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THERMOELASTIC PROBLEM OF A FINITE LENGTH HOLLOW CYLINDER

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ABSTRACT

In this paper, an attempt has been made to solve inverse problems of thermoelasticity of a finite length hollow cylinder occupying the space $D : \alpha \leq r \leq b, \; -h \leq z \leq h$. Marchi-Fasulo transform and Hankel transform techniques are used to obtain the general solution for the set of boundary value problems. Particular types of boundary conditions have been taken to illustrate the utility of the approach. The transformed components of the stresses and temperature distribution have been obtained. A numerical inversion technique is employed to invert the integral transform, and the resulting quantities are presented graphically.

Key words: Hollow cylinder, Thermoelastic problem, Marchi-Fasulo and Hankel transform techniques.

INTRODUCTION

The inverse thermoelastic problem consists of the determination of the temperature of the heating medium, the heat flux on the boundary surfaces of the solid when the conditions of the displacement and stresses are known at some points of the solid under consideration. The inverse problem is very important in view of its relevance to various industrial machines subjected to heating such as main shaft of Lathe and turbine and roll of a rolling mill.

M. Kandula[1] studied transient temperature distribution in a hollow cylinder with a linear variation in thermal conductivity with temperature. The boundary conditions considered were convective heating (Newton's law) at the exposed inner surface and adiabatic outer surface. The solution was obtained using the method of optimal linearization, with the initial solution given by the integral method.

Khobragade et al.,[2] discussed three dimensional coupled thermoelastic response of infinitely long hollow circular cylinder due to axisymmetrical heating, considered under the thermo-mechanical coupling effect. This approach was based upon integral transform techniques, to find the thermoelastic solution. The expression for both the temperature and the stress distribution were determined from field equation of motion. Numerical calculations are carried out and results are depicted graphically.

L. S. Chen & H. S. Chu[3,4] analyzed transient thermal stress distribution of a finite composite hollow cylinder and ring-stiffened hollow cylinder, which was heated by a periodically moving line source on its

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inner boundary and cooled convectively on the outer surface, is analyzed in this paper. The heat sources are assumed to be axisymmetric, moving along the axis of the hollow cylinder with constant velocity. Laplace transform has been used to solve the temperature distribution of the hollow cylinder, and Eigen function expansion methods were used. The associated thermal stress distributions are then obtained by solving the thermoelastic displacement function and the Love function.

Xu-Long Peng et al.,[5] studied thermoelastic problems in a functionally graded hollow circular cylinder by using the Laplace transform techniques and convert the heat conduction equation as well as the thermal stress problem into Fredholm integral equations. By numerically solving the resulting equations and performing numerical inversion of the Laplace transform, the temperature change and thermal stress response in physical domain can be obtained.

Statement of the problem

Consider a hollow cylinder of length h occupying the space $D : \alpha \leq r \leq b, \; -h \leq z \leq h.$ Thermal expansion of the material of the cylinder r and T is the temperature of the cylinder satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\boldsymbol{\theta}}{k} = \frac{1}{\boldsymbol{\alpha}}\frac{\partial T}{\partial t} \quad \ldots(1)$$

where θ is the internal heat source subject to the initial conditions $T(r,z,0) = F(r,z)$ and the boundary conditions

$$T(a,z,t) = 0 \quad \ldots(2)$$

$$T(b,z,t) = 0 \quad \ldots(3)$$

$$T(r,z,t)\,|_{z=0} = 0 \quad \ldots(4)$$

and

$$\left.\begin{aligned} \left(T + k_1 \frac{\partial T}{\partial z}\right)_{z=-h} &= 0 \\ \left(T + k_2 \frac{\partial T}{\partial z}\right)_{z=h} &= 0 \end{aligned}\right\} \quad \ldots(5)$$

Equations (1) t0 (5) constitute the Mathematical formulation of the problem under consideration

Determination of temperature solution

Apply Hankel and Marchi Fasulo transform and taking their inversion of them, we obtain

$$\mathrm{T}(\mathrm{r},\mathrm{z},\mathrm{t}) = \sum_{n=0}^{\infty} \frac{\mathrm{P_m(z)}}{\mu_m} \sum_{n=1}^{\infty} \frac{\mathrm{J_n^2 J_0^2(\lambda_n a)}}{(\lambda_n^2 + \mathrm{r}^2)[\mathrm{J_0^2(\lambda_n a) - J_0^2(\lambda_n^2 b)}]}$$
$$\times[\mathrm{J_0(\lambda_n r)G_0(\lambda_n b) - J_0(\lambda_n b)G_0(\lambda_n r)}] \times \left\{ \frac{\mathrm{K}\bar{\theta}}{\mu_m^2 \mathrm{K} + \lambda_n^2 \mathrm{K}} - \frac{\mathrm{K}\bar{\theta}}{\mu_m^2 \mathrm{K} + \lambda_n^2 \mathrm{K}} \frac{1}{e^{(\mu_m^2 \mathrm{K} + \lambda_n^2 \mathrm{K})t}} \right\} \quad \ldots(6)$$

Determination of displacement solution

$$\varphi(r,z,t) == \frac{r^2}{4}\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{J_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}$$
$$\times[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\} \quad \ldots(7)$$

Determination of stresses components

$$U=\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{J_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times\left\{\frac{r}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]\ +\frac{r^2}{4}[\lambda_n J_0'(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G_0'(\lambda_n r)]\right\} \quad \ldots(8)$$

$$W=\sum_{m=1}^{\infty}\frac{P_m'(z)}{\mu_m}\left\{\frac{r^2}{4}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}$$
$$\times[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\} \quad \ldots(9)$$

$$\sigma_r=(\lambda+2G)\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$

$$\times\left\{\left\{\frac{1}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r}{2}[\lambda_n J_0'(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G_0'(\lambda_n r)]\right\}\right.$$

$$\left.+\left\{\left\{\frac{r}{2}[\lambda_n J_0'(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r^2}{4}[\lambda_n J_0''(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G_0'(\lambda_n r)]\right\}\right.\right.$$

$$+\lambda\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times\left\{\left\{\frac{1}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]\quad+\frac{r}{4}[\lambda_n J_0'(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G_0'(\lambda_n r)]\right\}\right.$$
$$+\left\{\sum_{m=1}^{\infty}\frac{P_m''(z)}{\mu_m}\left[\frac{r^2}{4}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right]\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)] \quad \ldots(10)$$

$$\sigma_z=(\lambda+2G)\left\{\sum_{m=1}^{\infty}\frac{P_m''(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}$$
$$\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}\times[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]$$

$$+\lambda\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times\left\{\left\{\frac{1}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r}{2}[\lambda_n J_0'(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G_0'(\lambda_n r)]\right\}\right.$$

$$+\left\{\sum_{m=1}^{\infty}\frac{P''_m(z)}{\mu_m}\left[\frac{r^2}{4}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right]\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)+\frac{r^2}{4}[\lambda_n^2 J''_0(\lambda_n r)G_0(\lambda_n b)-\lambda_n^2 J_0(\lambda_n b)G'_0(\lambda_n r)]$$

$$+\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}$$
$$\times\left\{\left\{\frac{1}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r}{4}[\lambda_n J'_0(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G'_0(\lambda_n r)]\right\}\right.$$

$$\sigma_\theta=(\lambda+2G)\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times\left\{\left\{\frac{1}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r}{4}[\lambda_n J'_0(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G'_0(\lambda_n r)]\right\}\right.$$

$$+\lambda\left\{\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times\left\{\left\{\frac{1}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]\quad+\frac{r}{4}[\lambda_n J'_0(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G'_0(\lambda_n r)]\right\}\right.$$
$$\times\left\{\left\{\frac{r}{2}[\lambda_n J'_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r^2}{4}[\lambda_n J''_0(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G'_0(\lambda_n r)]\right\}\right.$$

$$+\left\{\sum_{m=1}^{\infty}\frac{P''_m(z)}{\mu_m}\left[\frac{r^2}{4}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right]\right\}$$
$$\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}\times[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]$$

$$\tau_{rz}=G\left\{\sum_{m=1}^{\infty}\frac{P'_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times\left\{\left\{\frac{r}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r^2}{4}[\lambda_n J'_0(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G'_0(\lambda_n r)]\right\}\right.$$

$$+\left\{\sum_{m=1}^{\infty}\frac{P'_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}\right\}\times\left\{\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{\theta}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$
$$\times\left\{\left\{\frac{r}{2}[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]+\frac{r^2}{2}[\lambda_n J'_0(\lambda_n r)G_0(\lambda_n b)-\lambda_n J_0(\lambda_n b)G'_0(\lambda_n r)]\right\}\right.$$

Special Case -

$$F(z,t)=(1-e^{-t})(z+h)^2(z-h)^2(r-a)(r-b)$$

$$\overline{F}(m,t)=\int_{-h}^{h}[(1-e^{-t})(z+h)^2(z-h)^2(r-a)(r-b)]dz$$

$$= (1-e^{-t})(r-a)(r-b)\int_{-h}^{h}(z+h)^2(z-h)^2\,dz$$

$$= (1-e^{-t})(r-a)(r-b)4(k_1+k_2)\left\{\frac{a_n b\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right\}$$

Substituting in equation (4), we get -

$$T(r,z,t)=\sum_{n=0}^{\infty}\frac{P_m(z)}{\mu_m}\sum_{n=1}^{\infty}\frac{J_n^2 J_0^2(\lambda_n a)}{(\lambda_n^2+r^2)[J_0^2(\lambda_n a)-J_0^2(\lambda_n^2 b)]}$$
$$\times[J_0(\lambda_n r)G_0(\lambda_n b)-J_0(\lambda_n b)G_0(\lambda_n r)]\times\left\{\frac{K\bar{F}}{\mu_m^2 K+\lambda_n^2 K}-\frac{K\bar{F}}{\mu_m^2 K+\lambda_n^2 K}\frac{1}{e^{(\mu_m^2 K+\lambda_n^2 K)t}}\right\}$$

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