ISSN : 0974 - 7435 Volume 10 Issue 2

BioTechnology
An Indian Journal

FULL PAPER

BTAIJ, 10(2), 2014 [324-330]

The design of ultimate brownie pan based on mathematical model

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ABSTRACT

In electric oven, pan shape is rectangular or round in general, when a rectangular pan is used to roast food, heat is concentrated on four corners, as a result, the food will be scorched due to overheating, therefore, the rectangular pan can use the space of oven efficiently, but the food can not be roasted well. On a contrary, if a round pan is used, the heat can be distributed evenly on the outer rim of the pan, so the edge of food will not be overheated. This situation causes a lot of trouble to the user of oven. It has become a worthy issue that how to ensure the quality of roasted food, while both the oven space can be used most efficiently while the food visual effect can be the best.

In order to solve the quality problem that is to achieve the largest average thermal distribution $\boldsymbol{H}$ of a pan, we have compared the phenomenon of a temperature field tip thermal effect to that of an electric field tip thermal effect, and we have also built the tip thermal effect model of temperature distribution function, and through theoretical derivation, we have obtained the relation equations between temperature distribution function $\boldsymbol{T}$, aaverage heat distribution function $\boldsymbol{H}$ and tip distance $\boldsymbol{r}$, semi-apex angle $\boldsymbol{\alpha}$. And their monotonous relations has been analyzed and we have obtained when the semi-apex angle is $\alpha_{\max} = \frac{\pi}{2}$, which means that a round pan has a minimum curvature and max $\boldsymbol{H}$. The model is simulated by using Matlab, and we know through three dimensional simulation temperature profile, the curvature is max, in other word, is max when a round tessellation is used, which is in consistent with the result of theoretical derivation. And through using existing radiation thermal conduction model, we make a further verification of the tip thermal effect model, which approves the rationality of the model.

KEYWORDS

Tessellation;
Tip thermal effect;
Bi-objective programming.

INTRODUCTION

Owing to its usefulness, household electric oven is accepted by a growing number of people, and has become a rookie in household appliance industry. In recent years, new type of electric oven has been usually adopting new technologies including far-infrared heating technology that has such features as short heating time and lower energy consuming, which is very welcomed by a large number of users. Theoretically, heating by using infrared ray is to use the theory that when radiation wavelength of object is in consistent with that of receiving wavelength, the object is able to absorb a lot of infrared energy so as to accelerate the molecular movement within the object, then the object can be heated to a higher temperature, and finally, to fulfill a heating purpose.

In general, an aluminum pan is used as a tool and medium to roast in an electric oven, the shape of roast pan is decided by the inner shape of oven, and most of such shapes are rectangular. However, when a rectangular pan is used to roast food, heat is concentrated on four corners, as a result, the food will be scorched due to overheating, therefore, the rectangular pan can use the space of oven efficiently, but the food can not be roasted well. On a contrary, if a round pan is used, the heat can be distributed evenly on the outer rim of the pan, so the edge of food will not be overheated. However, the shape of most of electric oven is rectangular, because using a round pan can waste the space of oven, although it can ensure the quality of roasting food. This situation causes a lot of trouble to the user of oven. It has become a worthy issue that how to ensure the quality of roasted food, while both the oven space can be used most efficiently and the food visual effect can be the best.

HEAT TRANSFER ANALYSIS AND BASIC KNOWLEDGE WITHIN OVEN

Heat conduction analysis within oven

Through reading the instruction of the electric ove], we know that: electric resistance wire is heated by electricity that is transferred into thermal energy, and the inner temperature of the oven is increased, therefore the food in the oven can be heated. The whole process is from outside to inside. In brief, the electric oven is just like a closed stove of which the inner part is heated by thermal radiation.

Through observing the inner structure of the oven, we know easily that the oven is equipped with fan or air inlet and outlet, therefore the inside part of the oven can achieve a temperature uniformity due to convective heat transfer. Then, what factors lead to scorching a cake?

Definition and classification of tessellation

Tessellation is the process of creating a two-dimensional plane using the repetition of a geometric shape with no overlaps or no gaps. Namely, one or several two-dimensional geometric shapes with identical shape and size are spliced into a pattern without leaving any gaps or overlapping. This is the tessellation of plane Fig., also known as mosaic of plane figure[1,2]. In a tessellated graphic pattern, if there is only one element pattern, it is called single tessellation; otherwise known as multiple tessellation. In a single tessellation, if the element pattern is a regular polygon (equilateral triangle, square, regular pentagon, regular hexagon, etc), this tessellation is called (single) regular tessellation. If the edges of each element pattern match exactly with each other when tessellated, it is edge-to-edge tessellation, shown in Figure 1 and Figure 2. If the tessellation is both regular and edge-to edge, it is a single regular edge-to-edge tessellation.

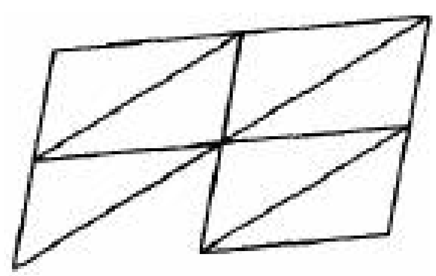

Figure 1 : Edge-to-edge tessellation

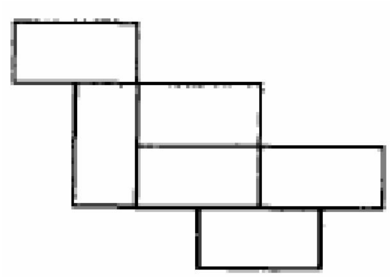

Figure 2 : Non edge-to-edge tessellation

SINGLE REGULAR TESSELLATION MODEL BASED ON PAN ISSUES

In order to maximize the number of pans accommodated in oven, the identity and regularity should be first considered when selecting the shape of pans. Then consideration should be given to maximizing the cover area of pans without overlapping. Based on the above thinking, we establish a single regular tessellation model, specifically derived as follows[3]:

In the process of tessellation, the measure of each interior angle of a regular polygon with *n* sides is

$$\frac{(n-2)\cdot 180^\circ}{n}$$

To realize tessellation, first we have to ensure that the sum of an integer number of interior angles is 360°, assuming the number of apex angles of a fixed point is k, then

$$k\cdot\frac{(n-2)180^\circ}{n}=360$$

and

$$nk-2k=2n$$

$$nk-2k-2n=0$$

$$nk-2k-2n+4=4$$

We have

$$(n-2)\cdot(k-2)=4$$

Therefore

$$\begin{cases}n=3\\k=6\end{cases}\begin{cases}n=4\\k=4\end{cases}\begin{cases}n=6\\k=3\end{cases}$$

That is to say, in regular polygon, only equilateral triangle, square, and hexagonal can be tessellated seamlessly. The effect figure is as follows:

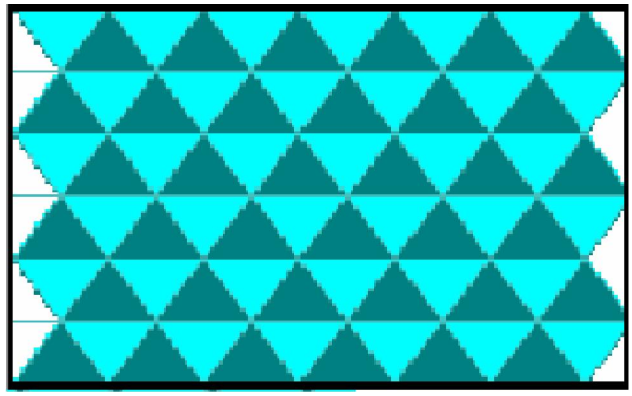

Figure 3

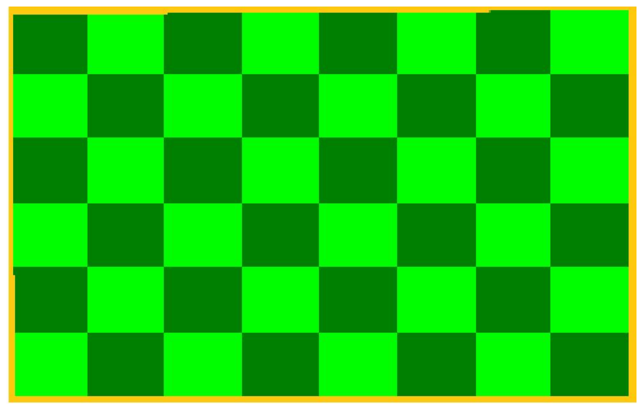

Figure 4

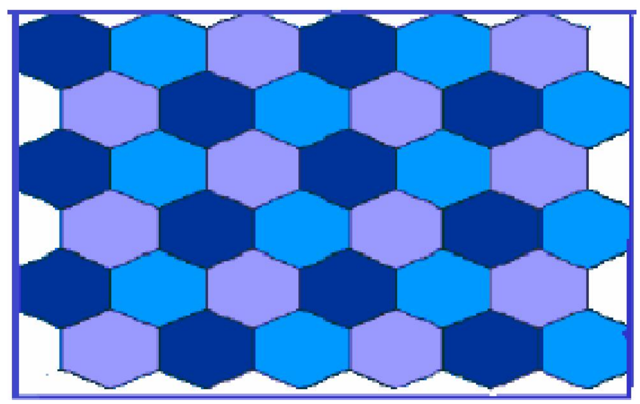

Figure 5

As the Figure transits from rectangle to circle in this case, equilateral triangle does not have to be considered. When tessellated, the sides of rectangle and the oven boundary coincide seamlessly while gaps must exist between sides of regular hexagon and the oven boundary[4]. In the process of tessellation, there are gaps not only at the boundaries but also between Figures when using other transitional Figures such as regular pentagon, regular octagon, circle, etc. Bellow we give the number of units of square in tessellated state and the numbers of units of regular hexagons in non tessellated state and make a comparison[5].

First, let's assume an ideal state, i.e. the inwall of the oven and the pan completely match without gaps, the number of tessellation units is

$$N=\left\lfloor\frac{W\cdot L}{A}\right\rfloor$$

However, in the actual situation, the pans cannot cover the oven completely. When the gap left at the boundary are not enough for a new unit, the number of units we find $\geq$ real number. The gap on the boundary will cause large errors. Therefore, we can use round-to-integer model to improve the tessellation model and further obtain the relationship between the oven length, width and the number of pan units.

(1) Square unit

$$N=\left\lfloor \frac{W}{\sqrt{A}}\right\rfloor \cdot \left\lfloor \frac{L}{\sqrt{A}}\right\rfloor$$

(2) Regular hexagon unit

$$\begin{cases} h=\sqrt{\dfrac{2\sqrt{3}}{3}A} \\ N=\left\lceil \dfrac{\left\lfloor \frac{W}{h}\right\rfloor}{2}\right\rceil \cdot \left\lfloor \dfrac{L}{h}\right\rfloor+\left(\left\lfloor \dfrac{L}{h}\right\rfloor-1\right)\cdot\left(\left\lfloor \dfrac{W}{h}\right\rfloor-\left\lceil \dfrac{\left\lfloor \frac{W}{h}\right\rfloor}{2}\right\rceil\right) \end{cases}$$

(3) Circle unit

$$\begin{cases} A=\pi r^2 \\ N=\left\lceil \dfrac{\left\lfloor \frac{W}{2r}\right\rfloor}{2}\right\rceil \cdot \left\lfloor \dfrac{L}{2r}\right\rfloor+\left(\left\lfloor \dfrac{L}{2r}\right\rfloor-1\right) \\ \cdot\left(\left\lfloor \dfrac{W}{2r}\right\rfloor-\left\lceil \dfrac{\left\lfloor \frac{W}{2r}\right\rfloor}{2}\right\rceil\right) \end{cases}$$

$\lceil\ \rceil$, $\lfloor\ \rfloor$ represent floor and ceiling functions.

Combined with the actual situation, we refer to the household oven whose actual size is $500mm\times 400mm\times 500mm$ and area is $80cm^2$. Substituting into the above formula, we get the number of rectangle is 25, regular hexagon is 18, circle is 16. Thus we come to a conclusion that rectangle is the best choice.

THE TIP THERMAL EFFECT MODEL BASED ON MAXIMUM AVERAGE HEAT DISTRIBUTION

Temperature field and electric field has a lot of similar physical quantities such as isotherm and equipotent line, isothermal surface and equipotent surface, temperature gradient and field gradient. Through reading material, we know that there is tip effect in electric field[6], there is tip thermal effect in temperature field[7,8], both the effects have a lot of similar features, so the tip thermal effect in the temperature field can be compared to the tip effect of the electric field.

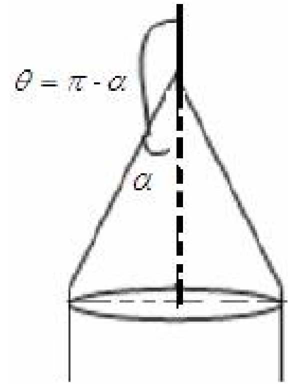

Figure 6 : A spire Figure

Tip thermal effect model

First, let's analyze a simple spire shape, please look at the following Figure:

The function of surface temperature field is:

$$T_r=-Avr^{v-1}[1+2v\ln(\cos\frac{\beta}{2})] \quad (1)$$

$$T_\beta=Avr^{v-1}\tan\frac{\beta}{2} \quad (2)$$

$$T=\sqrt{T_r^{\ 2}+T_\beta^{\ 2}} \quad (3)$$

To plug (1), (2) into (3), the following function of temperature distribution can be obtained:

$$T=Avr^{v-1}\sqrt{[1+2v\ln(\cos\frac{\beta}{2})]^2+\tan^2\frac{\beta}{2}} \quad (4)$$

The r is the distance to the spire tip, α, θ are respective radian value and angle value, $v=\frac{1}{2\ln\frac{2}{\theta}}$, $\beta=\pi-\theta$, and average heat distribution function is:

$$H=\overline{T} \quad (5)$$

In order to simplify calculation, the intensity is obtained by assuming tip temperature as 0, A is an undetermined constant. Considering that the study is the tip, so $\alpha\le 1$, that is $\theta\le 57.3^\circ$. According to equation (4), the relation between tip temperature field and semi-apex angle (illustrated by using radian) can be obtained, and through equation, when α is certain, $r\uparrow$ when $T\downarrow$, when r is certain, $\alpha\uparrow$ when $T\downarrow$, $T\downarrow$ when $r\uparrow \cup \alpha\uparrow$ 时 $T\downarrow$, and

$$\lim_{\substack{\alpha\to a_1\\ r\to r_1}} T = \lim_{\substack{\beta\to\beta_1\\ r\to r_1}} T = \lim_{\substack{\beta\to\beta_1\\ r\to r_1}} Avr^{v-1}$$

$$\sqrt{[1+2v\ln(\cos\frac{\beta}{2})]^2+\tan^2\frac{\beta}{2}} = T_0$$

In this equation, α_1, r_1, T_0 are constant, and $\alpha_1 >> 0,\ r_1 >> 0$. Through the above equation, when α, r is far from constant 0, the temperature T tends to be stable, so the value of average heat distribution function H is mainly determined by the nearby temperature value when α, r are 0.

Under the premise of constant total capacity of thermal conduction, the following can be obtained:

(1) when $\begin{cases}\alpha\to 0\\ r\to 0\end{cases}$ and temperature $T\uparrow$, total average thermal distribution $H\downarrow$.

(2) Near the tip $r\to 0$, if the semi-apex angle $\alpha\uparrow$, curvature $\downarrow$, T changes slowly, $H\uparrow$.

Through the above calculation, we can obtain that when the semi-apex angle $\alpha_{\max} = \frac{\pi}{2}$ (round shape), the curvature is minimum, and the average heat distribution function H is maximum.

The Matlab simulation of the tip thermal effect model

Through Matlab simulation[9,10], we achieve the change process between temperature T and distance r when semi-apex angle is certain, please look at Figure 7(a), and achieve the change process between temperature T and semi-apex angle α when the distance r is certain, please look at 7(b), and achieve the change process between temperature T and semi-apex angle α and between temperature T and distance r, please look at 7(c).

According to the Figure 7(a): with the enlarging of the distance to the tip, temperature change tends to be stable, which means the temperature distribution is relatively even, in another word, the temperature change is more violent near tip area with uneven temperature distribution.

According to Figure 7(b): with the enlarging of the tip angle and the decreasing of the curvature, temperature change tends to be stable, which means temperature distribution is relative even, in another word, the smaller the tip angle is, the larger the curvature is, and the temperature change is relatively violent with an uneven temperature distribution.

In conclusion, the closer the tip is, the smaller the angle is, which means that the larger the curvature is, the more uneven temperature distribution is[12-14]; on the contrary, the more distant to the tip, the larger the angle is, which means that the smaller the curvature is, the more uneven the temperature distribution is, please look at the Figure 7(c).

Thus, it can be concluded that: from rectangular to round, the round shape has a smallest curvature, so the edge of a round pan has the most even thermal distribution, which is in consistent with theoretical derivation.

The verification result of radiation thermal conduction model

In order to verify the rationality and correctness of the tip thermal effect model, a conventional radiation thermal conduction model is used to consider the issue of average heat distribution.

The working principle of electric oven is that the resistance device of the electric oven emits heat to radiate the object in the oven. Through the equation of radiation heat thermal conduction capacity:

$$\Phi_r = h_r \Delta t F$$

In this equation: Φ_r is radiation thermal conduction capacity, h_r is radiation thermal conduction coefficient, Δt is temperature gradient, F and is thermal conduction area in the direction of thermal conduction. We can see that the heat absorbed by the object in the electric oven is related to heat conduction area, and when the heat conduction area F is enlarged, the heat absorption capacity is also increased. When pan shape is changed from rectangular to round, and under the circumstance of a certain pan area A, $S_{Side} = AL$, $L\uparrow, S_{Side}\uparrow$, $\Phi_r\uparrow$. As a result, for rect-

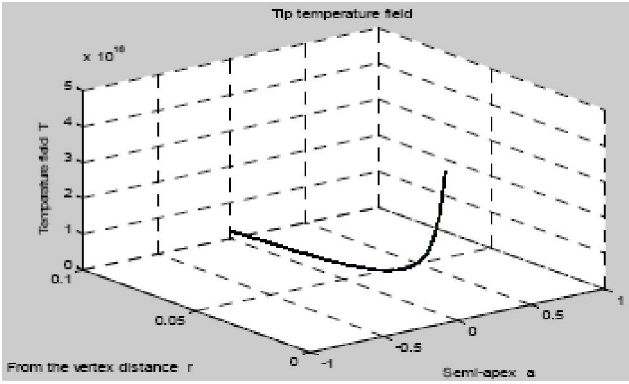

Figure 7(a)

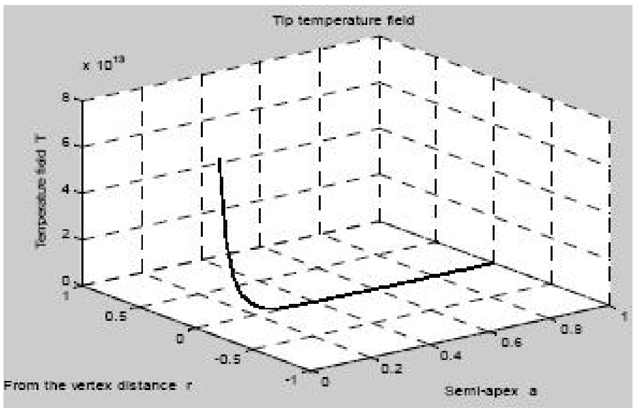

Figure 7(b)

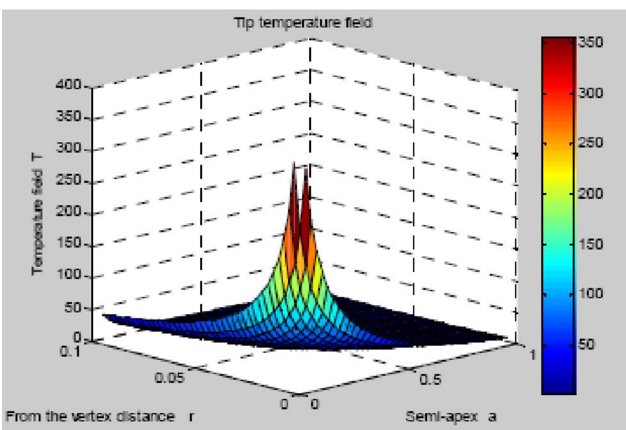

Figure 7(c)

angular pan and round pan, when the heights h are the same, the rectangular pan can absorb more heat than that of the round pan($\Phi_{r-\mathrm{Re}c} > \Phi_{r-Rou}$), so, within the same time, (to roast a bread), a rectangular pan has a higher temperature inside, which lead to scorching the edge and the corner of the bread.

The conclusion can explain the phenomenon in the issue, which is in consistent with the conclusion of the tip thermal effect model at the same time. This illustrates the feasibility of comparing temperature field to electric field and designing a tip thermal effect model.

BI-OBJECTIVE PROGRAMMING MODEL BASED ON NUMBER OF PANS AND HEAT DISTRIBUTION

When we select the shape of pans, two factors need to be taken into account, namely, pattern tessellation effect and heat distribution at the oven edges. When we choose a rectangular pan whose side ratio is consistent with the oven, there exists the situation that the oven can be completely tessellated by the pans, which takes full advantage of the oven space. Hence, in the case of equal area A, when the pan is rectangular, the number of pans that can fit in the oven is maximal.

However, we can learn from tip thermal effect model that sharper pan corner and smaller curvature (i.e. smaller angle) lead to larger temperature gradient change and more uneven temperature distribution. Meanwhile, the superposition of temperature field at corners causes temperature rise, which in turn result in overcooking at corners. We use PDE toolbox comes Matlab to make the rectangular heat map, see Figure 8.

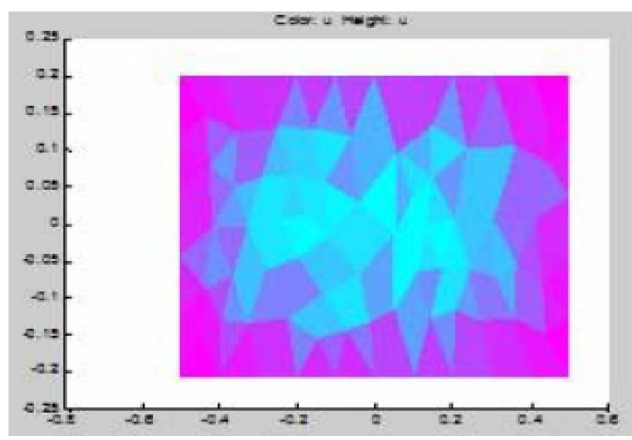

Figure 8 : Rectangular heat distribution

Analyzing the result of tip thermal model, we learn that the curvature of circle is maximal, its temperature gradient changes smoothly and temperature is distributed evenly. Therefore, the heat distribution across the edges of round pan is most uniform and the food is not easy to be overcooked. Thinking from this angle, we should choose round pan.

ACKNOWLEDGEMENTS

This paper is supported by the Scientific Technology Research and Development Plan Project of Tangshan(2012cx-11;12140201B-3;12140201B-7),Hebei Province Natural Science Fund Project(E2013209215).

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