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The solution to matrix equation in complex domain

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ABSTRACT

The article illustrates the matrix decomposition method applied to finding total solutions to matrix equation. It is known that, there are minimal polynomial and characteristic polynomial for matrix order n they played an important part in matrix theory. The subject of this article is the inverse problem of above questions, the solution to this question should have certain contribution in matrix theory and calculation.

KEYWORDS

Matrix equation; Jordan block; Formal derivative.

INTRODUCTION

This paper get the following theorem

Theorem if $a_i\left(i=0,1,\cdots,n\right)$ is any plural, matrix equation $a_nA^n+a_{n-1}A^{n-1}+\cdots+a_1A+a_0I=0$ all the solution of the Matrix equation has the form[1,2]

$$P\begin{pmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_k \end{pmatrix}p^{-1}$$

The "P" is invertible matrix,

$$J_i=\begin{pmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{pmatrix}$$ is Jordan block $\left(i=1,2.\cdots,k\right)$,

λ_i is the solution of the N degree univariate polynomial equation $f\left(\lambda\right)=a_n\lambda^n+a_{n-1}\lambda^{n-1}+\cdots+a_1\lambda+a_0=0$; And r_i , as the stage of J_i , is no more than λ_i , which is the root of the multiplicity of the $f\left(\lambda\right)=0$.

EMPRICAL STUDY

In order to prove this theorem, we need to prove the following lemma firstly.

Lemma 1 Set A as n stage phalanx, there is invertible matrix P makes[3]

$$P^{-1}AP=\begin{pmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_k \end{pmatrix}$$ $J_1,J_2,\cdots,J_k$ as Jordan block

Prove see in Page45～47.

Lemma 2 if $J=\begin{pmatrix} \lambda & 1 & & 0 \\ & \lambda & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda \end{pmatrix}$ is r stage Jordan block, k is Positive integer, then for any positive integer k has

$$J^k=\begin{pmatrix} \lambda^k & C_k^1\lambda^{k-1} & C_k^2\lambda^{k-2} & \cdots & C_k^{r-1}\lambda^{k-(r-1)} \\ & \lambda^k & C_k^1\lambda^{k-1} & & \vdots \\ & & \ddots & \ddots & C_k^2\lambda^{k-2} \\ & & & \ddots & C_k^1\lambda^{k-1} \\ 0 & & & & \lambda^k \end{pmatrix}$$

$$=\begin{pmatrix} \lambda^k & \frac{(\lambda^k)'}{1!} & \frac{(\lambda^k)''}{2!} & \cdots & \frac{(\lambda^k)^{(r-1)}}{(r-1)!} \\ & \lambda^k & \frac{(\lambda^k)'}{1!} & & \vdots \\ & & \ddots & \ddots & \vdots \\ & & & \ddots & \frac{(\lambda^k)'}{1!} \\ 0 & & & & \lambda^k \end{pmatrix}$$

Prove using mathematical induction card

We can get lemma 3 from lemma 2 directly.

Lemma 3 If $f(\lambda)=a_n\lambda^n+a_{n-1}\lambda^{n-1}+\cdots+a_1\lambda+a_0$ is Complex coefficients polynomial, J is r stage Jordan block,

Then,

$$f(J)=\begin{pmatrix} f(\lambda) & \frac{f'(\lambda)}{1!} & \frac{f''(\lambda)}{2!} & \cdots & \frac{(f(\lambda))^{(r-1)}}{(r-1)!} \\ & f(\lambda) & \frac{f'(\lambda)}{1!} & \cdots & \frac{(f(\lambda))^{(r-2)}}{(r-2)!} \\ & & \ddots & \ddots & \vdots \\ & & & \ddots & \frac{f'(\lambda)}{1!} \\ 0 & & & & f(\lambda) \end{pmatrix}$$

Lemma 4 if λ_0 is the multiple solution of r, which is the solution of $f(\lambda)=a_n\lambda^n+a_{n-1}\lambda^{n-1}+\cdots+a_1\lambda+a_0=0$, then $f(\lambda_0)=f'(\lambda_0)=\cdots=f^{(r-1)}(\lambda_0)=0$,

But $f^{(r)}(\lambda_0)\neq 0$.

Prove is omit.

From lemma 3 and lemma 4, we can get:

Lemma 5 The Jordan block $J=\begin{pmatrix} \lambda_0 & 1 & & 0 \\ & \lambda_0 & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_0 \end{pmatrix}$ to meet the Matrix equation $a_nA^n+a_{n-1}A^{n-1}+\cdots+a_1A+a_0I=0$ and the necessary and sufficient condition is: λ is the solution of $f(\lambda)=a_n\lambda^n+a_{n-1}\lambda^{n-1}+\cdots+a_1\lambda+a_0=0$, and the stage J is no more than the multiple number of λ_0, which is the solution of $f(\lambda)=0$.

CONCLUSIONG

Sufficiency. If $A=P\begin{pmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_k \end{pmatrix}P^{-1}$, and $P, J_1, J_2, \cdots, J_k$ as the established condition of the theorem $f(J_1)=f(J_2)=\cdots=f(J_k)=0$, so

$$f(A) = f\left[P\begin{pmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_k \end{pmatrix}P^{-1}\right]$$

$$= P\begin{pmatrix} f(J_1) & & & 0 \\ & f(J_2) & & \\ & & \ddots & \\ 0 & & & f(J_k) \end{pmatrix}P^{-1} = 0$$

Necessity. If matrix A meets $f(A) = a_n A^n + a_{n-1}A^{n-1} + \cdots + a_1 A + a_0 I = 0$, we can get the following information from the lemma 1 exiting the invertible matrix P.

$$A = P\begin{pmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_k \end{pmatrix}P^{-1}$$

then

$$f(A) = P\begin{pmatrix} f(J_1) & & & 0 \\ & f(J_2) & & \\ & & \ddots & \\ 0 & & & f(J_n) \end{pmatrix}P^{-1} = 0$$

So, $f(J_1) = f(J_2) = \cdots = f(J_k) = 0$, from lemma 5, $J_1, J_2, \cdots, J_k$ meet the condition of the Lemma.

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