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Weyl electrovacuum solutions and gauge invariance (Preliminary research for gravity control)

Abstract

It is argued that in Weyl electrovacuum solutions the linear term in the metric cannot be eliminated just on grounds of gauge invariance. Its importance is stressed.

Keywords

Root gravity; Condenser plates; Biefeld-Brown effect; Einstein's unified theories; Alternative gravitational theory.

INTRODUCTION

In general relativity static electric fields alter the metric of spacetime through their energy-momentum tensor[1]

$$T^{\mu}_{\nu} = \frac{\varepsilon}{4\pi}\left(F^{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta^{\mu}_{\nu}F^{\alpha\beta}F_{\alpha\beta}\right) \qquad (1)$$

where

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \qquad (2)$$

is the electromagnetic tensor, $A_{\mu} = (\bar{\varphi},0,0,0)$ is the four-potential and ε is the dielectric constant of the medium.
T^{μ}_{ν} enters the r.h.s. of the Einstein equations

$$R^{\mu}_{\nu} = \kappa T^{\mu}_{\nu} \qquad (3)$$

where κ is the Einstein constant. We have taken into account that $T^{\mu}_{\mu} = 0$. In addition, the Maxwell equations are coupled to gravity through the covariant derivatives of $F_{\mu\nu}$

$$F^{\mu\nu}{}_{;\nu} = \frac{1}{\sqrt{-g}}\left(\sqrt{-g}\ F^{\mu\nu}\right)_{\nu} = 0 \qquad (4)$$

where g is the metric's determinant and usual derivatives are denoted by subscripts. The electric field is $E_{\mu} = F_{0\mu} = -\bar{\varphi}_{\mu}$. Obviously, T^{μ}_{ν} from Eq. (1) contains only quadratic terms in $\bar{\varphi}_{\mu}$.

This allows to hide κ and ε by normalizing the electric potential to a dimensionless quantity

$$\phi = \sqrt{\frac{\kappa\varepsilon}{8\pi}}\bar{\phi} \qquad (5)$$

The factor 8π is chosen for future convenience and we use CGS units. This is a much more efficient way to get rid of the constants in the Einstein-Maxwell equations than the choice of relativistic units.

ROOT GRAVITY

Let us confine ourselves to the axially-symmetric static metric[2]

$$ds^2 = f(dx^0)^2 - f^{-1}[e^{2k}(dr^2 + dz^2) + r^2 d\varphi^2] \qquad (6)$$

where $x^0 = ct$, $x^1 = \varphi$, $x^2 = r$, $x^3 = z$ are cylindrical coordinates, $f = e^{2u}$ and u is the first, while k is the second gravitational potential. Both of them depend only on r and z.
For the electric field one has

$$E_r = -\bar{\phi}_r,\ E_z = -\bar{\phi}_z \qquad (7)$$

The field equations read

$$\Delta u = e^{-2u}(\phi_r^2 + \phi_z^2) \qquad (8)$$

$$\Delta\phi = 2(u_r\phi_r + u_z\phi_z) \quad (9)$$

$$\frac{k_r}{r} = u_r^2 - u_z^2 - e^{-2u}(\phi_r^2 + \phi_z^2) \quad (10)$$

$$\frac{k_z}{r} = 2u_r u_z - 2e^{-2u}\phi_r\phi_z \quad (11)$$

where $\Delta = \partial_{rr} + \partial_{zz} + \partial_r / r$ is the Laplacian. We have used the definition given in Eq. (5).

The first two equations determine and f. After that k is determined by integration.

Weyl electrovacuum solutions[3] are obtained when the gravitational and the electric potential have the same equipotential surfaces, $f = f(\phi)$. Eqs. (8-9) yield

$$(f_{\phi\phi} - 2)(\phi_r^2 + \phi_z^2) = 0 \quad (12)$$

which gives

$$f = A + B\phi + \phi^2 \quad (13)$$

where A and B are arbitrary constants. Replacing it in Eqs. (8-9) one comes to an equation for ϕ.

$$\Delta\phi = \frac{B + 2\phi}{A + B\phi + \phi^2}(\phi_r^2 + \phi_z^2) \quad (14)$$

Let us make one more assumption, that ϕ depends on r, z through some function $\psi(r, z)$ which satisfies the Laplace equation $\Delta\psi = 0$. Then $\phi(\psi)$ is determined implicitly from

$$\psi = \int \frac{d\phi}{A + B\phi + \phi^2} \quad (15)$$

An important equality follows

$$\phi_i = f\psi_i \,, \; \overline{\phi}_i = f(\phi)\overline{\psi}_i \quad (16)$$

where i = r, z.

Eqs. (10-11) become

$$k_r = \frac{D}{4}r(\psi_r^2 - \psi_z^2) \,, \; k_z = \frac{D}{2}r\psi_r\psi_z \quad (17)$$

where $D = B^2 - 4A$. Thus in Weyl electrovac solutions the harmonic master potential determines the electric and the gravitational fields.

The theory should be invariant under gauge transformations, which in this case are simply translations: $\phi' = \phi + a$ with a being an arbitrary constant.

Eqs.(1-4) and (7-11) are gauge invariant, but Eq.(13) is not because A, B change into

$$A' = A + Ba + a^2, \; B' = B + a \quad (18)$$

This happens because f depends directly on the electric potential and not on its derivatives.

In some papers this is used to set B′ to zero and eliminate the linear term.

In this paper we shall show that this is not correct. In fact, the general solution (13) stays gauge invariant because A′, B′ are also arbitrary constants. In a particular solution A′, B′ should be fixed and should not change under a gauge transformation. This is possible when after a is selected one compensates its effect by choosing A, B in such a way that A′, B′ stay fixed at any particular value. Eq.(18) shows that this always can be done and in this way the gauge invariance of f is restored. For example, due to Eq.(5), the electric potential is very small everywhere for realistic fields and it is natural that it should go to zero at infinity or when the field is turned off. Then asymptotic flatness requires to set A′ = 1 and this condition can be kept in spite of possible gauge transformations. The coefficient B′ is not determined by the system of equations (8-9) and the Weyl conditions. One can not just put it to zero by a gauge transformation. In fact, arguments were given in[4,5] that its value is 2. Then f becomes a perfect square, while k vanishes and the space part of the metric is conformally flat. It should be noticed that D′ = D so that the vanishing of k is gauge invariant.

The presence of the linear term in f with a coefficient of order unity is not just of academic interest. Because of the gravitational potential a particle at rest feels an acceleration[1]

$$g_i = \frac{c^2}{2}(\ln g_{00})_i = c^2 f^{-1}\left(\frac{B'}{2}\sqrt{\frac{\kappa\varepsilon}{8\pi}}\overline{\phi}_i + \frac{\kappa\varepsilon}{8\pi}\overline{\phi\phi}_i\right) \quad (19)$$

Covariant and contravariant components coincide in practice because for realistic electric fields the metric is almost flat. Eq.(7) shows that the first term is proportional to the electric field, which due to Eq.(16) may be derived also from the master potential because f is extremely close to one.

Let us note that

$$c^2\sqrt{\frac{\kappa}{8\pi}} = \sqrt{G} = 2.58\times10^{-4} \,, \; c^2\frac{\kappa}{8\pi} = \frac{G}{c^2} = 7.37\times10^{-27} \quad (20)$$

where $G = 6.674 \times 10^{-8}$ cm^3 / g · s^2 is the Newton constant and $c = 2.998 \times 10^{10}$ cm / s is the speed of light. Due to the square root, the first coefficient is 10^{23} times bigger than the second and for realistic fields and media this cannot be compensated by the squares of potentials and the additional $\sqrt{\varepsilon}$ factor in the second term. The latter is typical for linear perturbation theory.

In relativistic units G = c = 1 the difference does not show up. Thus, provided that B′ = 2, the linear term is essential and the coupling of electromagnetism to gravity appears to be much stronger than it is usually thought. It causes a number of effects, the most prominent being the movement of a usual capacitor towards one of its poles. In this case there is plane symmetry in the bulk, f and depend only on z, which means they are functionally related and the general solution belongs to the Weyl class. However, Eqs.(10,17) show that k depends on r and breaks the symmetry unless D = 0, which gives B′ = ±2. Putting the usual formula for the electric field inside a capacitor into Eq.(19) gives for the acceleration which acts on the dielectric inside it

$$g_z = \pm\sqrt{G\varepsilon\, f}\,\frac{\overline{\psi}_0}{d} \approx \pm 2.58\times10^{-4}\frac{\sqrt{\varepsilon}}{d}\overline{\psi}_0 \quad (21)$$

where ψ_0 is the potential difference between the plates and d is the distance between them.
A more detailed derivation can be found in Refs.[4,5].

RESULT AND DISCUSSION

If the capacitor is hanging freely, this effect may be tested experimentally. To increase the acceleration it is advantageous to make d small (typically $0.1cm < d < 1cm$), to raise ψ_0 up to 2×10^4 *CGS* (six million volts, which is possible) and to take a ferroelectric material with ε in the range of 10^4, like barium titanate ($B_aT_iO_3$) or many others. Thus $\sqrt{\varepsilon}/d$ may reach in principle 10^3 and the maximum acceleration $g_{z,max} = 5.2g_{earth}$ is more than enough to counter Earth's gravity.
This effect has been discovered by the prominent electrical engineer Thomas Townsend Brown (1905-1985) already in 1923 together with Prof. P. A. Biefeld and called the Biefeld-Brown effect[6]. Brown worked on his own on it up to the sixties with high voltage equipment in the range *70 - 300kV*. He didn't give a formula like Eq.(21) but stressed that the effect is bigger the closer the condenser plates, the higher the voltage and the greater the ε, which is in accord with Eq.(21). He also found that the capacitor moves towards its positive pole, resolving experimentally the sign ambiguity in the above formula. There have been speculations that the effect might follow from some of the Einstein's unified theories. Today one would mention string theory or some other alternative gravitational theory. However, it appears that the effect is a part of usual General Relativity due to its strong nonlinearity. It is worth to repeat Brown's experiments in different laboratories and check formula (21).

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