



The critical values of ising model with the interaction of the second neighbor in the presence of DM interaction and magnetic field

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ABSTRACT

The Ising three qubit model has been studied by the interaction of the second neighbor in the presence of DM interaction and external longitudinal uniform magnetic field. We've studied the critical values of DM interaction in the absence of magnetic field and also in the presence of magnetic field. In this case, it has been shown that the critical value of DM interaction depends on the exchange coefficient parameters and also the magnetic field. We've studied the critical value of the external longitudinal magnetic field per different cases of the exchange coefficients and DM interaction power. It has been shown that in some cases, there is no critical value of magnetic field. © 2014 Trade Science Inc. - INDIA

INTRODUCTION

In recent decade, the spin quantum systems have been attended in the low dimensions and specially frustrated systems. $\frac{1}{2}$ spin chain with the interaction of the second nearest neighbor (that it equals the zigzag ladder model) is one of the models that has got frustrated effects. The Hamiltonian of this model is as bellow Figure

$$H = J_1 \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + J_2 \sum_j \vec{S}_j \cdot \vec{S}_{j+2} \quad (1)$$

where \vec{S} is the operator of $\frac{1}{2}$ spin. $J_1 \approx J_2$ are respectively represent of the exchange interaction power between the nearest neighbors and the second nearest neighbors. This model has been studied with anti ferromagnetic interactions ($J_1, J_2 > 0$)^[1-4]. The critical value $J_{2c} = 0.2411J_1$ has been obtained which separates the

double degenerate binary phase $J_2 > J_{2c}$ with the excited gap from spin – liquid phase without gap $J_2 < J_{2c}$. But there is less information with anti ferromagnetic and ferromagnetic interactions ($J_1 < 0, J_2 > 0$)^[4-9] and it hasn't been offered a complete picture of the phases of this model so far. The importance of anisotropy exchange interactions in spin systems is that leads to a weak ferromagnetic or Helical magnetic disorder in quantum anti ferromagnetic systems. The first time, it is introduced with phenomenological approach by Dzyaloshinskii^[10]. A microscopic model also suggested with anisotropy interaction for the first time by Moriya^[11] that showed such interactions which enter in perturbation theory is resulted from coupling of spin – orbit in magnetization systems with low symmetry and basically is a expansion of Anderson exchange mechanism^[12], which is related to jump and electron Philip. The Hamiltonian of

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DM interaction is as $\vec{D} \cdot (\vec{S}_i \times \vec{S}_j)$. Since DM interaction break the symmetry of $SU(2)$ in Heisenberg model, we can know it as a factor of some unusual behavior of Heisenberg model such as bending^[13] or small gaps^[14-18]. Some of the systems expected to be described with this interaction are as $Cu(C_6D_5COO)_2 3D_2O$ ^[14,19], Yb_4As_3 ^[20-22], $BaCu_2Si_2O_7$ ^[23]. In reference^[24], the writers have offered a plan for phase diagram of Heisenberg model in the presence of spin-orbit interaction with the approach of the quantum renormalization. This model has got three ferromagnetic phase, spin liquid and Neel phase that have been separated from each other by critical lines (depend on DM interaction constant). Saeed MahdaviFar and his partners also have studied antiferromagnetic Heisenberg model in the presence of external magnetic field and interaction of the alternative spin – orbit on any site by Lanczos numerical approach and reported the existence of four luttinger liquid phases, alternating chiral, spin– flop and ferromagnetic^[25]. On one hand, the entanglement of two and three qubit systems have completely been studied in the presence of DM interaction^[26-30]. By adding DM interaction to Hamiltonian (1), the effect of this interaction can be studied on the frustrated model. In this case the Hamiltonian is as

$$H = J_1 \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + J_2 \sum_j \vec{S}_j \cdot \vec{S}_{j+2} + \vec{D} \cdot \sum_j \vec{S}_j \times \vec{S}_{j+1} \quad (2)$$

It is a specially case of Hamiltonian (2) of Ising model with the interaction of second neighbor in the presence of DM interaction and external longitudinal uniform magnetic field that the Hamiltonian is as

$$H = J_1 \sum_j S_j^z S_{j+1}^z + J_2 \sum_j S_j^z S_{j+2}^z + \vec{D} \cdot \sum_j \vec{S}_j \times \vec{S}_{j+1} +$$

$$h \sum_j S_j^z \quad (3)$$

In this article, in the second section, we compute the eigenvalues and eigenstates of three qubit model of Hamiltonian (3). In the third section, we study the phase transition points in the absence of magnetic field. In the fourth section, we study the quantum phase transition points in the presence of magnetic field.

Ising three qubit model with the interaction of the second neighbors in the presence of DM interaction and external longitudinal uniform magnetic field

The Hamiltonian of Ising model with interaction of the second neighbor in the presence of DM interaction and longitudinal uniform magnetic field is as:

$$H = J_1 \sum_j S_j^z S_{j+1}^z + J_2 \sum_j S_j^z S_{j+2}^z + \vec{D} \cdot \sum_j \vec{S}_j \times \vec{S}_{j+1} + h \sum_j S_j^z \quad (4)$$

where J_1, J_2 are the exchange energy coefficients and \vec{D} represents the vector of DM interaction power. By choosing $\vec{D} = D \hat{z}$, we can obtain the Hamiltonian (4) for three qubit case as bellow:

$$H = J_1 (S_1^z S_2^z + S_2^z S_3^z) + J_2 S_1^z S_3^z + D (S_1^x S_2^y - S_1^y S_2^x) + D (S_2^x S_3^y - S_2^y S_3^x) + h (S_1^z + S_2^z + S_3^z) \quad (5)$$

We have selected the eigenstates of the total S_z as the basekets of the Hilbert space.

The critical value of DM interaction in the absence of magnetic field $h=0$

In this case, if $J_1, J_2 > 0$, there isn't any critical point. If $J_1 < 0, J_2 > 0$ and $J_1 > 0, J_2 < 0$, we've got the critical value for as

$$D_c = \sqrt{J_1^2 - \frac{1}{2} |J_1 J_2|} \quad (6)$$

and its diagram has come in diagram 1-a. In the case $J_1, J_2 < 0$, the critical value D is as

$$D_c = \sqrt{J_1^2 + J_1 J_2} \quad (7)$$

and its diagram has come in Figure 1-b.

The critical value D and h in the case of $h \neq 0$.

We study the critical value of magnetic field and DM interaction power:

1) $J_1, J_2 > 0$

In this case, the critical value (D) is as:

$$D_c = \sqrt{J_1^2 - \frac{3}{2} h J_1 - \frac{1}{2} h J_2 + J_1 J_2} \quad (8)$$

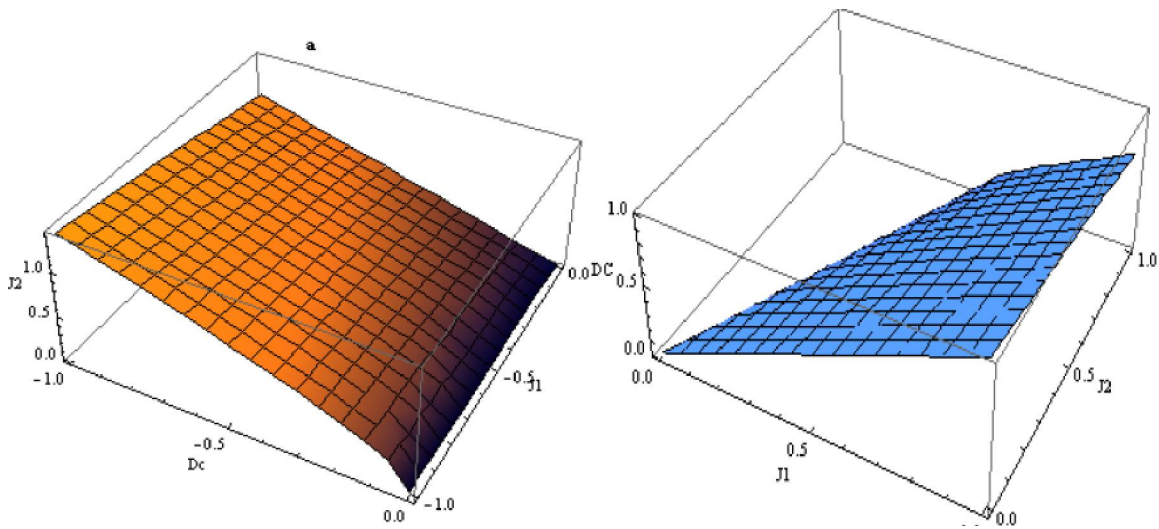


Figure 1 : The critical value of the DM interaction, D_c , in respect to the exchanges J_1, J_2 .

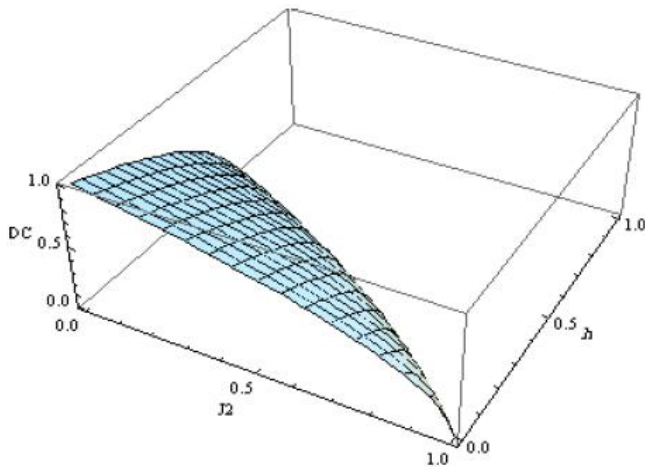


Figure 2 : The critical value of the DM interaction, D_c , in respect to the exchange J_2 and h . ($J_1=1$)

and its diagram has come according to J_2 and h for $J_1=1.0$ in Figure 2 and the critical value of magnetic field is as

$$h_c = \frac{1}{2}(3J_1 + J_2 + \sqrt{J_-^2 + 8D^2}) \tag{9}$$

Where $J_- = J_1 - J_2$ and its diagram has come in Figure 3.

2) $J_1 > 0, J_2 < 0$

In this case the critical value of D is

$$D_c = \sqrt{\frac{h^2}{2} + J_1^2 - \frac{3}{2}hJ_1 + (\frac{1}{2}h - J_1)|J_2|} \tag{10}$$

and its diagram has come in Figure 4, and the critical magnetic field will be as

$$h_c = \frac{1}{2}(3J_1 - J_2 + \sqrt{J_+^2 + 8D^2}) \tag{11}$$

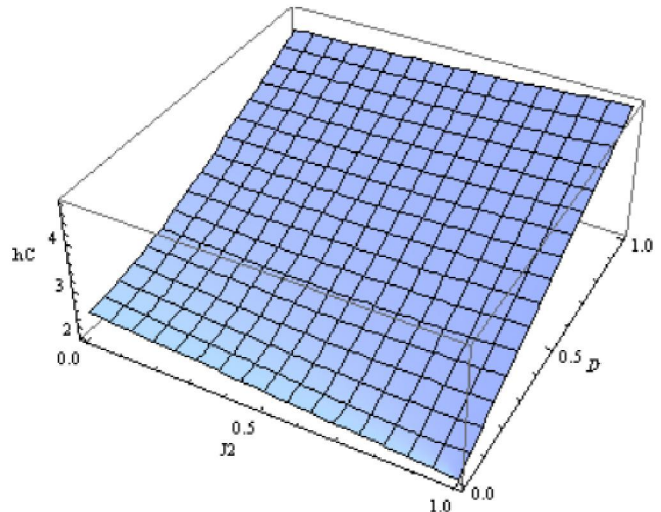


Figure 3 : The critical value of the h_c , in respect to the exchange J_2 and D . ($J_1=1$)

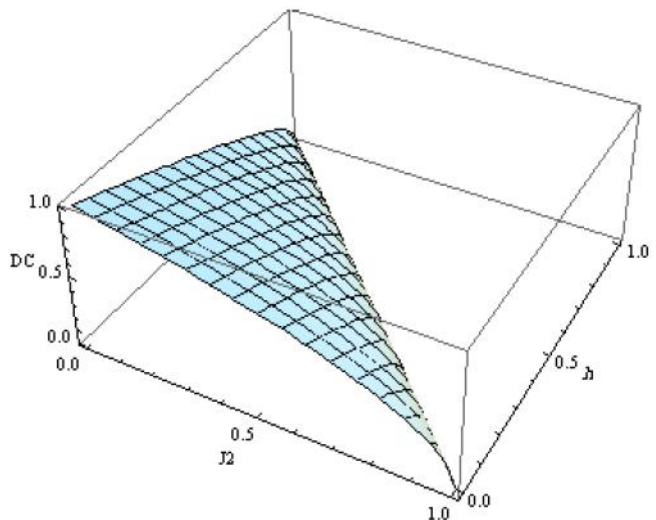


Figure 4 : The critical value of the DM interaction, D_c , in respect to the exchanges J_1, J_2 .

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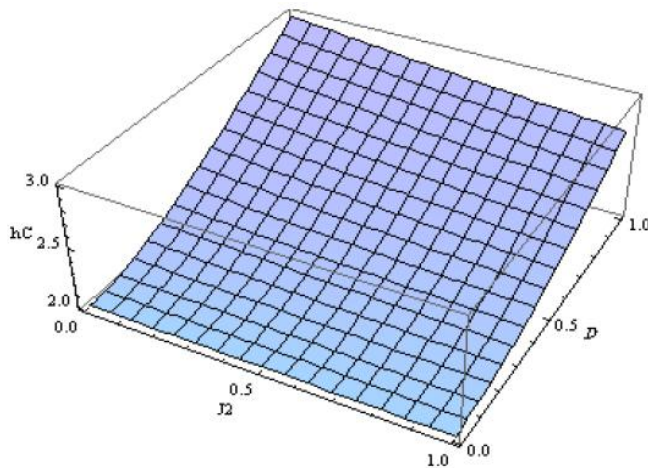


Figure 5 : The critical value of the h_c in respect to the exchanges J_2 and D . ($J_1=1$)

and its diagram has come in Figure 5. ($J_1 = 1.0$)

3) $J_1 < 0, J_2 > 0$

In this case, we've got the lack of magnetic field and the critical value for DM interaction is as the two case: first

$$D_c = \sqrt{J_1^2 - \frac{1}{2}|J_1|J_2} \quad (12)$$

that is similar to the diagram 1. The second case

$$D_c = \sqrt{\frac{1}{2}|J_1|J_2} \quad (13)$$

that its diagram has come in Figure 6.

4) $J_1, J_2 < 0$

In this case, we've got the lack of the critical magnetic field and the critical value of DM interaction power will be

$$D_c = \sqrt{|J_1|(|J_1| + |J_2|)} \quad (14)$$

that its diagram is similar to diagram 1-b.

CONCLUSION

Critical values of the three qubit Ising model with next nearest neighbor interaction in presence of the DM interaction and longitudinal magnetic field (h) is investigated. Critical values D_c, h_c in different cases are evaluated and it is shown that they are related to the interaction coefficients D and h . In the case $J_1, J_2 > 0$, there is not any critical value for the DM interaction.

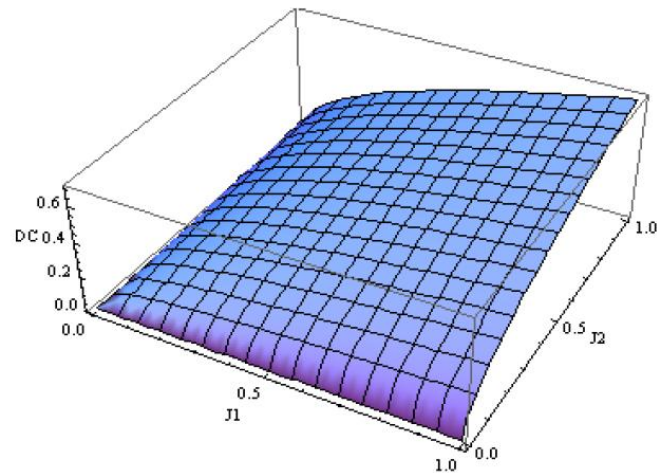


Figure 6 : The critical value of the D_c in respect to the exchanges J_1, J_2 .

ACKNOWLEDGMENT

The authors gratefully acknowledgment the financial and support of this research, provided by the Islamic Azad University, Saveh Branch, Saveh, Iran.

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