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Unsteady MHD Flow Past Impulsively Started Vertical Plate in Porous Medium with Heat Source and Chemical Reaction

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Abstract

An unsteady MHD free convective flow of an incompressible, electrically conducting, viscous fluid along an infinitely long vertical plate with an impulsive motion in the presence of heat source and homogeneous binary chemical reaction in a rotating system is studied here. The dimensionless form of governing equations is obtained and solved analytically with the help of Laplace transform method. The solutions obtained for concentration, temperature and velocity distribution are analyzed graphically. The numerical values of the skin friction at the plate are shown in the table. It is noticed that for a strong magnetic field the flow pattern is affected significantly by Hall current at a different rotation. The results obtained may be useful for applications in the area related to the structure of rotating magnetic stars, the solar physics which deals the sunspot development, and the solar cycle, etc.

Keywords: MHD flow; Porous medium; Vertical plate

Introduction

The convective boundary layer flow along an infinite flat plate is one of the important problems. An impulsive motion of a plate in a viscous fluid was studied by Stewartson [1,2]. His study was completely based within the context of boundary layer equations. If the fluid is electrically conducting then the magnetic field can stabilize such a flow within a porous and non-porous medium. And the magneto hydrodynamic flow with heat generation and chemical reaction is widely used in many engineering processes with applications in industries. In recent years, considerable progress has been made in the study of the thermo physical properties affecting magneto hydrodynamic flow. For instance, the magneto hydrodynamic flow of a vertical permeable uniformly stretched surface with chemical reaction and heat absorption/generation was studied by Chamka [3]. He solved the problem analytically and observed that the Prandlt number, Schmidt number and the strength of magnetic field retard the fluid velocity. Prasad et al. [4] analyzed the 2-D impulsive motion of an infinite vertical plate with mass transfer and radiation. They solved the model by finite-difference method and observed that when the radiation

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parameter increases, the fluid velocity decreases near the plate. Further, Ibrahim et al. [5] studied the effect of chemical reaction on magneto hydrodynamic flow with mass and heat transfer along a moving vertical porous plate. Further Ibrahim et al. [6] extended their work by considering the radiation on the same model.

Hall current term in generalized Ohm's law cannot be neglected for the problems having a strong magnetic field. Also, the rotating flow of incompressible, electrically conducting and viscous fluid has abundant geophysical and astrophysical applications. So, many scholars have studied such models, for instance, Agarwal et al. [7] studied the combined effect of Hall current and dissipation on rotating fluid with free convective motion. Seth et al. [8] worked on unsteady hydro magnetic free convective flow along a moving vertical plate with rotation and thermal radiation in porous medium and observed that rotation retards fluid velocity in primary flow direction; whereas it accelerates fluid flow in the secondary flow direction in the boundary layer region. Some other scholars such as Prakash et al. [9], Ramana et al. [10] and Reddy MG [11] have studied the MHD flow with radiative heat-mass transfer and chemical reaction. The present model analyzes the combined effect of rotation and Hall current on unsteady magneto hydrodynamic convective flow past along impulsively started vertical flat plate with a heat source and chemical reaction in a porous medium.

Mathematical Model

Consider viscous, electrically conducting and incompressible fluid through porous medium. Let $x'-$ axis be chosen vertically upward along the motion of the plate. And the normal direction of the plate is taken along the $z'-$ axis. Also, the plate and the fluid are rotating together as a rigid body about $z'-$ axis with a constant angular velocity Ω'. A constant magnetic field B_o is applied along $z'-$ axis. As the plate is infinitely long lying in the $x'-y'$ plane; so, various physical variables involved in the problem are considered as the functions of t' and z' only.

Initially at a time $t' \le 0,$ the system is at rest; and has a constant temperature and concentration T_∞ and C_∞ respectively. At the time $t' > 0$, the plate starts moving with a constant velocity u_o along $x'-$ axis. Also, the concentration and temperature of the plate are raised to C_p and T_p respectively. The impulsive motion of the plate and the free convection causes the disturbance in the fluid. From electric charge conservation equation $\nabla.\vec{\mathbf{J}}=0$, we have $J_{z'}=$ constant, where $\vec{\mathbf{J}}=(J_{x'},J_{y'},J_{z'})$ is the current density vector. As the plate is non-conducting, so at the plate $J_{z'}=0.$ Thus $J_{z'}=0$ everywhere in the fluid. Under the above assumptions, the governing equations with Boussinesq's approximation are as follows:

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = \upsilon\frac{\partial^2 u'}{\partial z'^2} + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) + \frac{B_o}{\rho}J_{y'} - \frac{\upsilon}{K'}u', \quad (1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \upsilon\frac{\partial^2 v'}{\partial z'^2} - \frac{B_o}{\rho}J_{x'} - \frac{\upsilon}{K'}v', \quad (2)$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z'^2} + \frac{Q_o}{\rho c_p}(\mathrm{T}-\mathrm{T}_\infty), \quad (3)$$

$$\frac{\partial C}{\partial t'} = D\frac{\partial^2 C}{\partial z'^2} - K_o(C-C_\infty). \quad (4)$$

The boundary conditions taken are as under:

$$\left.\begin{array}{l} t'\le 0: u'=0, v'=0, T=T_\infty, C=C_\infty \quad \forall\ z',\\ t'>0: u'=u_o, v'=0, T=T_p, C=C_p \ \ at\ z'=0,\\ u'\to 0, v'\to 0, T\to T_\infty, C\to C_\infty \quad as \quad z'\to\infty. \end{array}\right\} \qquad (5)$$

Taking Hall current into account and neglecting the electron pressure gradient, the ion slips and the thermo-electric effects, the generalised Ohm's law is given as-

$$\vec{\mathbf{J}}+\frac{\omega_e\tau_e}{B_o}(\vec{\mathbf{J}}\times\vec{\mathbf{B}})=\sigma(\vec{\mathbf{E}}+\vec{\mathbf{q}}\times\vec{\mathbf{B}}) \quad (6)$$

On solving (6), we get $J_{x'}=\dfrac{\sigma B_o(v'+m\,u')}{1+m^2}, J_{y'}=\dfrac{\sigma B_o(m\,v'-u')}{1+m^2}$

To obtain the equations in non-dimensional form, the following dimensionless parameters are introduced:

$$\left.\begin{array}{l} u=\dfrac{u'}{u_o},\ v=\dfrac{v'}{u_o},\ t=\dfrac{u_o^2}{\upsilon}t',\ z=\dfrac{u_o}{\upsilon}z',\ \theta=\dfrac{(T-T_\infty)}{(T_p-T_\infty)}, \phi=\dfrac{(C-C_\infty)}{(C_p-C_\infty)}, \mathrm{P}_r=\dfrac{\upsilon}{\alpha},\\ Q=\dfrac{\upsilon}{\rho c_p u_o^2}Q_o,\ M=\dfrac{\sigma B_o^2\ \upsilon}{\rho u_o^2},\ G_r=\dfrac{g\beta\upsilon(T_p-T_\infty)}{u_o^3}, G_m=\dfrac{g\beta^*\upsilon(C_p-C_\infty)}{u_o^3},\\ \Omega=\dfrac{\upsilon}{u_o^2}\Omega',\ \mathrm{c}_r=\dfrac{K_o}{u_o^2}\upsilon,\ K=\dfrac{u_o^2}{\upsilon^2}K',\ S_c=\dfrac{\upsilon}{D}. \end{array}\right\} \qquad (7)$$

Using equation (7), equations (1), (2), (3), (4) and (5) respectively, become:

$$\frac{\partial u}{\partial t}-2\Omega\ v=\frac{\partial^2 u}{\partial z^2}+\frac{M}{(1+m^2)}(m\ v-u)+G_m\phi+G_r\theta-\frac{u}{K}, \qquad (8)$$

$$\frac{\partial v}{\partial t}+2\Omega\ u=\frac{\partial^2 v}{\partial z^2}-\frac{M}{(1+m^2)}(v+m\ u)-\frac{v}{K}, \qquad (9)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2}+Q\theta, \qquad (10)$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{S_c}\frac{\partial^2\phi}{\partial z^2}-c_r\phi. \qquad (11)$$

$$\left.\begin{array}{l} t\le 0: u=0,\ v=0,\ \theta=0,\ \phi=0\ \ \forall\ z,\\ t>0: u=1,\ v=0,\ \theta=1,\ \phi=1\ \ at\ \ z=0,\\ u\to 0,\ v\to 0,\ \theta\to 0,\ \phi\to 0\ \ as\ \ z\to\infty. \end{array}\right\} \qquad (12)$$

To solve above system, assume $\mathbf{V}=u+iv$. Then using equations (8) and (9), we get,

$$\frac{\partial\mathbf{V}}{\partial t}=\frac{\partial^2\mathbf{V}}{\partial z^2}-b\mathbf{V}+G_r\theta+G_m\phi, \qquad (13)$$

The boundary conditions (12) are transformed:

$$\left.\begin{array}{l} t\le 0:\mathbf{V}=0,\ \theta=0,\ \phi=0 \qquad \forall\ z, \\ t>0:\mathbf{V}=1,\ \theta=1,\ \phi=1\ \ at\ z=0, \\ \mathbf{V}\to 0,\ \theta\to 0,\ \phi\to 0 \quad as\ z\to\infty. \end{array}\right\} \tag{14}$$

The dimensionless PDEs (10), (11) and (13), with the boundary conditions (14) are solved with the help of Laplace transform method ($P_r \neq 1$ & $S_c \neq 1$).

$$\begin{aligned} \mathbf{V}(z,t) &= \alpha_3 \left\{2Cosh(a_1 z) + e^{-a_1 z} erf\,(a_1\sqrt{t}-\eta) - e^{a_1 z} erf\,(a_1\sqrt{t}+\eta)\right\} + \alpha_1 e^{-B_1 t}\left\{e^{-a_2 z} erfc(\eta - a_2\sqrt{t}) - e^{-a_2 z} erfc(\eta + a_2\sqrt{t})\right\} \\ &+\alpha_1 \left\{2Cosh(a_3 z) + e^{-a_3 z} erf\,(a_4\sqrt{t}-a_5\eta) - e^{a_3 z} erf\,(a_4\sqrt{t}+a_5\eta)\right\} - \alpha_1 e^{-B_1 t}\left\{2Cosh(a_6 z) + e^{-a_6 z} erf\,(a_7\sqrt{t}-a_5\eta)\right\} \end{aligned} \tag{15}$$

$$\begin{aligned} &+\alpha_2 e^{-B_2 t}\left\{e^{-b_2 z} erfc(\eta - b_2\sqrt{t}) - e^{-b_2 z} erfc(\eta + b_2\sqrt{t})\right\} + \alpha_2\left\{2Cosh(b_3 z) + e^{-b_3 z} erf\,(b_4\sqrt{t}-b_5\eta) - e^{b_3 z} erf\,(b_4\sqrt{t}+b_5\eta)\right\} \\ &-\alpha_2 e^{-B_2 t}\left\{2Cosh(b_6 z) + e^{-b_6 z} erf\,(b_7\sqrt{t}-b_5\eta) - e^{b_6 z} erf\,(b_7\sqrt{t}+b_5\eta)\right\} + \alpha_1 e^{a_6 z - B_1 t} erf\,(a_7\sqrt{t}+a_5\eta) \end{aligned}$$

$$\theta(z,t) = \frac{1}{2}\left\{2Cosh(a_3 z) + \mathrm{e}^{-a_3 z} erf\,(a_4\sqrt{t}-a_5\eta) - \mathrm{e}^{a_3 z} erf\,(a_4\sqrt{t}+a_5\eta)\right\}, \ (16)$$

$$\phi(z,t) = \frac{1}{2}\left\{2Cosh(b_3 z) + \mathrm{e}^{-b_3 z} erf\,(b_4\sqrt{t}-b_5\eta) - \mathrm{e}^{b_3 z} erf\,(b_4\sqrt{t}+b_5\eta)\right\}. \ (17)$$

The skin-friction components τ_x and τ_y are obtained as:

$$\tau_x + i\tau_y = -\left.\frac{\partial \mathbf{V}}{\partial z}\right|_{z=0}$$

Results and Discussion

To examine the influence of chemical reaction, magnetic field, heat source, rotation, Hall current and time on flow, a number of selected graphs for velocity versus boundary layer length are shown in FIGURES 1 to 9. These graphs show that the magnitude of primary velocity *u* and secondary velocity *v* decrease rapidly on increasing boundary layer length to approach free stream value. From the FIGURES 1 and 2, it is observed that with the increase in c_r (chemical reaction parameter) both the components of velocity go on decreasing. The rate of decrease becomes sharp with an increase in rotation. Effect of bouncy force can be seen from FIGURE 3, which shows that the buoyancy force accelerates the flow in both the directions. This is because of an increase in the values of thermal and mass Grashof number. FIGURES 4, 5 and 6 show the effect of Hall current on the fluid velocity at the different rotation. It is noticed that the velocity of the fluid in the primary direction increases, whereas secondary velocity decreases on increasing Hall current parameter *m*. This shows that Hall current tends to accelerate fluid flow along the primary direction; whereas it tends to retard secondary velocity throughout the boundary layer region. It can be observed that the Hall current stabilizes the fluid velocity only if the strength of applied magnetic field is strong (FIGURE 6). FIGURE 7 shows that the magnetic field parameter retards the flow in the primary direction while accelerates the flow in the secondary direction. It is because of an applied transverse magnetic field which produces a resistive type force known as the Lorentz force. FIGURES 8 and 9 illustrate the influence of heat source parameter *Q* at different instant of time. In both the cases, *Q* accelerates the flow in both the directions. But as the time increases the rate of increase becomes high. FIGURES 10 and 11 displays the variation of the temperature distribution in the fluid near the plate with heat source parameter at a different time for $P_r = 0.71$. It is found that at a particular instant of time temperature in the system increases with the increase in the heat source parameter. FIGURES 12 and 13 depict concentration distribution at different times which shows that an increase in S_c or c_r reduces the thickness of the concentration boundary layer. TABLE 1 display the variation of skin friction due to variation in chemical reaction parameter c_r at different rotation. It is found that for a fixed rotation, τ_x increases with c_r , and τ_y decreases with increase in c_r .

TABLE 1. **Skin-fiction for c_r .**

$$\begin{pmatrix} P_r = 0.71, S_c = 2.01, M = 2, G_r = 5, m = 0.5, \\ K = 0.5, G_m = 5, Q = 0.5, t = 0.2 \end{pmatrix}$$

$\Omega \downarrow$ / $c_r \rightarrow$	$-\tau_x(10^{-3})$			$\tau_y(10^{-3})$		
	1	**3**	**5**	**1**	**3**	**5**
2	1035	938	862	192	168	149
4	949	863	795	289	252	222

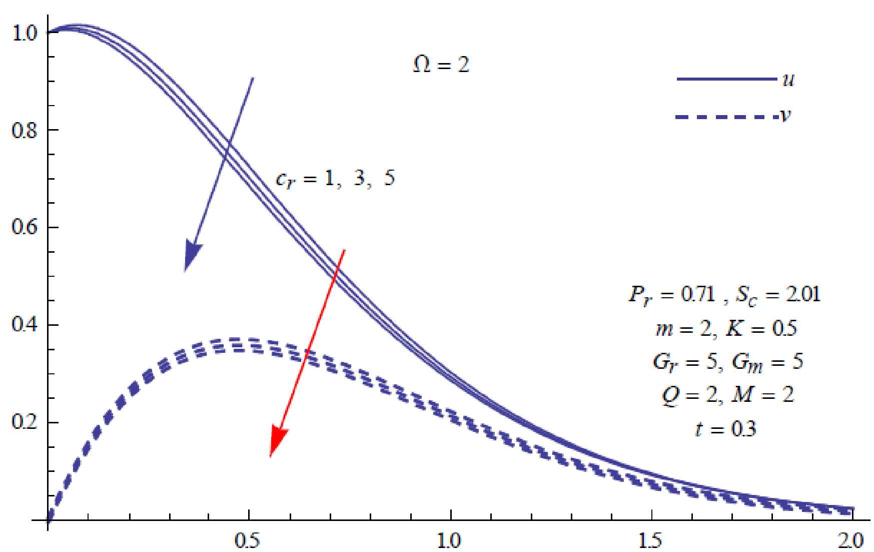

FIG. 1. **Velocity profile for c_r at $\Omega = 2$.**

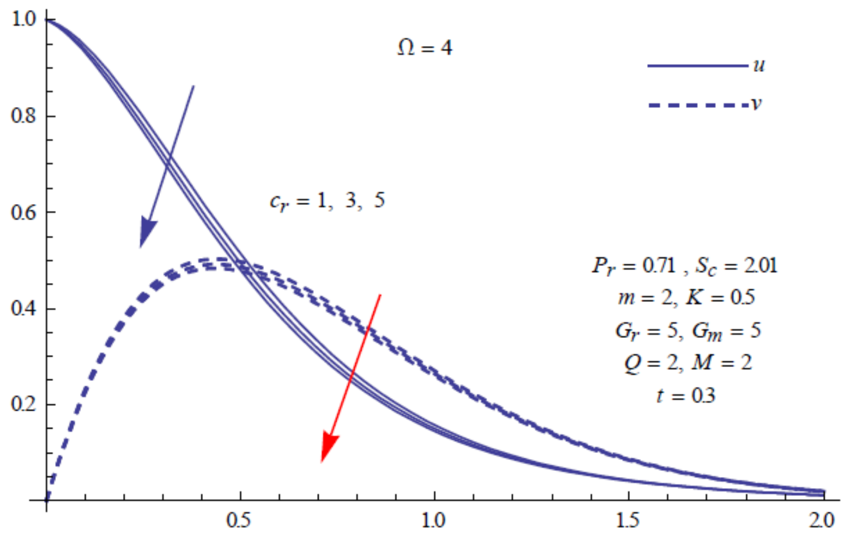

FIG. 2. **Velocity profile for c_r at $\Omega = 4$**

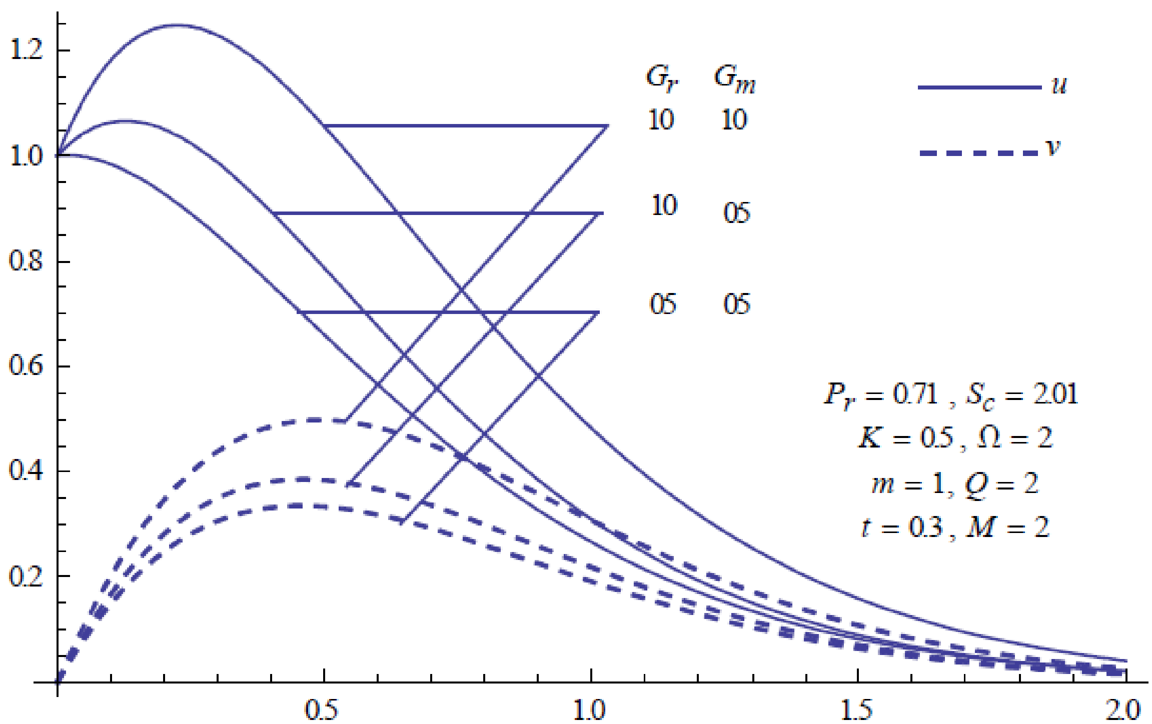

FIG. 3. **Velocity profile for G_r and G_m .**

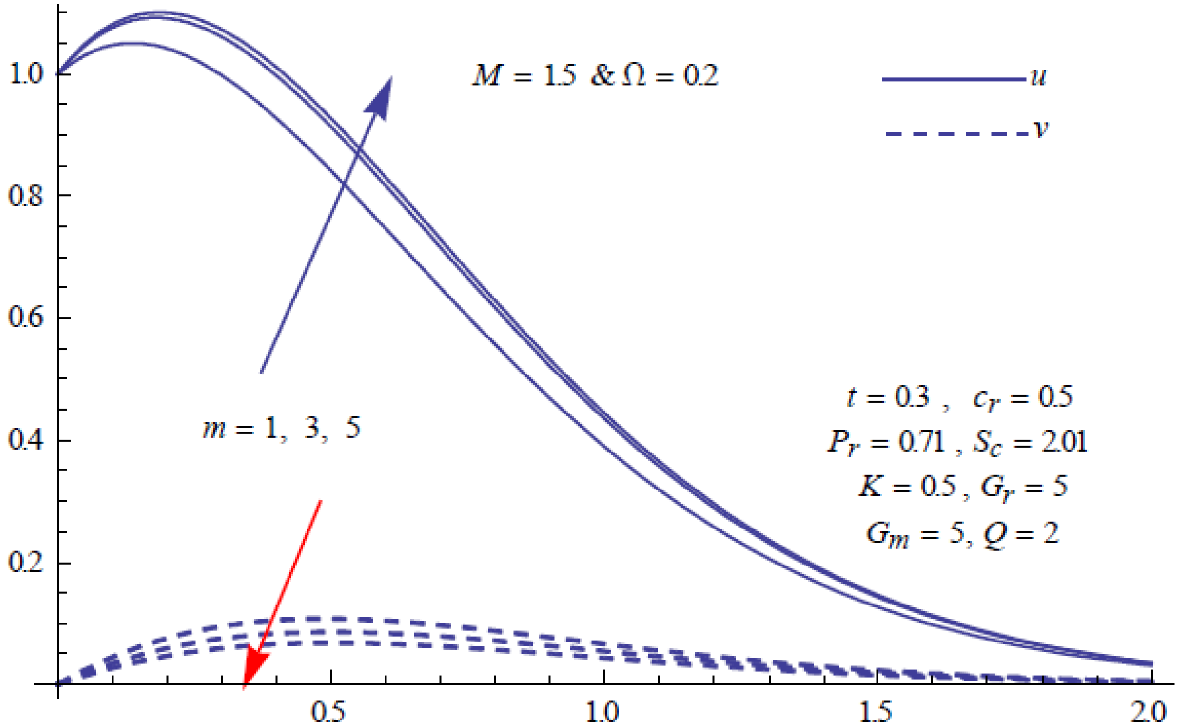

FIG. 4. **Velocity profile m at M = 1.5 and $\Omega = 0.2$**

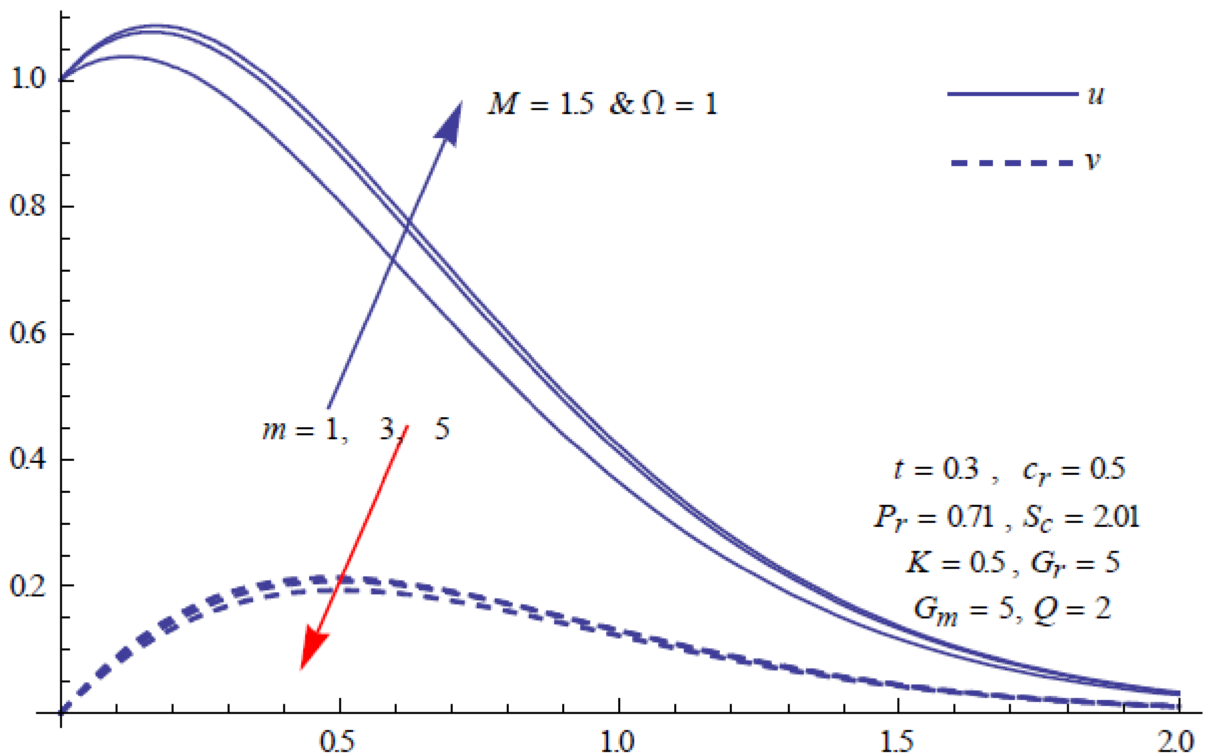

FIG. 5. **Velocity profile for m at M = 1.5 and $\Omega = 1$.**

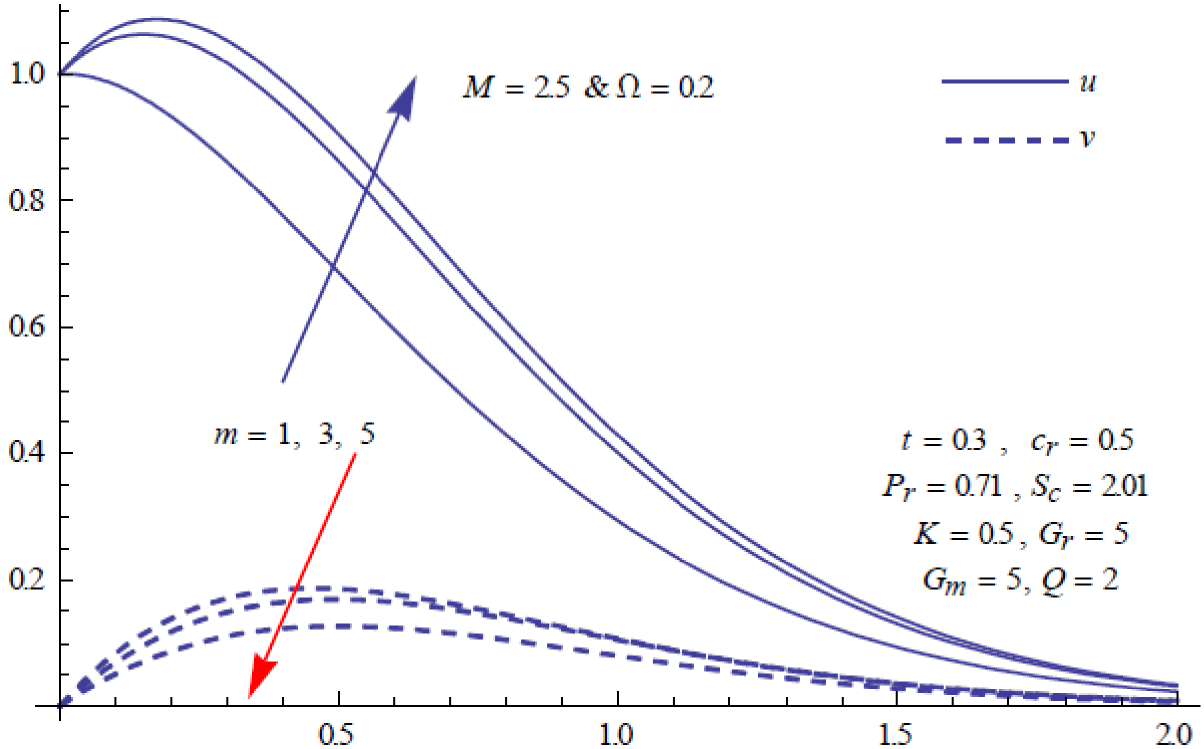

FIG. 6. **Velocity profile for m at M = 2.5 and $\Omega = 0.2$.**

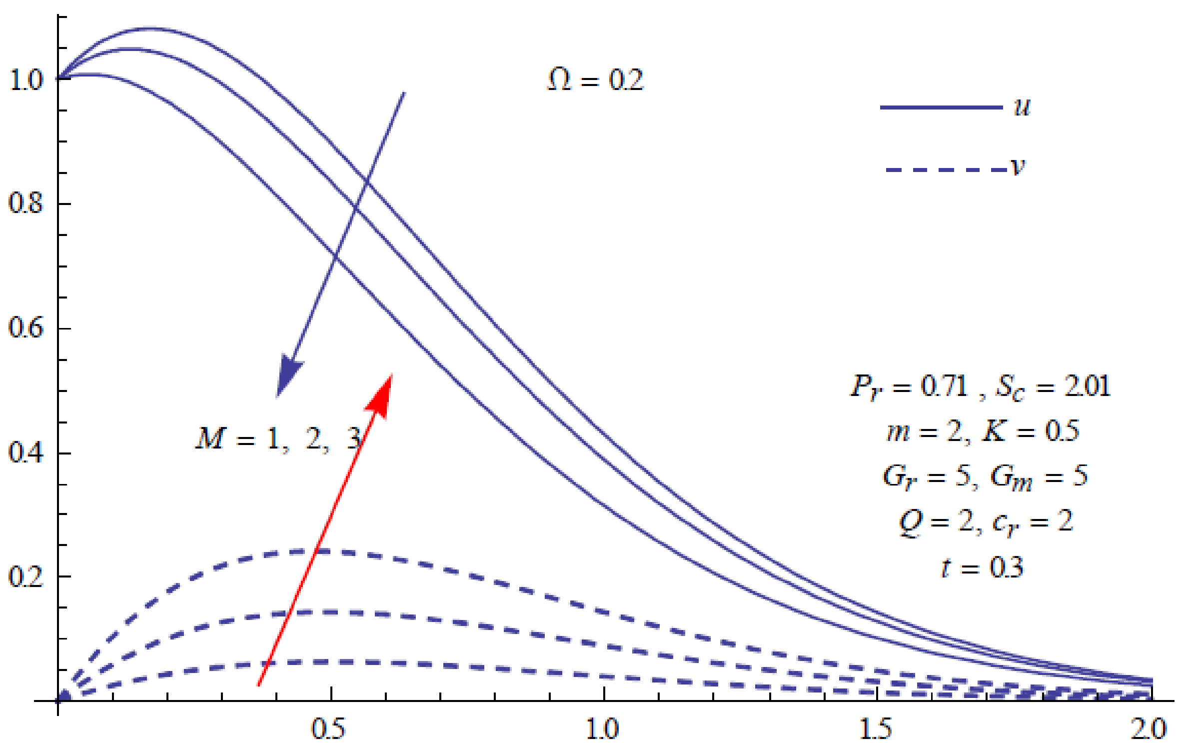

FIG. 7. **Velocity profile for M .**

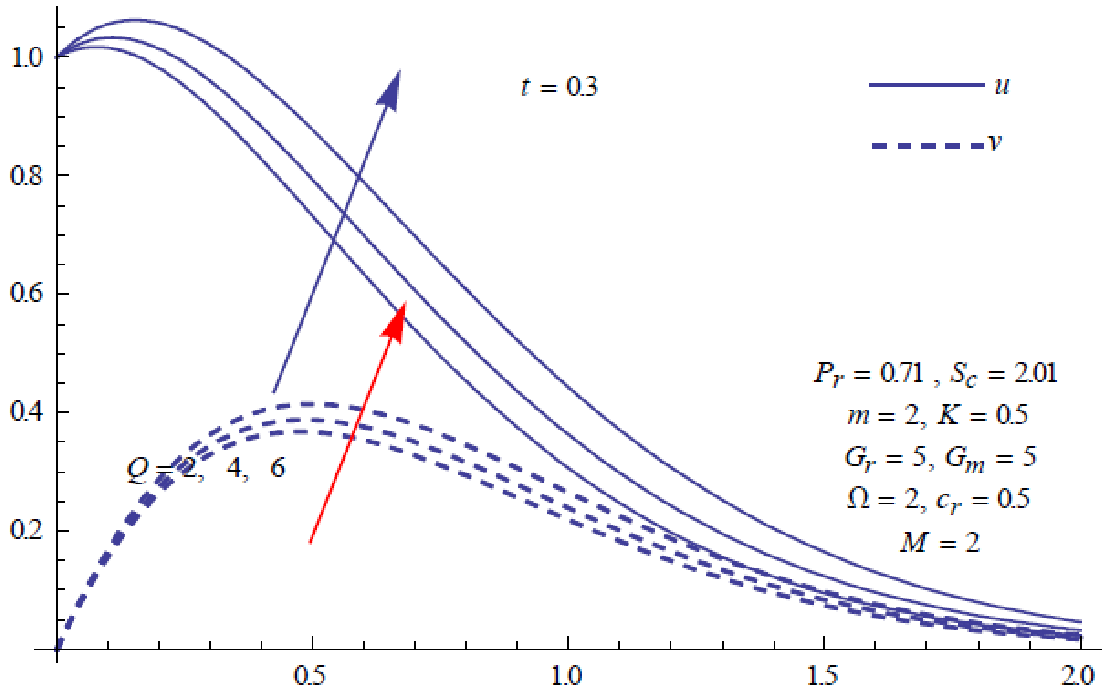

FIG. 8. **Velocity profile for Q at t = 0.3**

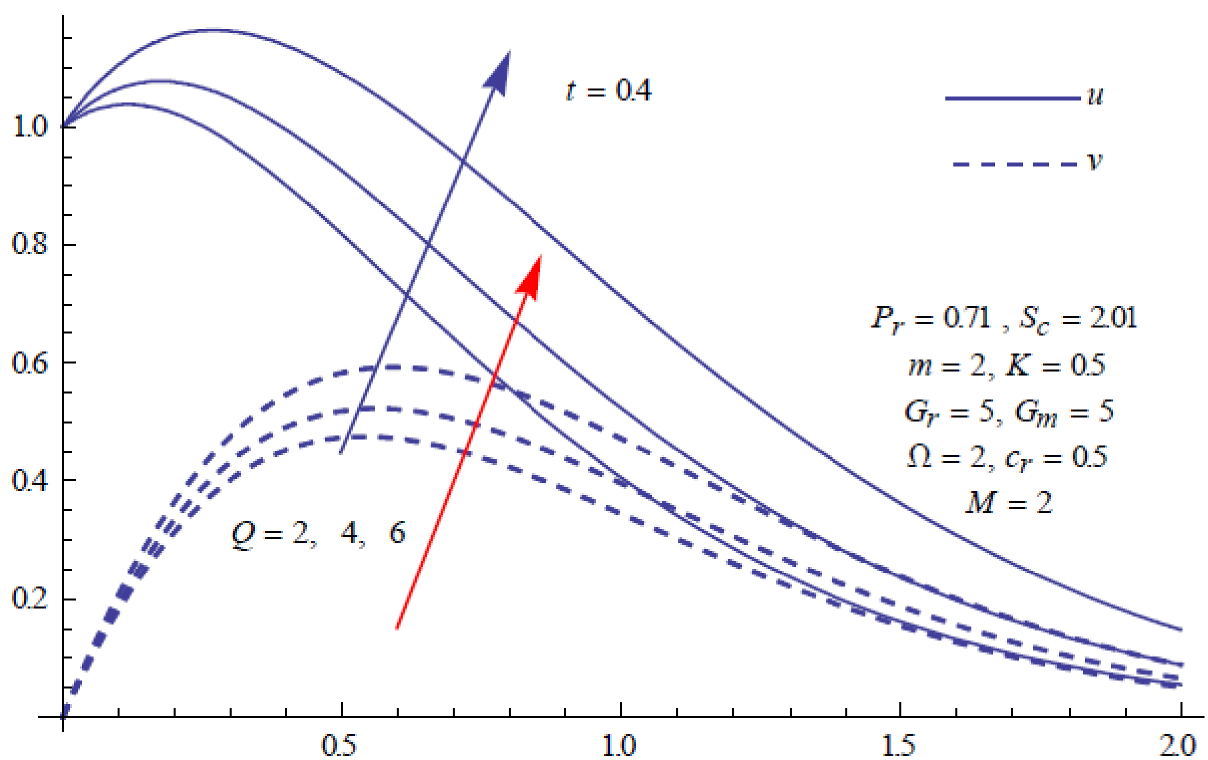

FIG. 9. **Velocity profile for Q at t = 0.4.**

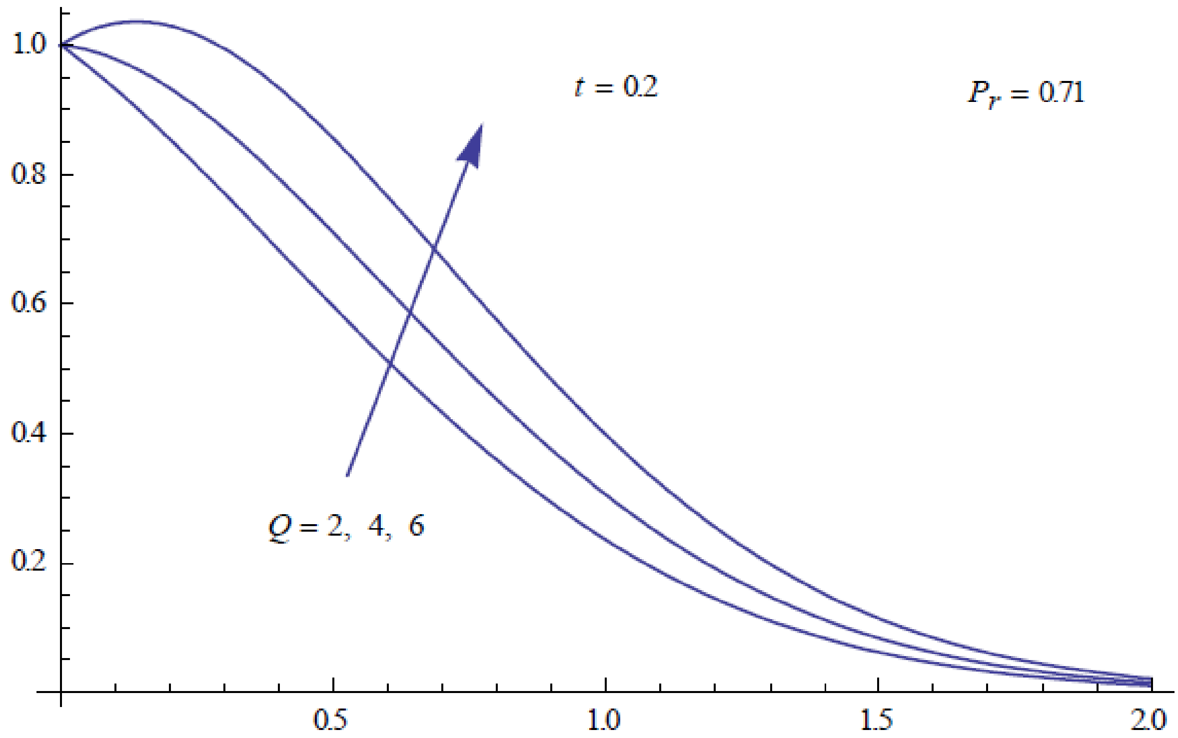

FIG. 10. **Temperature profile *Q* at *t* = 0.2.**

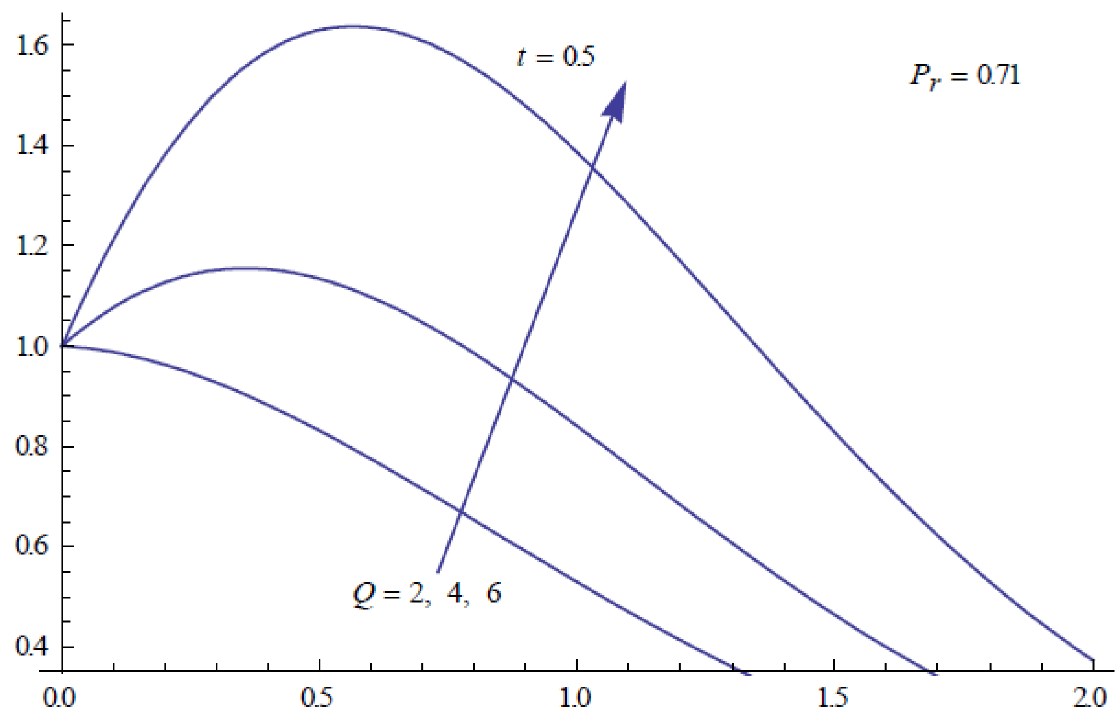

FIG. 11. **Temperature profile for *Q* at *t* = 0.5.**

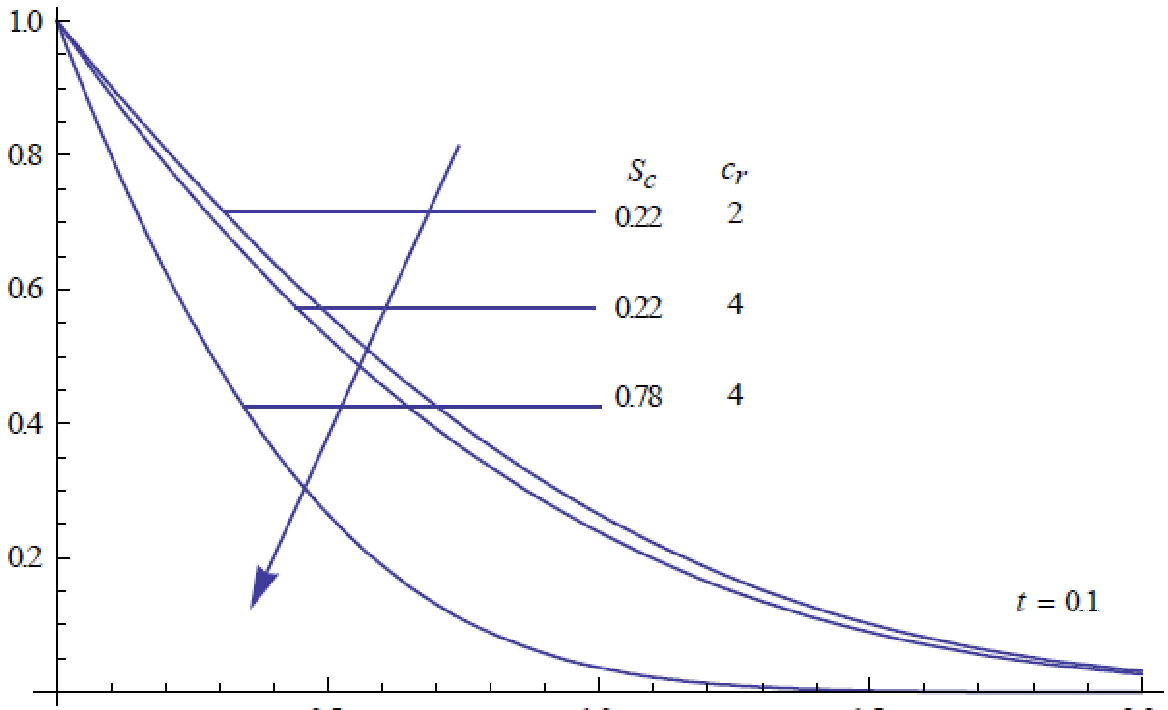

FIG. 12. **Concentration profile for S_c and c_r at t = 0.1**

Conclusion

It is found that Hall current has a tendency to accelerate the primary flow while it retards secondary flow. An increase in the chemical reaction parameter corresponds to decrease in both the component of the fluid velocity as well as the concentration in the system. However, the fluid velocity in both the directions as well as the temperature increases with the increase in heat source parameter. Also, the solution (15) obtained is valid only for $P_r \neq 1$ and $S_c \neq 1$. For the case when $P_r = 1$ and $S_c = 1$, the thicknesses of concentration, momentum and thermal boundary layers are of the same order of magnitude. It is also found that τ_x increases and τ_y gets decreased with c_r .

Symbols

C_p: specific heat at constant pressure
C_r: dimensionless chemical reaction parameter
D: mass diffusion coefficient
g: acceleration due to gravity
G_m: mass Grashof number
G_r: thermal Grashof number
K_e: mean absorption coefficient
K´: permeability parameter
m: Hall parameter
M: magnetic field parameter
P_r: Prandlt number
Q_o: heat source coefficient
Q: dimensionless heat source coefficient
S_c: Scthimth number
t: dimensionless time
u´: primary velocity of the fluid
u: dimensionless primary velocity of the fluid
v´: secondary velocity of the fluid
v: dimensionless secondary velocity of the fluid
z: dimensionless spatial coordinate normal to the plate
α: thermal diffusivity
β: volumetric coefficient of thermal expansion
β*: volumetric coefficient of concentration expansion
θ: dimensionless temperature
ν: kinematic viscosity
ρ: density of fluid
σ_s: Stefan-Boltzmann constant
τ_e: electron collision time
ϕ: dimensionless concentration
ω_e: cyclotron frequency of electron
Ω: dimensionless rotation parameter

τ_e electron collision time

ω_e cyclotron frequency of electron

Ω dimensionless rotation parameter

Appendix

$$b=\frac{M^2 i}{m+i}+2i\Omega+\frac{1}{K}, A_1=\frac{G_r}{1-P_r}, A_2=\frac{G_m}{1-S_c}, A_3=\frac{G_r}{b+Q},\ A_4=\frac{G_m}{b-c_r}, B_1=\frac{b+P_r Q}{b+Q}, B_2=\frac{b-S_c c_r}{1-S_c},\ a_1=\sqrt{b},$$

$$a_2=\sqrt{b-B_1}, a_3=\sqrt{-QP_r}, a_4=\sqrt{-Q}, a_5=\sqrt{P_r}, \mathrm{b}_6=\sqrt{-S_c(B_2-c_r)}, \mathrm{b}_7=\sqrt{c_r-B_2}, \alpha_1=\frac{A_1}{2B_1}, \alpha_2=\frac{A_2}{2B_2},$$

$$a_6=\sqrt{-P_r(Q+B_1)}, a_7=\sqrt{-Q-B_1}, \eta=\frac{z}{2\sqrt{t}}, \alpha_3=\frac{1}{2}-\frac{A_1}{2B_1}-\frac{A_2}{2B_2},\ b_2=\sqrt{b-B_2}, b_3=\sqrt{c_r S_c}, b_4=\sqrt{c_r}, b_5=\sqrt{S_c}.$$

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