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A NOTE ON RADIUS AND DIAMETER OF A GRAPH W.R.T. D-DISTANCE

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ABSTRACT

The *D*-distance between vertices of a graph is obtained by considering the path lengths and as well as the degrees of vertices present on the path. In this article, we study the relations between radius and diameter of a graph with respect to *D*-distance, and also we obtain some results on trees with respect to *D*-distance.

Key words: D-distance, D-radius, D-diameter.

INTRODUCTION

By a graph G = (V, E), we mean a finite undirected graph without loops and multiple edges. The concept of distance is one of the important concepts in study of graphs. It is used in isomorphism testing, graph operations, hamiltonicity problems, extremal problems on connectivity and diameter, convexity in graphs etc. Distance is the basis of many concepts of symmetry in graphs.[1]

In addition to the usual distance, d(u, ν), between to vertices u, ν $\in$ V(G), we have detour distance[2], superior distance[3], signal distance[4], etc.

In an earlier article[5], the authors have introduced the concept of *D-distance* by considering not only path length between vertices, but also the degrees of all the vertices present in a path while defining the D-distance.

Preliminaries

Definition 2.1: If u, ν are vertices of a connected graph G, the *D-length* of a u – ν

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path s is defined as $l(s) = d(u,v) + \deg(u) + \deg(v) + \sum \deg(w)$ where sum runs over all intermediate vertices w of s.

Definition 2.2: The D-*distance* $d^D(u, v)$ between two vertices u, v of a connected graph G is defined as $d^D(u,v) = \min\{l(s)\}$ if u, v are distinct and $d^D(u,v) = 0$ if $u = v$, where the minimum is taken over all $u - v$ paths s in G.

Definition 2.3: The *D-eccentricity* of any vertex v, $e^D(v)$, is defined as the maximum distance from v to any other vertex, i.e., $e^D(v) = \max\{d^D(u,v) : u \in V(G)\}$

Definition 2.4: Any vertex u for which $d^D(u,v) = e^D(v)$ is called D-*eccentric* vertex of v. Further, a vertex u is said to be D-*eccentric vertex* of G, if it is the D-eccentric vertex of some vertex.

Definition 2.5: The D-*radius,* denoted by $r^D(G)$, is the minimum D-eccentricity among all vertices of G i.e., $r^D(G) = \min\{e^D(v) : v \in V(G)\}$. Similarly the D-*diameter,* $d^D(G)$, is the maximum *D*-eccentricity among all vertices of G

Definition 2.6: The D-*center* of G, $C^D(G)$, is the sub graph induced by the set of all vertices of minimum D-*eccentricity*. A graph is called D-*self centered* if $C^D(G) = G$ or equivalently $r^D(G) = d^D(G)$. Similarly, the set of all vertices of maximum D-eccentricity is the *D-periphery* of G.

Results on D-radius and D-diameter

In this section, we prove relation between D-radius and D-diameter and also we prove the results on trees with respect to D-distance.

Theorem 3.1: For a connected graph G the D-radius satisfies the inequality $r^D(G) \le d^D(G) \le 2r^D(G) - 2$.

Proof: The first inequality follows from the definition. For the second inequality let u and v be the vertices of G such that $d^D(u,v) = d^D(G)$ and let $w \in V(G)$ such that $e^D(w) = r^D(G)$. We have $d^D(G) = d^D(u,v) \le d^D(u,w) + d^D(w,v) - \deg(w)$ (see 5) as w is an internal vertex we obtain $d^D(u,v) \le e^D(w) + e^D(w) - 2$ and hence $r^D(G) \le d^D(G) \le 2r^D(G) - 2$.

Next, we prove the sharpness of the bounds of theorem 3.1.

Theorem 3.2: There exists a class of connected graph G_K. Such that $d^D(G_k) = 2r^D(G_k) - 2$.

Proof: If $G = P_5$ where v_1, v_2, v_3, v_4 and v_5 are the vertices define G_K to be the graph which is obtained by adding k pendent edges to each v_i for i = 2, 4.. Let $u_1, u_2, u_3, ..., u_k$ be the vertices adjacent to v_2 and let $w_1, w_2, w_3, ..., w_k$ be the vertices adjacent to v_4. From these construction we get the following relations $e^D(v_i) = 12 + 2n$, i = 1 and 5 $e^D(v_3) = 7 + n$, $e^D(u_i) = e^D(w_i) = 12 + 2n$, $\forall i = 1,2,3,...k$. Hence $d^D(Gk) = 12 + 2n = 2(7 + n) - 2 = r^D - 2$.

Proposition 3.3: There exists a class of connected graph G such that $r^D(G) = d^D(G)$.

Proof: Let K_n be a complete graph with n vertices then the D-radius and D-diameter are given by $r^D(K_n) = d^D(K_n) = 2n - 1$.

Theorem 3.4: The vertex set of every graph G with at least two vertices is the vertex-to-vertex center of some connected graph.

Proof: Let G be a graph with n vertices. We show that G is the center of some connected graph H (i.e., vertex set of G is the center of H). First if G is not complete make G is complete graph then add two new vertices u and v to G and join them to every vertex of G but not each other. Next, we add two new vertices p and q and join p to u and q to v the resulting graph H is given Fig. (1) then it is clear that $e^D(p) = e^D(q) = 3n + 9$, $e^D(u) = e^D(v) = 3n + 7$ and $e^D(w) = 2n + 5$. For every vertex w in G. Hence the vertex set of G is the vertex-to-vertex center of H.

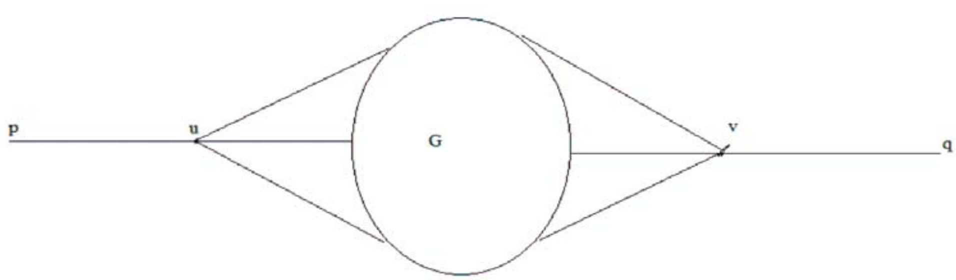

Fig. 1: The center $C^D(G)$ of a graph G contained in block of a graph

Theorem 3.5: The center $C^D(G)$ of any connected graph G lies within a block of G.

Proof: suppose the center $C^D(G)$ of a connected graph G lies in more than one block. Then G contains a cut vertex v such that $G-v$ has two components G_1 and G_2 each of which contains a central vertex of G. Let u be an eccentric vertex of v and let P be a G path of length $e^D(v)$. Then v contains no vertex from at least one of G_1 and G_2, say G_1. Let x be a central vertex of G_1 and let P′ be a $x-v$ geodesic in G. Then $e^D(x) \geq d^D(x,v)+d^D(v,u) \geq +e^D(v)$. So x is not a central vertex, which is a contradiction because $x \in C^D(G)$. Thus all central vertices must be lie in a single block.

Now, we prove the results on trees.

Theorem 3.6: All eccentric vertices of a tree w.r.t. D-distance are end vertices.

Proof: In a tree there is a unique path between any two vertices and hence the maximum D-distance from one vertex to any other vertex occurs at end vertices only. Hence, all eccentric vertices of tree are end vertices.

Theorem 3.7: The center of tree, w.r.t. D-distance consists of either a single vertex or two adjacent vertices.

Proof: The result is trivial for the trees K_1 and K_2. We show that any other tree T has the same center as the tree T′ obtain by removing all end vertices of T. Clearly, for each vertex v of T, only an end vertex can be an eccentric vertex of v. Thus the eccentricity of each vertex in T′ will be exactly $1+\deg(v)+\deg(u)$ less than the same vertex in T. Hence the vertices with minimum eccentricity in T′ are the same, vertices of minimum eccentricity in T, that is T and T′ have the same center. If the process of removing end vertices is repeated, we obtain successive trees having the same center as T. since T is finite, we eventually obtain a sub tree of T, which is either K_1 or K_2. In either case, the vertices in this ultimate tree constitute the centre of T, which thus consists of a single vertex or a pair of adjacent vertices.

Theorem 3.8: The periphery of tree consists of end vertices only.

Proof: Science the maximum D-distance from one vertex to other vertex in a tree occurred at end vertices only. The maximum eccentricity of a vertex is the peripheral vertex, that occurs only at end vertices. Therefore, the periphery of tree consists of end vertices only.

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