ISSN : 0974 - 7435 Volume 10 Issue 2

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(2), 2014 [259-265]

# The study of the keep-right-except-to-pass rule

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## ABSTRACT

The rule of driving automobiles on the right requires drivers to drive in the right-most lane unless they are passing another vehicle. When overtaking, the drivers move one lane to the left and retune to their former travel lane after passing. First, this paper presents two mathematical models to analyze this rule. Second, the integral equation between the distance which the overtaking vehicle has driven and the distance which the previous $i$ vehicles have driven is formed and the driving hours $t_0$ which the changing line vehicle spends on the changing line conforms to the following condition:

$$t_0 > \frac{v_0 - a_m t_c + \sqrt{(a_m t_c - v_0)^2 + 2a_m C}}{a_m}$$

It justifies that the overtaking vehicles should spend some time on the changing line before returning to the original lane which can reduces the frequency of lane changing and is conducive to the improvement of traffic flow. 

## KEYWORDS

Keep right rule;
The number of the traffic flow;
The traffic flow model;
The congestion index model.

## ASSUMPTIONS

1 The highway in this paper is no ramp and no traffic accident;
2 The highway in the model ignores the influence of other vehicles of the overtaking lane on the overtaking vehicle;
3 The Safety distance between two adjacent car is a constant.

## MODEL ONE

On the basis of the above conditions, the feasibility of traffic rule- the keep right rule can be analyzed by establishment of traffic flow model and traffic congestion index model on the condition of light traffic and congestion.

### The preparation of the models

### Definition

The study of vehicles on the road, if all the vehicles are driving along a single path to the same direction, there is neither overtaking in the fleet, nor vehicles entering or leaving the fleet. Then the traffic flow model-namely traffic flow model through can be built up by comparison of traffic movement on the road to the fluid

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movement in the pipeline.

**Definition**

Suppose at the location of $x$ whose distance from the standing point of the road is, in the period of $(t, t+\Delta t)$ ,the number of vehicles passing through the point is $Q(x,t,t+\Delta t)$, then the instantaneous traffic flow at the location $x$ at the time $t$ is:

$$q(x,t)=\lim_{\Delta t\to 0}\frac{Q(x,t,t+\Delta t)}{\Delta t} \quad (1)$$

Called as the traffic flow and vehicle flow for short.

**Definition**

$Q(x,x+\Delta x,t)$ is the number of vehicles passing through the road section $x$ and $x+\Delta x$ respectively at the time of $t$, then the density of the Instantaneous traffic flow is:

$$\rho(x,t)=\lim_{\Delta x\to 0}\frac{Q(x,x+\Delta x,t)}{\Delta x} \quad (2)$$

Called as the density of the Instantaneous traffic flow at the location $x$ at the time $t$.

**Definition**

Suppose the equation of the motion for a vehicle is $x = x(t)$, then the speed of the vehicle is

$$\dot{x}(t)=u(x(t),t) \quad (3)$$

**The traffic flow model**

First, the change on the number of the vehicles on the road section $(x, x+\Delta x)$ in the time period $(t, t+\Delta t)$ needs to be considered. Then at the location of $x$, the approximate number of cars into the road section is $q(x,t)\Delta t$ ,the approximate number of vehicles out of the road section from the location of $x+\Delta x$ is $q(x+\Delta x,t)\Delta t$. Thus the number of vehicles in this road section in the time period of $(t, t+\Delta t)$ has increased to $(q(x,t)-q(x+\Delta x,t))\Delta t$. Second, the density of the traffic flow has changed from $\rho(x,t)$ to $\rho(x,t+\Delta t)$ during the time period of $\Delta t$. Therefore, the approximate number of vehicles which has increased in the road section is $(\rho(x,t+\Delta t)-\rho(x,t))\Delta x$. On the condition that no vehicles can enter or leave the road section from any other places, so the following equation can hold water:

$$(\rho(x,t+\Delta t)-\rho(x,t))\Delta x=(q(x,t)-q(x+\Delta x,t))\Delta \quad (4)$$

Divide $\Delta x\cdot\Delta t$ on the both sides of the above equation, $\Delta x\to 0, \Delta t\to 0$, then the following equation can be concluded:

$$\frac{\partial\rho}{\partial t}+\frac{\partial q}{\partial x}=0 \quad (5)$$

The above equation is traffic flow continuity equation.

Considering the lane the vehicles will occupy during their overtaking, the mechanical model for traffic flow is following:

$$\begin{cases}\dfrac{\partial\rho}{\partial t}+\dfrac{\partial q}{\partial x}=s\\ \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+\dfrac{1}{\rho}\cdot\dfrac{\partial q(u_I-u)}{\partial x}\\ +\dfrac{\partial q}{\partial x}\cdot\dfrac{(u_w-u)}{\rho}-\dfrac{(u_w-u)s}{\rho}=0\end{cases} \quad (6)$$

In the above equation, $u_w$ is wave velocity and $u_I$ is the maximum of the wave velocity.

The analytic solutions of equation (6)can be calculated by the Analysis of characteristic line. The relation among the traffic flow, density and the rate of line changing is as follows:

$$q=\begin{cases}u_f(m\rho+(1-m)\rho_f e^a), & \rho_f e^a\le\rho<\rho'',0<a\le 1\\ u_f(m\rho+(1-m)\rho_f e^a), & \rho_f e^a\le\rho<\rho' e^a,-1\le a\le 0\\ \dfrac{1}{4}mu_f\rho_j e^a, & \rho' e^a\le k<0.5\rho_j e^a,-1\le a\le 0\\ mu_f(\rho-\dfrac{\rho^2}{\rho_j}e^{-a}), & 0.5\rho_j e^a\le\rho\le\rho_j e^a,-1\le a\le 0\end{cases} \quad (7)$$

In above equation, $u_f$ is free velocity, namely the speed limit of the road. $\rho_f$ is the maximum density at the speed of free velocity. $m$ is the coefficient of wave velocity. $\rho_j$ is the jam density at the speed of $0$.

$$\rho'=\frac{\rho_j}{4}-\frac{\rho_f(1-m)}{m},\ \rho''=\frac{\rho_j}{4}-\frac{\rho_f(1-m)e^a}{m}$$

$a$ **is the rate of line changing, namely the ratio between the flow of the overtaking vehicles and the traffic flow on the lane.**

## The solution of the traffic flow model

It is known from the equation(7)and Figure 1, if the number of the lane changing( $a > 0$ ), it will only increase the throughput at the high speed and low density and it will not affect the throughput of the road. If the number of the lane changing $(a < 0)$ , it will reduce the throughput of the road. On the basic road section of a closed highway, each line changing is the process that the vehicle leaves one lane and then enters another lane. If the vehicle leaves the lane, it will reduce the capacity of throughput of the original lane. If the vehicle enters another lane, it will have no effect on the capacity of the occupied lane. Therefore, the line changing on the basic road section (including the whole process of the vehicle leaving one lane and then entering another lane) will only reduce the capacity of throughput of the whole road. On the basis of (7), the relationship between the average frequency of lane changing per lane one the whole road and the capacity of throughput. It is known from equation(8), the increase on the frequency of lane changing will reduce the capacity of throughput of the road.

$$C = q_{max} = \frac{1}{4} m u_f \rho_j e^a \quad -1 \le a \le 0 \qquad (8)$$

## Data analysis

The data in chart 1 is from 12 observation sites of Beijing Shenyang expressway for 48 hours. The number of the traffic flow is $/(pcu \cdot h^{-1} \cdot \ln^{-1})$.

Chart 2 is about the relation between the number of the traffic flow and the frequency of the line changing. According to the statistic data, the frequency of the line changing is at the maximum when the traffic flow is neither in congestion nor sparsity. the frequency of the line changing is less frequent, when the traffic flow is sparse or smooth and the frequency of the line changing become the least when the traffic flow is in congestion. When the vehicle is changing the line, it will cause conflicting points which will affect the traffic, reducing the traffic capacity, lowering the quality of the road service and the operation safety of the freeway, resulting in traffic congestion and accident.

When the traffic flow is sparse or smooth, there are a lot of gaps on the driving lane. The road condition can meet the drivers' needs and their expectations of speed, therefore, they do not need to overtake, which reduces the frequency of lane changing.

When the traffic flow is in the intermediate state of smooth and jam, the vehicle at the slow speed have negative effect on the rear vehicle which can not meet the drivers' expectations of the speed, so the overtaking lane provides the possibility of overtaking and boosts the drivers' expectation of the speed. Overtaking and lane changing become frequent.

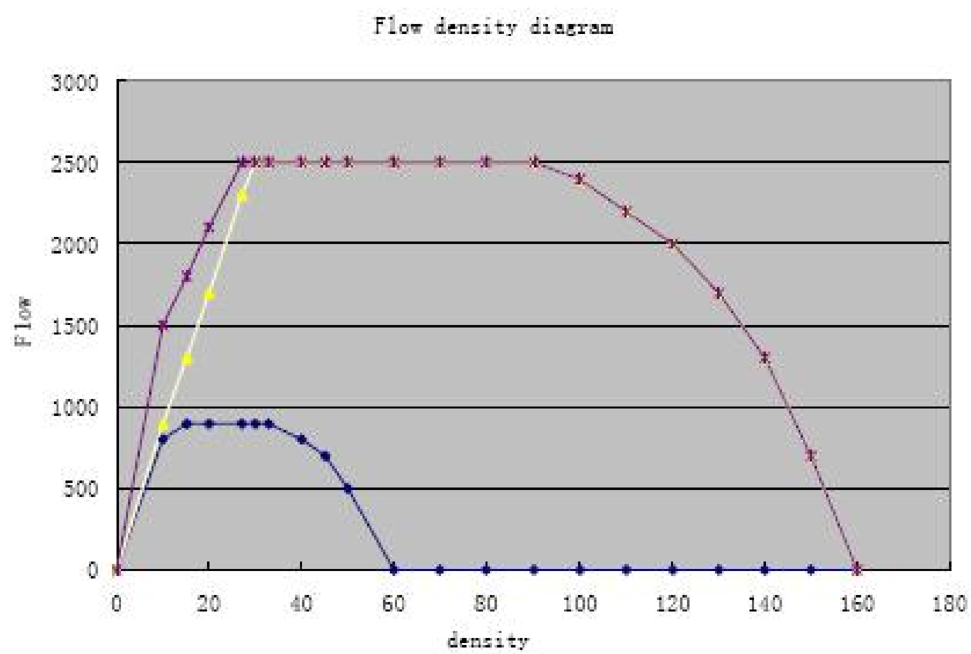


**Chart 1 : Flow density diagram**

TABLE 1

| The number of the traffic flow | The frequency of the line changing | The number of the traffic flow | The frequency of the line changing | The number of the traffic flow | The frequency of the line changing |
|---|---|---|---|---|---|
| 230 | 0.090 | 458 | 0.160 | 696 | 0.140 |
| 256 | 0.093 | 468 | 0.180 | 716 | 0.160 |
| 288 | 0.095 | 480 | 0.220 | 736 | 0.130 |
| 350 | 0.200 | 486 | 0.180 | 756 | 0.130 |
| 370 | 0.260 | 505 | 0.230 | 772 | 0.140 |
| 385 | 0.230 | 516 | 0.160 | 790 | 0.130 |
| 396 | 0.240 | 532 | 0.170 | 800 | 0.110 |
| 408 | 0.170 | 548 | 0.160 | 828 | 0.130 |
| 413 | 0.160 | 564 | 0.200 | 840 | 0.140 |
| 422 | 0.200 | 588 | 0.140 | 852 | 0.130 |
| 428 | 0.240 | 600 | 0.150 | 876 | 0.090 |
| 438 | 0.240 | 614 | 0.140 | 889 | 0.076 |
| 445 | 0.250 | 656 | 0.130 | 896 | 0.060 |
| 449 | 0.180 | 672 | 0.110 | 908 | 0.058 |

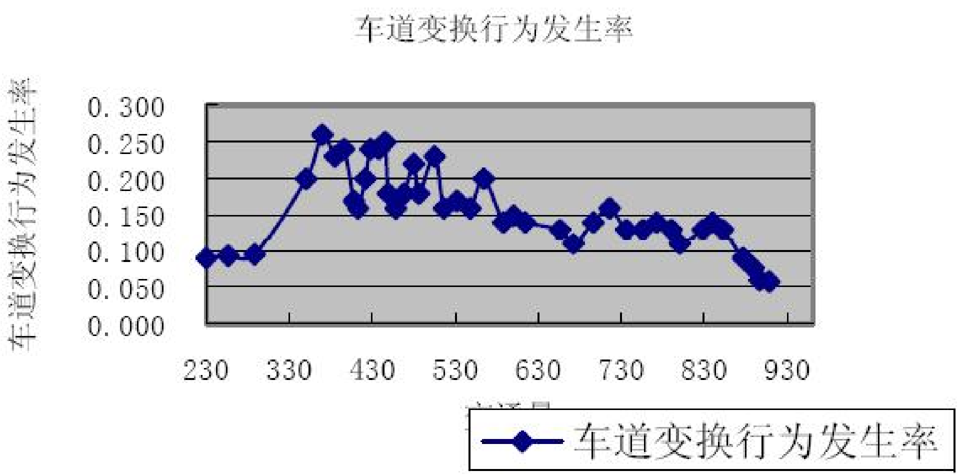


**Chart 2 : The diagram of relations**

When the traffic is in the congestion, the driving speed on the driveway is low and the gap between the vehicles are small which prevent the drivers from over-taking. Thus the frequency of the line changing is less frequent.

## Traffic congestion index model

Suppose at the time of $t$ the number of the vehicles is $Q$ on the highway $\Delta x$ in length.

### Definition

On the highway, $Q_i(x, x+\Delta x, t)$ symbols the time $t$. The number of the vehicles on the lane $i$ from the starting point $x$ to the ending point $x+\Delta x$ can be as follows:

$$\rho_i = \frac{Q_i(x, x+\Delta x, t)}{\Delta x}$$

The equation is at the moment of $t$ the traffic density on the lane $i$ which is $\Delta x$ in length.

### Definition

For a two-way four lane highway, traffic density is $\rho_1$. The traffic density of the overtaking lane is $\rho_2$.

The ratio is $R = \frac{\rho_2}{\rho_1 + \rho_2}$. $R$ is called as the degree of the traffic congestion assessment index at the moment of $t$ on the road section $\Delta x$ of a two-way four lane highway to the same direction.

The above assessment index reflects the degree of the congestion and sparsity on the highway at the moment of $t$. $R = R(t)$ is a new method to assess the degree of the congestion. For bidirectional multi lane, the traffic density of the overtaking lane needs to be considered as numerator. Thus

$$R = \frac{\rho_n}{\sum_{i=1}^{n} \rho_i} \qquad (9)$$

When the ratio approaches to 0, the vehicles on the road are sparse. It can be concluded that the number of the overtaking vehicles is sparse at the moment of $t$ on the road section. The traffic is smooth on the lane. When the ratio is increasing, it can be concluded that the number of the overtaking vehicles is increasing. When the ratio is around $\frac{1}{n}$, it can be concluded that it is congestion on this road section.

**The solution of the traffic congestion index model**

On the condition of sparse traffic, the distance between vehicles is larger and there is less occurrence of overtaking. $R$ is small and approaches to 0. On the condition of congestion, the distance between vehicles is narrower which can not meet the drivers' expectations of the driving at the high speed and the drivers will tend to drive on the overtaking lane to meet their expectations on speed. On a one way four lane highway, $R$ is approaching to $\frac{1}{2}$ to one direction. The drivers start to overtake other vehicles and change to another lane which will increase the traffic pressure of the overtaking lane and also cause the traffic jam on the overtaking lane. At the same time it will affect the capacity of the throughput.

On the basis of the traffic flow at the different time of Chengdu freeway (Figure 2), the chart of time and congestion index can be drawn as follows (chart 2)

**The ratio of congestion**

On the actual condition of the freeway, the traffic flow is comparatively heavier in the morning and in the afternoon comparing to the other time period. The road capacity of throughput is reducing. The frequency of the lane changing and overtaking are both increasing. It can be seen from the chart that the higher of the fre-

**TABLE 2**

| time | traffic flow | The flow of overtaking vehicles | The ratio of congestion |
|---|---|---|---|
| 7:00 | 6145 | 701 | 11.40% |
| 8:00 | 11513 | 2372 | 20.60% |
| 9:00 | 11885 | 2520 | 21.20% |
| 10:00 | 10710 | 2121 | 19.80% |
| 11:00 | 9459 | 1286 | 13.60% |
| 12:00 | 8559 | 1061 | 12.40% |
| 13:00 | 9697 | 1503 | 15.50% |
| 14:00 | 10401 | 1966 | 18.90% |
| 15:00 | 10669 | 2070 | 19.40% |
| 16:00 | 11608 | 2310 | 19.90% |
| 17:00 | 12713 | 2771 | 21.80% |
| 18:00 | 10459 | 1998 | 19.10% |
| 19:00 | 6878 | 798 | 11.60% |

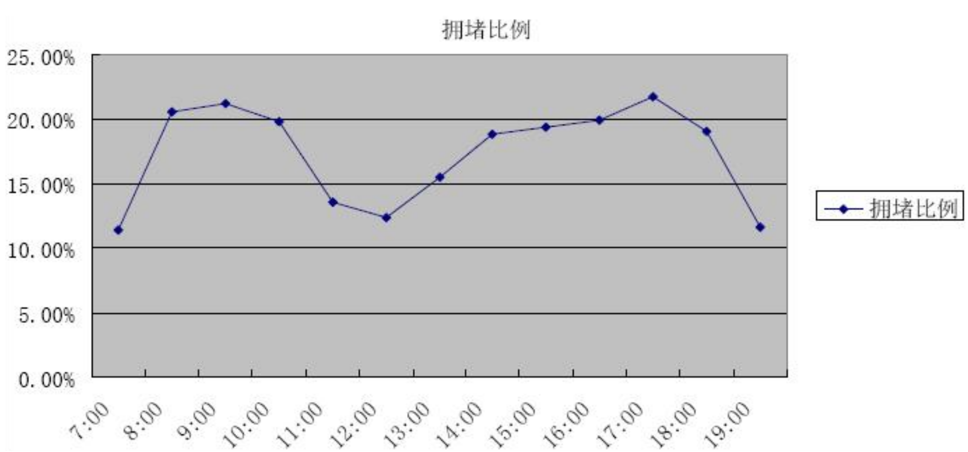


**Chart 3 : Time and congestion index**

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quency of the lane changing is, the higher the congestion index will be. So the conclusion can be made as follows: In a certain range the frequency of the lane changing is in direct proportion to the congestion index. When the congestion index is increasing, the road capacity will decrease.

## MODEL TWO

Multi –lane highway requests the driver to drive on the most right lane. When overtaking, the drivers move to a lane on the left. After the overtaking, the driver drives back to the original lane which will occur two lane changes in the overtaking process. From the analysis of the problem four, the increasing of the lane changing frequency will be reduce the road capacity.

Suppose the two vehicles A and B drive on the highway at the initial speed of $v_1$ and $v_2$ respectively, $v_2 < v_1$, and the two vehicles need to keep the safety distance $L_s$. B starts to overtake at the accelerated speed $a$ and moves to the left lane. The driving distance of B can be calculated by the physical kinematics is $v_2 t + \frac{1}{2}at^2$. A is driving at the same initial speed and its driving distance is $v_1 t$. When B is overtaking A, $v_2 t + \frac{1}{2}at^2 - v_1 t = 2L_s$ and $(v_2 - v_1)^2 - 4\times\frac{1}{2}a\times(-2L_s) > 0$ is known, so the overtaking time is $t = \frac{-(v_2 - v_1) + \sqrt{(v_2 - v_1)^2 + 4aL_s}}{a}$. suppose the highway is $L$ in length. B is driving at the steady speed of $v_2$. If A can overtake safely constantly, then the number of overtaking of A in the whole journey is:

$$N = \frac{L}{v_2 \cdot t} = \frac{L \cdot a}{v_2(-(v_2 - v_1) + \sqrt{(v_2 - v_1)^2 + 4a \cdot L_2})} \tag{10}$$

The accelerated speed $a$ has a greater influence on the number of overtaking comparing to other factors. Thus the maximum number of overtaking $N = \frac{L}{2L_s}$ can be analyzed on the distance of overtaking. The condition of successive overtaking is also included in the equation. So the distance of the overtaking must be limited.

The limited driving distance is transformed to the limited minimum driving time. So the integral equation between the driving distance of the overtaking vehicle and the driving distance of the number of $i$ vehicles can be set up.

$$\int_{t_c}^{t_c+t_0} (v_{t_c} + a_m t)dt > v_0 t_0 + (i+1)L_s + \sum_{i=1}^{n} L_i \tag{11}$$

$t_c$ represents the time for line changing $(s)$. $t_0$ represents the time length of overtaking $(s)$, $t$ is time, represents the speed of the overtaking vehicle at the time of $t_c$ $(m/s)$. $v_0$ is the initial speed of the car in front $(m/s)$. $a_m$ the maximum accelerated speed of the overtaking vehicle $(m/s^2)$. $L_s$ is the safety distance between two vehicles $(m)$. $L_i$ is the length of the vehicle $(m)$. $v_0 t_0$ is the driving distance of the vehicle in front. $\sum_{i=1}^{n} L_i$ is the total length of the previous $i$ vehicles

The solution of(11) is:

$$\frac{1}{2}a_m t_0^2 + (a_m t_c - v_0)t_0 > (i+1)L_s + \sum_{i=1}^{n} L_i \tag{12}$$

the driving time of the overtaking vehicle on the overtaking line $t_0$ from the solution of (12)meets

$$t_0 > \frac{v_0 - a_m t_c - v_{t_c} + \sqrt{(a_m t_c - v_0 + v_{t_c})^2 + 2a_m C}}{a_m} \tag{13}$$

Where $C = (i+1)L_s + \sum_{i=1}^{n} L_i$

In all, the overtaking vehicle can not go back to the original lane immediately after the overtaking. It should remain on the overtaking lane for a period of time and then go back its original lane. The frequency of lane changing can be reduced and the number of the traffic flow can be improved by this way.

## REFERENCES

[1] Yongjie Tan, Zhijie Cai; Mathematic models [M]. Shanghai:Fudan university Press, 389-391 **(2011)**.

[2] Bai Wei, Li Cunjun; Overtaking Model Based on Different Limiting Speed [J]. Journal of Transportation Systems Engineering and Information Technology, **13(2)**, 64-95 **(2013)**.

[3] Xu Huizhi, Cheng Guozhu, Pei Yulong; Study on effect of lane-changing behavioral characteristic to velocity [J]. Sciencepaper online, **5(10)**, 749-753 **(2010)**.

[4] Yang Xiaobao, Zhang Ning; The Mathematic Analysis of the Number of Lanes in Road Capacity [J]. Journal of Wuhan University of Technology, **32(4)**, 604-605 **(2008)**.

[6] Xiong Lieqiang, Wang Fu, Li Jie, Peng Shaomin; The Relationship between the Parameters of Mixed Traffic Flow [J], Journal of Wuhan University of Technology (Transportation Science&Engineering), **33(7)**, 98-99 **(2005)**.

[8] D.D.Salvucci, A.Liu; The time course of lane-changing: Driver control and eye-movement behavior [J]. Transportation Research part F: Traffic Psychology and Behavior, **5(2)**, 123-132 **(2002)**.