

Thermal Gradients on Planetary Bodies and the Molar Mass Ideal Gas Law

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Abstract

It has always been complicated mathematically, to calculate the average near surface atmospheric temperature on planetary bodies with a thick atmosphere. Usually, the Stefan Boltzmann (S-B) black body law is used to provide the effective temperature, then debate arises about the size or relevance of additional complicating factors, including the albedo and the greenhouse effect. Presented here is a simple and reliable method of accurately calculating the average near surface atmospheric temperature on planetary bodies which possess a surface atmospheric pressure of over 10 KPa. The formula used is the molar mass version of the ideal gas law. This method requires a gas constant and the measurement of only three gas parameters; the average near-surface atmospheric pressure, the average near surface atmospheric density and the average mean molar mass of the near-surface atmosphere. This indicates that all information on the effective plus the residual near-surface atmospheric temperature on planetary bodies with thick atmospheres, is automatically 'baked-in' to the three mentioned gas parameters. It is known that whenever an atmospheric pressure exceeds 10 KPa, convection then dominates over radiative interactions as the main method of energy transfer, and a rising thermal gradient is formed. This rising thermal gradient continues on down, (if there is a depression or a mine shaft) to even below the average surface level. Given this thermodynamic situation, it is very likely that no one gas has an anomalous effect on atmospheric temperatures that is significantly more than any other gas. In short; there is unlikely to be any significant warming from the greenhouse effect on any planetary body in the parts of atmospheres which are >10 KPa. Instead, it is proposed that the residual temperature difference between the S-B effective temperature and a measured near-surface temperature (the atmospheric effect) is a thermal enhancement which is actually caused by auto-compression.

Keywords: Greenhouse effect; Temperatures of planets; Earth temperature; Venus temperature; Auto-C; Climate sensitivity; Global climate change; Global warming; Atmospheric thermal gradient

Introduction

It is a postulate of this work that any warming effects arising from atmospheric greenhouse gases such as CO₂, are subjected to a 100% rate of negative feedbacks which are inherent in the climate system. The detail of these feedbacks will not be

outlined here, but the open nature of the atmosphere, coupled with the following thermodynamic arguments, indicate that this postulate makes sense.

It is known that planetary bodies which have thick atmospheres, naturally set up a rising thermal gradient in that part of the atmosphere which is higher than a pressure of 10 KPa, (0.1bar) until that bodies’ surface is reached [1] FIG. 1. Less well known is that this rising temperature gradient continues even below the surface [2] making it difficult to attribute to the greenhouse effect. In this denser part of the atmosphere, (on Earth, the troposphere) convection and adiabatic auto-compression effects rule over radiative or ‘greenhouse’ effects in the determination of atmospheric temperatures and the thermal gradient. However, higher up in the atmosphere, once the atmospheric pressure drops below 10 KPa (0.1 bar) then radiative effects dominate. This is because the atmosphere there is too thin to initiate convection or any warming due to auto-compression. Although the term ‘auto-compression’ may be unfamiliar to some, this can be seen as simply an engineering term for what meteorologists call the ‘lapse rate’ and in astronomy is called the ‘Kelvin-Helmholtz’ contraction. Under the latter, the contraction and so compression of a large inter-stellar molecular gas cloud under gravity, achieves such high temperatures that nuclear fusion initiates, and a star is born [3].

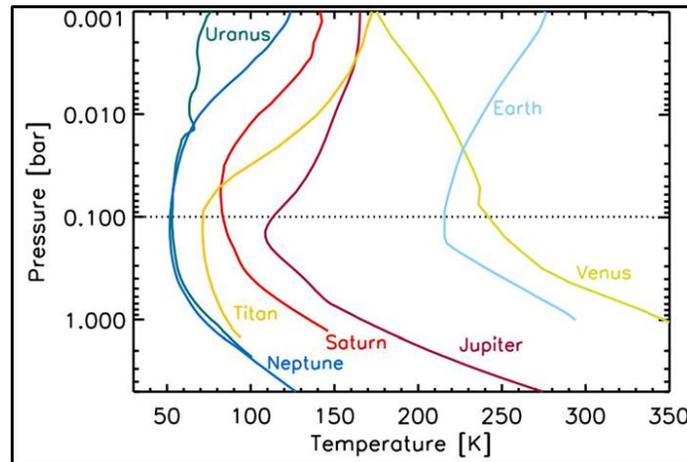


FIG 1. A thermal gradient appears in all planetary atmospheres >10 KPa (0.1 bar) [1]

Using this knowledge, an exacting yet simple method is introduced, which enables the average near-surface atmospheric temperature of any planetary body with an atmospheric pressure of over 10 KPa, to become easily and quickly calculated. A molar version of the ideal gas law is utilised formulas 5 and 6, which consists of one gas constant and three basic atmospheric gas parameters; the average near-surface atmospheric pressure, the average near-surface atmospheric density and the mean molar mass of the near-surface atmosphere.

This formula proves itself here, to be not only more accurate than any other method heretofore used, but is far simpler to calculate. It requires no input from parameters previously thought to be essential for the calculation of atmospheric temperatures; for example, solar insolation, albedo, greenhouse gas content, ocean circulation and cloud cover among many others. The reason these are not required, is because they, (and all others) are already automatically ‘baked-in’ to the three

gas parameters mentioned. Note: although terms for insolation intensity and auto-compression are not used in the formula, it is proposed that these two are still what virtually determine the average near-surface planetary atmospheric temperature.

Venus is the planet which has been hard to explain

There has always been difficulty in explaining, or in formulating a simple method to satisfactorily explain or calculate the very high surface atmospheric temperature of the planet Venus using conventional mathematical means or by employing the greenhouse gas hypothesis. Here, the molar mass version of the ideal gas law will be used to simply and accurately determine the surface temperature of this planet, by the measurement of just three variable gas parameters and the knowledge of one fixed gas constant.

Molar mass version of ideal gas law calculates planetary surface temperatures

The ideal gas law may be used to more accurately determine surface temperatures of planets with thick atmospheres than the S-B black body law [4], if a density term is added; and if kg/m³ is used for density instead of gms/m³, the volume term V may be dropped. This formula then may be known as the molar mass version of the ideal gas law.

The ideal gas law is; $PV = nRT$ (1)

Convert to molar mass; $PV = m/M.RT$ (2)

Convert to density; $PM/RT = m/V = \rho$ (3)

Drop the volume term; $\rho = P/(R.T/M)$ (4)

Find for temperature;
$$T = \frac{P}{\left(R \times \frac{\rho}{M} \right)}$$
 (5)

V= Volume

m= Mass

n= Number of moles

T= Near-surface atmospheric temperature in Kelvin

P= Near-surface atmospheric pressure in KPa

R= Gas constant (m³, KPa, Kelvin⁻¹, mol⁻¹) = 8.314

ρ= Near-surface atmospheric density in kg/m³

M= Near-surface atmospheric mean molar mass (gm/mol⁻¹)

Alternatively, the molar mass version of the ideal gas law can be written thus

$$T = \frac{PM}{R\rho} \tag{6}$$

Methodology involves calculating average near-surface temperature of planets

Formula 5 is here used throughout.

Using the properties of Venus [5],

$$T = \frac{9200}{\left(8.314 \times \frac{65}{43.45}\right)}$$

Venus calculated surface temperature = 739.7K

Using the properties of Earth from Wiki, [6]

$$T = \frac{101.3}{\left(8.314 \times \frac{1.225}{28.97}\right)}$$

Earth calculated surface temperature = 288.14K

Venus is calculated at 739.7K, which is given by NASA as ~740K. Earth is calculated at 288K, currently its quoted by NASA [7] at 288K. It will be noted that the average temperature of the surface of Titan was measured by the Voyager 1, and by the Huygens lander [8] and was probably used as an input to find the surface density; (the independently-measured surface density on Titan could not be found in the literature). The 94K will therefore come out of the below formula, since it is a rearrangement of formula 1. This could be seen as a circular argument. However, it is unlikely that if and when the density of Titan is directly measured, for instance by the use of a dasymeter or similar, it will be significantly different from the 5.25kg/m³ stated here.

Calculate for Titan, data [9];

$$T = \frac{146.7}{\left(8.314 \times \frac{5.25}{28.0}\right)}$$

Titan calculated surface temperature = 93.6K

Calculate for Earth's South Pole, data [10];

$$T = \frac{68.13}{\left(8.314 \times \frac{1.06}{28.97}\right)}$$

Earth's South Pole average calculated temperature = 224K (-49°C) Calculate for Mars;

$$T = \frac{0.69}{\left(8.314 \times \frac{0.02}{43.34}\right)} \qquad T = \frac{0.9}{\left(8.314 \times \frac{0.02}{43.34}\right)}$$

Mars calculated surface temperature = 180K to 234K

The average temperature on Mars is 210K; as suspected from other work [1] this method is inaccurate for Mars, due to the very low and variable atmospheric pressure. Pressures here were measured at the Viking 1 landing site and varied between 690Pa and 900Pa according to the season. It is only in atmospheres with a pressure of over 10 KPa (0.1bar) that strong convection and a troposphere/tropopause is formed, and its associated thermal gradient. Mars is included to demonstrate the

validity of the >10 KPa rule. For Mars, the mid-point between the summer and the winter pressures is used, which results in a temperature of 207K. The gas giants will now be assessed; note that these planets do not have a defined surface like the terrestrials planets have, so here they are given a ‘surface’ by using the Earth’s surface pressure of 101.3 KPa (1 atm) as a level to use for this calculation.

Calculate for Jupiter [7];

$$T = \frac{101.3}{\left(8.314 \times \frac{0.16}{2.2}\right)}$$

Jupiter calculated temperature at 1atm of pressure = 167K

Calculate for Saturn [7];

$$T = \frac{101.3}{\left(8.314 \times \frac{0.19}{2.07}\right)}$$

Saturn calculated temperature at 1atm of pressure = 132.8K

Calculate for Uranus [7];

$$T = \frac{101.3}{\left(8.314 \times \frac{0.420}{2.64}\right)}$$

Uranus calculated temperature at 1atm of pressure = 76.6K

Calculate for Neptune [7];

$$T = \frac{101.3}{\left(8.314 \times \frac{0.450}{2.53}\right)} \qquad T = \frac{101.3}{\left(8.314 \times \frac{0.450}{2.69}\right)}$$

In the case of Neptune, NASA gave two values for mean molar mass; 2.53 and 2.69, this necessitated two separate calculations to give a high and a low of calculated temperatures. Neptune’s calculated temperature at 1atm of pressure = 68.5K to 72.8K. The temperature on Neptune at 1atm of pressure is 72K; this lies quite between the two calculated temperatures (TABLE 1).

TABLE 1. Comparison of calculated and actual average surface temperatures.

Planetary body	Calculated temperature Kelvin	Actual temperature Kelvin	Error
Venus	739.7	740	0.04%
Earth	288.14	288	0.00%
South Pole of Earth	224	224.5	0.20%
Titan	93.6	94	0.42%
Mars (low pressure)	180 to 234	210	1.40%
Jupiter	167	165	1.20%
Saturn	132.8	134	0.89%
Uranus	76.6	76	0.79%
Neptune	68.5 to 72.8	72	0.00%

Analysis about the postulate and hypothesis being presented herein

If this simple relationship between surface atmospheric density, pressure and molar mass is an accurate method of predicting surface temperatures on bodies with a thick atmosphere, it will necessarily be informative about what actually determines these planetary surface temperatures, and will have important implications for climate sensitivity.

In short, a postulate being put forward here, is that in the case of Earth, solar insolation provides the ‘first’ 255 Kelvin – in accordance with the black body law [11]; this being the ‘effective’ or the ‘base’ level. Then gravitationally induced auto-compression provides the ‘other’ 33 Kelvin, termed the ‘residual’, to arrive at the known and measured average global temperature of 288 Kelvin. The ‘other’ 33 Kelvin is not hypothesised to be provided by the greenhouse effect, because if it was, the molar mass version of the ideal gas law would not then work to accurately calculate real planetary temperatures in the case of small incremental changes to gas levels, as it clearly does here.

Temperature in a gas is a measure of the average kinetic energy of the particles in the gas. When atmospheric gas pressure exceeds 10 KPa, a temperature gradient is set up from that pressure level [1], down to the planetary surface (this thermal gradient is known and measured to continue even below the surface, if there is for example, a mine shaft). It is postulated here, that the cause of this thermal gradient is gravity-induced auto-compression. In general terms, the surface temperature sets up convective overturning of the troposphere, which is adiabatic through much of the convection cycle [2], and this combines with gravitationally induced atmospheric auto-compression to create the observed tropospheric thermal enhancement and temperature gradient.

The origins of this thermal effect on gases go back to James Maxwell, who, in his 1872 book ‘Theory of Heat’ [12] demonstrated that the formation of the thermal gradient from the tropopause downwards is assisted by convection and more particularly, the increasing atmospheric pressure, which itself is a result of a combination of the Earth’s gravitational field and the atmospheric density.

“In the convective equilibrium of temperature, the absolute temperature is proportional to the pressure.” [12].

The idea of a thermal gradient naturally forming in any column of gas in a gravitational field was first proposed in the 1860’s by Loschmidt [13]. At the time, Maxwell thought that this idea violated the second law of thermodynamics, yet as has been shown here, derivations of Maxwell’s own ideal gas law is an excellent predictor of temperatures – when the atmosphere is thick enough to be compressed by a gravitational field.

The controversy between Loschmidt on one side, with Maxwell and Boltzmann on the other, raged for some time and was finally experimentally tested in 2007, with the results published by Graeff [14]. Graeff’s experiments concluded that a gravitationally-induced temperature gradient does spontaneously develop in sealed columns of both air and water – the bottom of the column being warmer than the top. The theoretical amounts of warming according to Graeff should be 0.07K/m and 0.04 K/m respectively. Graeff’s experimental apparatus reported 0.07K/m and 0.05K/m – so basically confirming

Loschmidt's predictions. The thermal gradient appeared, despite the reverse gradient being prevalent in the immediate environment of the experiment. Loschmidt originally has said that the second law of thermodynamics needed to be re-stated to include the effects of gravitational fields on fluids.

The adiabatic auto-compression hypothesis enunciated herein, states that convection/pressure/lapse rate effects dominate over radiative effects in regions of all planetary atmospheres >0.1 bar and a temperature gradient is naturally formed. In effect, gravity forms a density and a temperature gradient; pressure is a corollary.

Auto-compression is known and used daily in mining

Auto-compression is well known in underground mining, and is used by ventilation engineers to calculate how hot the mine air will get, so that they know how much cooling air to provide at each level. The effect of auto-compression can be calculated by the following relationship;

$$P_e = P_s \exp(gH/RT)$$

Where;

P_e = Absolute pressure at end of column (KPa)

P_s = Absolute pressure at start of column (KPa)

g = Acceleration due to gravity (m/s^2)

H = Vertical depth (m)

As can be clearly seen, this effect primarily relies on pressure and gravity, which will be different for each planetary body.

Mechanism is adiabatic

Note that we are examining an adiabatic process, and when a gas parcel expands adiabatically, as it does when rising in a gravitational field, it does positive work – and the kinetic energy drops and so the temperature drops. However, when a gas parcel is compressed, as it is when it descends in a gravitational field, then it does negative work, and its kinetic energy rises and so its temperature goes up. Why does the kinetic energy of the gas rise when descending? It's because some of its potential energy is converted to enthalpy, so producing an increase in pressure, specific internal energy and hence, temperature in accordance with the following equation;

$$H = PV + U$$

Where;

H = Enthalpy (J/kg)

P = Pressure (Pa)

V = Specific volume (m^3)

U = Specific internal energy (kinetic energy)

Discussion on Maxwell vs. Arrhenius and the 'Greenhouse effect'

Work in this area of gas physics was detailed in the 19th century. However, there is a strong difference between the work and the views of the researchers Maxwell and Arrhenius. Maxwell's work shows that temperatures in the lower troposphere of Earth are primarily determined by convection and the atmospheric mass/pressure/gravity relationship. Arrhenius's later work

[15] completely ignored this and determined that temperatures in the lower troposphere of Earth are caused by the radiative effects of greenhouse gases. There have been papers critical of Arrhenius’s radiative effects ideas since 1909 [16]. Who is correct is critical to the present, since if Arrhenius is correct, then there should be some concern about CO₂ emissions, if the climate sensitivity is high enough. If Loschmidt’s version of Maxwell’s work is correct, then more CO₂ will have no measurable effect on tropospheric atmospheric temperatures.

What do atmospheric measurements actually show? Measurements [17] of the effects of more CO₂ in the atmosphere appear to strongly support Maxwell. At pressures above 0.1 bar, “the extra CO₂ merely replaces water vapour” and little difference is seen in temperatures – but at pressures below 0.1 bar more CO₂ is measured to cause strong cooling. One of the main problems with the Arrhenius view, appears to be that convection is virtually ignored as a mode of heat transfer; later work shows that only 11% of heat transfer in the troposphere is actually carried by radiation [18]. Whether this can cause significant net warming in an open atmosphere is debatable. A recent paper has supported the Arrhenius view somewhat by quantifying a forcing due to increased atmospheric CO₂ [19]. But there remains a lack of any paper in the literature, which quantifies any warming attributable to increasing atmospheric CO₂ concentrations.

The accuracy and implications of formulas 5 and 6

It is apparent that this simple formula calculates the ‘surface’ temperatures of many planetary bodies in our Solar System accurately FIG 2. Specifically, those which have atmospheres thick enough to form a troposphere (i.e. possessing an atmospheric pressure of over 10 KPa or 0.1bar). These are; Venus, Earth, Jupiter, Saturn, Titan, Uranus and Neptune. All calculated temperatures are within 1.2% of the NASA reported ‘surface’ temperature (except for Mars, which is excluded because it has a much lower atmospheric pressure than 10 KPa). This accuracy is achieved without using the S-B black body law, or the need to include terms for such parameters as TSI levels, albedo, clouds, greenhouse effect or, for that matter, adiabatic auto-compression. All that is required to be able to accurately calculate the average near-surface atmospheric temperature, is the relevant gas constant and the knowledge of three variable gas parameters.

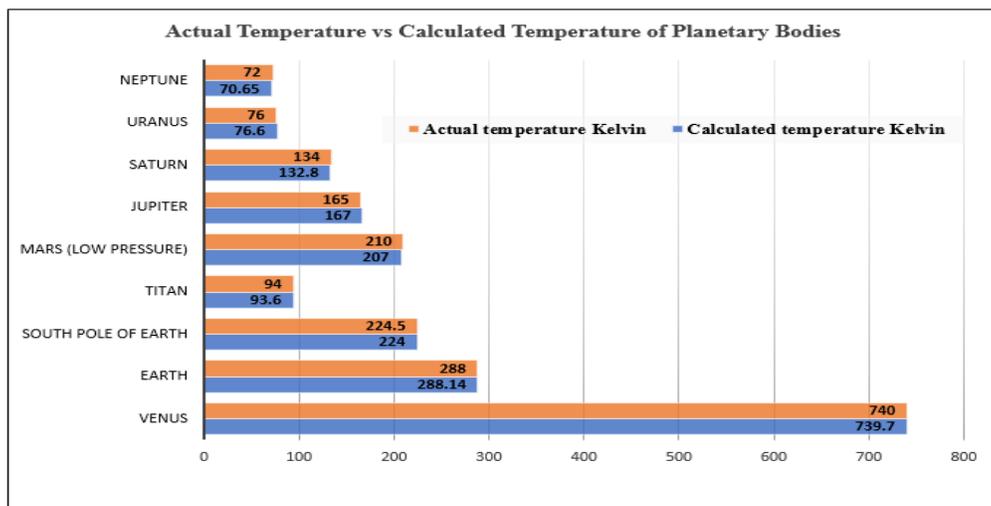


FIG. 2. Actual temperature vs. calculated temperature of 8 planetary bodies and the South Pole

Discussion on the implications for climate sensitivity to CO₂

The reporting of climate sensitivity in the literature has been steadily reducing for decades, with some recent papers pointing to a very low sensitivity, of much less than 1°C [20-22]. A careful reading of these papers, (for example the most recent one) clearly indicates that the 0.6°C cited, is in fact an absolute maximum. This present work, if not invalidated by subsequent work, clearly points to a climate sensitivity so low that it would not be possible to measure it in the real atmosphere; it could even be negative (i.e., more CO₂ creates cooling). This work leads directly to the conclusion that a small change in any single atmospheric gas, such as a doubling of the CO₂ level, (from the so-called 'pre-industrial' 0.03% to 0.06%) can have no measurable positive or negative effect on atmospheric temperatures.

Probable implications for the climate sensitivity to CO₂

Some reflection upon the simplicity and accuracy of these results, combined with knowledge of significant other factors such as the thermal gradient, should bring an unbiased person to some probable implications of this work. These are that the residual near-surface atmospheric temperatures on planetary bodies with thick atmospheres are not mainly determined by the greenhouse effect, but instead most likely by an effect from fluid dynamics, namely; auto-compression. This leads directly to the conclusion that the climate sensitivity to, for example, a doubling of the atmospheric carbon dioxide concentration has to be not only operating instantaneously, but also must be extremely low. Under this scenario, logic spells out that the temperature change caused by the addition of 0.03% of CO₂ in the lower troposphere, cannot be very different to the addition of a similar quantity of any other gas.

To be clear, formulas 5 and 6 appear to rule out any possibility that 33°C of global warming from a 'greenhouse effect' of the type proposed by the IPCC in their reports [23] can exist in the real atmosphere. The reason is that the IPCC state in their reports that a 0.03% increase in atmospheric CO₂, which represents a doubling from pre-industrial levels, must result in a global lower tropospheric near-surface temperature rise of ~3°C; (within a range of 1.5°C to 4.5°C) [24,25] and an even greater temperature rise in the upper troposphere.

This reported level of climate sensitivity to a doubling of atmospheric CO₂, has not changed significantly in the regular IPCC reports since 1990. Anything like this magnitude of warming caused by such a small change in gas levels appears to be completely ruled out by the molar mass version of the ideal gas law. The reason is that inserting any reasonable changes to the three gas parameters caused by the small 0.03% of extra CO₂ into the formula, leads to almost no change in atmospheric temperatures. A reasonable expectation would be that a 0.03% increase in atmospheric CO₂, which is a relatively heavy gas, would result in the following changes in the three gas parameters;

Pressure:	An increase of 0.03%
Density:	An increase of 0.05%
Molar Mass:	An increase of 0.05%

Calculate for a doubling of CO₂ from the pre-industrial level of 0.03% (by volume);

$$T = \frac{101.33}{\left(8.314 \times \frac{1.2256}{28.984}\right)}$$

Calculated temperature after doubling of CO₂ to 0.06% ≈ 288.23K

Climate sensitivity to CO₂ ≈ 288.23 - 288.14 ≈ 0.09K

The change would in fact be extremely small and difficult to estimate exactly, but would be of the order 0.09°C. That is, thirty-three times smaller than the stated ‘likely’ climate sensitivity of 3°C cited in the IPCC’s reports. Even that small number would likely be a maximum change, since if fossil fuels are burned to create the emitted CO₂, then atmospheric O₂ will also be consumed, reducing that gas in the atmosphere – and offsetting any temperature change generated by the extra CO₂. This climate sensitivity is already so low that it would be impossible to detect or measure in the real atmosphere, even before any allowance is made for the consumption of atmospheric O₂.

Conclusion

It has always been complicated mathematically, to calculate the average near surface atmospheric temperature on planetary bodies with a thick atmosphere. Usually, the (S-B) black body law is used to provide the effective temperature, then debate arises about the size or relevance of additional factors such as the greenhouse effect. Here is presented a simple and reliable method of calculating the average near surface atmospheric temperature on planetary bodies which possess a surface atmospheric pressure of over 10 KPa. This method requires knowledge of the gas constant and the measurement of only three atmospheric gas parameters; average near- surface atmospheric pressure, average near surface atmospheric density and the mean molar mass of the atmosphere.

The formula used is the molar mass version of the ideal gas law. It is here demonstrated that the information contained in just these three gas parameters alone is an extremely accurate predictor of average near-surface atmospheric temperatures, in atmospheres >10 KPa. Therefore, all information on the effective plus the residual near-surface atmospheric temperature on planetary bodies with thick atmospheres; residual meaning the difference between the effective, (that predicted by S-B black body law), and the measured actuality, must be automatically ‘baked-in’ to the three mentioned gas parameters.

This leads directly to the conclusion that a small change in any single atmospheric gas, not only has little effect on atmospheric temperatures, but has a very similar effect to the same percentage change in any other atmospheric gas. It is seen therefore, that no one gas particularly affects atmospheric temperatures more than any other gas; so, there can be no significant ‘greenhouse warming’ caused by ‘greenhouse gases’ on Earth, or for that matter on any other planetary body. Instead, it is hypothesised that the residual temperature differences, and the tropospheric thermal gradient observed on planetary bodies, are actually caused by auto-compression.

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