) method. The external magnetic field  $H$  is applied transverse to the direction of the initial muon spin polarization  $P_x(0)$ , which defines the x-axis. In high- $T_c$  superconductors, the positive ( $\mu^+$ ) forms an  $\sim 1\text{\AA}$  bond with an oxygen atom<sup>9,10</sup>, but in general the muon will stop at an interstitial site in the sample. There the muon spin precesses about the local magnetic

field  $B(r)$  in a plane perpendicular to the local field axis. The muon subsequently decays, emitting a fast positron. The angular dependence of the decay probability of the muon is given by –

$$W(E, \theta) = 1 + a(E) \cos(\theta) \quad \dots(1)$$

Where  $E$  is the kinetic energy of the decay positron,  $\theta$  is the angle between the directions of the muon spin and the emitted positron and  $a(E)$  is an asymmetry factor. When all positron energies are sampled with equal probability, the asymmetry factor has the value  $a = 1/3$ . The statistical average direction of the muon spin polarization is obtained by measuring the anisotropic angular distribution of decay positrons from an ensemble of implanted muons.

The  $\mu$ SR signal obtained by the detection of the decay positrons is given by

$$A(t) = a_0 P_x(t) \quad \dots(2)$$

Where  $A(t)$  is the  $\mu$ SR ‘asymmetry’ spectrum,  $a_0$  is the asymmetry maximum, and  $P_x(t)$  is the time evolution of the muon spin polarization

$$P_x(t) = \int_0^{\infty} n(B) \cos(\gamma_{\mu} B t + \theta) dB \quad \dots(3)$$

Here  $\gamma_{\mu} = 0.0852 \mu\text{S}^{-1}\text{G}^{-1}$  is the muon gyro magnetic ratio,  $\theta$  is a phase constant and

$$\langle B' \rangle = \langle \delta [B' - B(r)] \rangle \quad \dots(4)$$

is the probability of finding a local magnetic field  $B$  in the  $z$ -direction at a position  $r$  in the  $x$ - $y$  plane.

### Ginzburg-Landau models

In recent years, modified Landau models for  $B(r)$  have been abandoned in favour of models based on GL theory. The appealing aspect of the GL models is that the spatial variation of the order parameter is naturally built into the theory. The drawback is the GL theory assumes that the order parameter varies slowly in space and is strictly valid only near  $T_c$ . Despite these limitations, GL theory has proven to be highly successful in describing variations of  $n(B)$  as measured by  $\mu$ SR, yielding accurate quantitative values of  $\lambda$  and  $\xi$  in certain cases. As is the case in using modified Landau models, the key is to be careful with the interpretation of the fitted values.

The GL equations for the ideal Abrikosov vortex lattice can be solved by a variational method<sup>11</sup>. At low reduced field  $b = B/B_{c2} \ll 1$  and for  $\kappa \gg 1$ , an excellent analytical approximation to the spatial field profile in GL theory is<sup>12</sup> –

$$B(r) = B_0(1-b^4) \sum_G \frac{e^{-i\vec{G}\cdot\vec{r}} F(G)}{\lambda^2 G^2} \quad \dots(5)$$

Where  $F(G) = uK_1(u)$ ,  $u^2 = 2 \xi^2 G^2(1-b^4) [1-2b(1-b)^2]$  and  $K_1(u)$  is a modified Bessel function. Note the cutoff function  $F(G)$  depends on the local internal magnetic field  $B$ .

### Vortex core size

Superconductivity is strongly suppressed in the vortex core. Vortex core size is that it is a region of radius  $r \approx \xi$  where  $\xi$  is the coherence length. It is the length scale for spatial variation of the order parameter  $\psi$  which is the GL coherence length. The core radius is defined as –

$$\xi_1 = \frac{\Delta_0}{\lim_{r \rightarrow 0} \frac{\Delta(r)}{r}} \quad \dots(6)$$

Where  $\Delta_0 = \frac{1}{\xi_0}$  is the bulk superconducting energy gap at zero temperature and  $\xi_0$  is BCS coherence length  $\xi_{ab}$  is given by –

$$\xi_{ab} = \left( \frac{\phi_0}{2\pi Hc_2} \right)^{\frac{1}{2}} \quad \dots(7)$$

where  $\phi_0$  is flux quanta and  $Hc_2$  is the upper critical field.

## RESULTS AND DISCUSSION

In this paper, we have theoretically evaluated the magnetic field dependence of the vortex core size ' $r_0$ ' (Å) coherence length  $\xi_{ab}$  (Å) of superconductor  $V_3Si$ ,  $NbSe_2$  and  $LuNi_2B_2C$ . These parameters have been determined from muon spin rotation measurements. The coherence length  $\xi_{ab}$  has been determined from Ginzburg-Landau model. The equation for the ideal Abrikosov vortex lattice have been solved by a variational method. Magnetic field dependence of the vortex core size and coherence length are determined by taking  $\kappa \gg 1$ .  $\kappa$  is the GL parameter. The results for  $r_0$  (Å) and  $\xi_{ab}$  (Å) are shown in Table 1 and 2 with experimental data<sup>15-17</sup>.

**Table 1: An Evaluated results of magnetic field dependence of Vortex core size for superconductor V<sub>3</sub>Si, NbSe<sub>2</sub> and LuNi<sub>2</sub>B<sub>2</sub>C**

$\frac{H}{Hc_2}$	$r_0$ (Å)					
	V <sub>3</sub> Si (T = 0.22 Tc)		NbSe <sub>2</sub> (T = 0.03 Tc)		LuNi <sub>2</sub> B <sub>2</sub> C (T = 0.16 Tc)	
	Theo.	Expt. <sup>15</sup>	Theo.	Expt. <sup>16</sup>	Theo.	Expt. <sup>17</sup>
0.05	60.25	59.16	152.48	157.24	186.54	188.38
0.07	58.46	55.42	150.77	155.58	182.25	185.86
0.10	54.72	53.58	146.56	150.06	178.43	180.58
0.15	50.12	49.10	140.18	144.28	174.22	178.22
0.20	43.55	41.86	13.26	135.48	164.86	166.12
0.22	40.18	39.17	127.98	130.04	160.25	162.48
0.24	37.10	36.48	122.50	125.18	154.15	159.25
0.26	35.26	30.59	117.84	120.52	150.46	155.56
0.28	31.87	28.22	112.52	114.05	143.86	150.41
0.30	28.55	26.06	102.59	104.86	140.78	144.59
0.35	26.18	24.21	98.23	100.22	136.26	139.12
0.40	22.28	20.53	95.16	95.84	131.53	134.28

**Table 2: An Evaluated results of magnetic field dependence of coherence length  $\xi_{ab}$  (Å) for superconductor V<sub>3</sub>Si, NbSe<sub>2</sub> and LuNi<sub>2</sub>B<sub>2</sub>C**

$\frac{H}{Hc_2}$	$\xi_{ab}$ (Å)					
	V <sub>3</sub> Si (T = 0.22 Tc)		NbSe <sub>2</sub> (T = 0.03 Tc)		LuNi <sub>2</sub> B <sub>2</sub> C (T = 0.16 Tc)	
	Theo.	Expt. <sup>15</sup>	Theo.	Expt. <sup>16</sup>	Theo.	Expt. <sup>17</sup>
0.05	42.14	45.58	105.29	108.06	84.22	89.97
0.07	40.22	42.27	103.58	106.53	80.39	85.26
0.10	38.29	40.54	100.07	102.46	76.46	81.38
0.15	36.84	38.86	97.39	99.55	72.29	78.54

Cont...

$\frac{H}{Hc_2}$	$\xi_{ab}$ (Å)					
	V <sub>3</sub> Si (T = 0.22 Tc)		NbSe <sub>2</sub> (T = 0.03 Tc)		LuNi <sub>2</sub> B <sub>2</sub> C (T = 0.16 Tc)	
	Theo.	Expt. <sup>15</sup>	Theo.	Expt. <sup>16</sup>	Theo.	Expt. <sup>17</sup>
0.18	34.55	35.59	94.58	96.68	68.55	74.22
0.20	30.27	31.44	91.22	94.22	62.34	70.16
0.22	28.17	30.05	87.48	90.18	58.26	65.55
0.24	26.22	28.29	81.95	85.55	53.58	60.28
0.26	24.53	27.09	76.42	80.29	50.22	55.48
0.28	21.86	24.66	73.54	75.16	47.59	52.22
0.30	18.47	20.14	69.42	71.22	41.22	49.58
0.35	15.38	18.36	61.55	68.29	38.16	41.33
0.40	12.24	15.46	55.29	61.86	32.25	38.29

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