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## Table tennis flight trajectory influence factors research based on biomechanical model

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### ABSTRACT

With sports development, table tennis has already become a favorite event among broad masses, no matter in school, or in mass stadium, there are table tennis enthusiasts. Chinese table tennis level is always in the lead of the world, however to remain leading level without changing, it should make research and reform on table tennis training all the time. Based on this, the paper researches on table tennis air movement, with an aim to better reveal table tennis sports rules. The paper firstly makes decomposition on table tennis movement each phase, then successively makes force analysis of each phase, combines with mechanical knowledge and differential equation modeling relative knowledge to find out each phase mechanical relations' differential equations, and finally combines them and gets differential equation researches on table tennis air sports rules, which provides theoretical basis for Chinese table tennis training so that propel to Chinese table tennis development.

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### KEYWORDS

Bernoulli's theory;  
Differential equation;  
Table tennis trajectory;  
Magnus force;  
Biomechanics.

### PREFACE

Table tennis is Chinese national ball; it is well received by Chinese people. With social progress, table tennis has been widely developed in China, in school, nearly every primary school, junior school, high school, and university; all have table tennis fields and table tennis training. Outside of school, each major city stadium of course has enthusiasts, each major city also exists lots of table tennis clubs, which provides conveniences for massive table tennis enthusiasts exchanging. The paper researches on table tennis air movement rules, with an aim to build theoretical basis for table tennis training.

For table tennis movement rules, lots of predecessors have made their contributions, just by their unremitting efforts; it lets Chinese table tennis get widely development and popularization. Among them, Yin Yan (2011), made biomechanical analysis of table tennis movement three kinds of footwork's foot movement, he pointed out that foot part movement form after forehand striding step and forehand level step stroke exertion foot landing had similarities, and foot part movement form after backhand striding step, backhand level step and backhand crossover stroke exertion foot landing had similarities, in three kinds of footwork movement, when stroke exertion foot pedaling off the ground, foot part movement form had similarities<sup>[1]</sup>. Yao Rong-

Qi (2011), researched on Chinese national table tennis training base construction and development status and pointed out Chinese table tennis training base showed regional and seasonal features in layout, base could not meet modern sports training requirements in scientific research service aspect, compared to base hardware facilities, base occurred to obvious lagging in scientific research facilities, scientific research service and researchers' equipment. Therefore, it should strengthen construction on scientific research facilities, scientific research service and researchers' equipment<sup>[2]</sup>. Zhou Le(2013), made researches and analysis of Beijing amateur table tennis clubs' current status, he pointed out that most of clubs hardware facilities could meet consumers' demand, the clubs mostly aimed at profits, which had small contributions to improve amateur athletes' sports levels, therefore he made suggestions about organizing table tennis amateur athletes' competitions, and so strengthen broad table tennis enthusiasts positivity in improving sports levels<sup>[3]</sup>.

The paper on the basis of previous research, from the perspective of mechanics, it researches on table tennis air movement status, with an aim to build theoretical basis for Chinese table tennis training, so that propel to Chinese table tennis development and popularization.

### STROKING FORCE ANALYSIS

The paper assumes that when first time stroking table tennis, ball is static. Generally speaking, ball will suffer a resultant, its direction and table form into certain angles that is set as  $A$ . Its component forces: racket to table tennis elastic force  $f_1$  0 racket and ball friction force  $f_2$  0 racket bottom board acting force  $f_3$  as Figure 1 shows.

If  $f_2$  is smaller, then  $A$  is also smaller, so torque acting on table tennis is smaller, tablet tennis spinning property is poor, flight speed is fast, the ball is called quick attack; when  $f_2$  gradually increases, angle  $A$  will also gradually increase, so torque will also increase, spinning property will also gradually increase, flight speed is still fast, the ball is called accelerated loop; If changes ball force point, resultant direction points to oblique direction as Figure 1 shows, and flight speed

and spinning property are still very high, then it is called high loop.

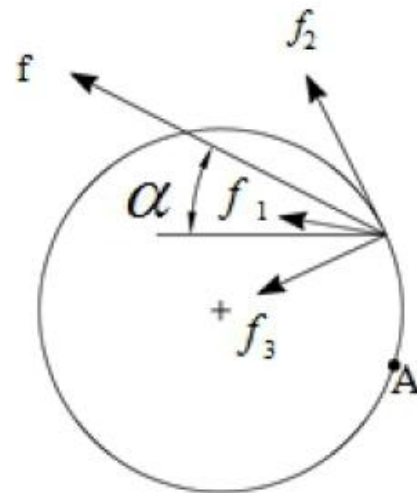


Figure 1 : Table tennis force analysis

### SPINNING BALL MOVEMENT ANALYSIS

Ball movement in the air is affected by many factors, the paper just discusses airflow such factor. When strokes ball, ball center is not in the force acting line, the force will let ball spin, ball spins around one axis at angular speed  $\omega$ . If axis of rotation is parallel to table tennis table, and then ball divides into topspin or backspin; if axis of rotation is vertical to table tennis table, then it is called sidespin; if ball's axis of rotation and table tennis neither be vertical nor parallel, then it divides into side topspin or side backspin. Ball movement trajectory in the air will suffer  $\omega$  size and spinning direction influences. The paper takes topspin as an example, assumes that plane goes through axis of rotation and vertical to ball center is  $n$ , spherical surface and plane  $n$  intersection's bottom and top are respectively as point  $b, a$ . When force acts on spherical surface first half part,  $a, b$  two points linear speed is  $V_a = V_b = \omega r$ , in spinning direction, assumes that ball speed is  $V$ . According to relative movement theorem, it can regard the movement's ball as zero, angular speed  $\omega$ , ball will suffer wind resistance, its size is  $V$ . In this way,  $a$  point air fluidity is  $(V - V_a)$ ,  $b$  point air fluidity is  $(V - V_b)$ . Therefore, ball spinning causes ball bottom and top air speed difference, and appears pres-

## FULL PAPER

sure difference  $\Delta f$ , because  $\Delta f$  let ball movement trajectory move down and show in arc. Spinning gets fiercer, and then trajectory's changes get more obvious. If the ball is sidespin, and then  $\Delta f$  will let ball trajectory bend laterally.

### Ball spinning and moment

As Figure 2 show, during ball movement process, it acts on a push force through sphere center to ball, the ball only moves along force direction, if acts on a force that deviates sphere center, the ball can make translation and also spins. Its movement trajectory shift extent is up to moment size.

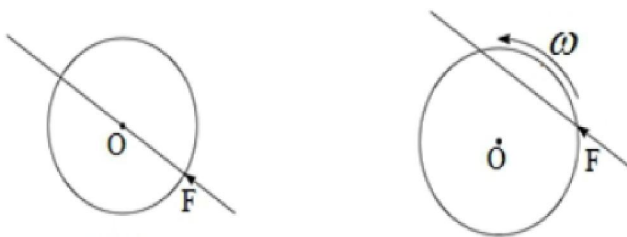


Figure 2 : Schematic diagram of ball movement with force crossing sphere center

On above, it can know if it wants table tennis to spin, it should give table tennis a force without acting on sphere center.

### Ball spinning and friction force

Force analysis of spinning ball and friction force is as Figure 3 show.

According to Figure 3, racket in stroking instant, it lifts racket up, and then it will suffer  $f_t$  and  $f_m$  two acting forces,  $f_t$  will go through sphere center, so it will not generate moment, and  $f_m$  doesn't go through sphere center, it will generate moment on ball, so ball will spin.

### Spinning ball and Bernoulli's theory as well as loop

In table tennis flight process, it will appear lots of

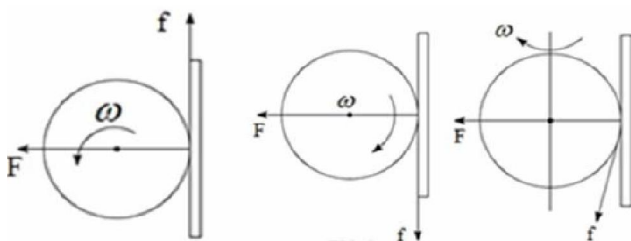


Figure 3 : Ball force in racket

loops that are not in the same vertical plane. Table tennis, due to suffer the factor influence, it will generate different loops. To make clear the problem, it should understand Bernoulli's theory. Blow between two vertical and parallel scrip, the two scrip will get closer internally, by Bernoulli's theory, it is clear that small pressure area fluid flow speed is big, and big pressure area fluid flow speed is small.

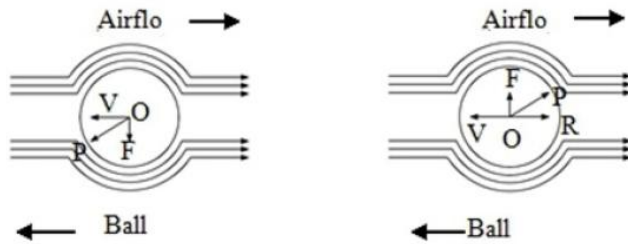
In table tennis movement process, spinning ball will generate above status, research on spinning ball, ball upper air has small flow speeds to ball, but following air has big flow speeds to ball, in this way, it has a downward lateral pressure, which will let ball movement trajectory become low, but topspin is just on the contrary, as Figure 4 showed trajectory.



Figure 4 : Ball movement trajectory

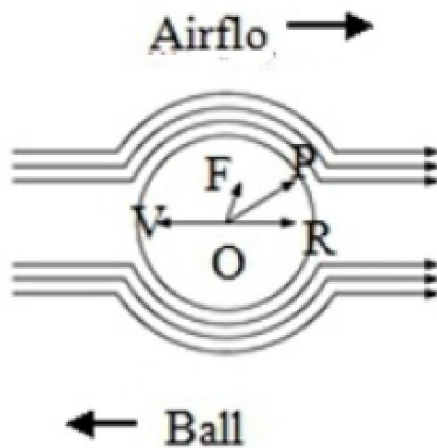
When there is a force acting on sphere and let it occur to spin, due to table tennis flight moment air and circulation mutual acting results, ball spinning extent and direction will have differences, generated effects are also different. Topspin is rotating forward along horizontal axis when moves in the air, because of viscosity, ball will drive its surrounding air spinning together. When ball moves, faced air resistance and ball top edge's air circulation are opposite, the region air flow speed will be small; but air resistance will be consistent to ball bottom edge air circulation, the region air flow speed will be big. According to Bernoulli's theory, ball top part and bottom part will generate pressure difference; because ball front and back circulation is vertical to air direction, it will not generate pressure difference, the direction is also downward. From above research, it is clear: ball will be in declining trend in movement process, movement trajectory is relative steep, and flight distance is shorter, as Figure 5.

Backspin will opposite to topspin, therefore, backspin will be in the rising trend in movement process, movement trajectory will get bigger, movement speed



**Figure 5 : Topspin, backspin force during running in the air**  
will become slower and slower.

When ball moves, top edge and bottom edge will not generate pressure difference; ball front edge and back edge will also not generate pressure difference; ball left side air circulation is opposite to air flowing direction, ball right side air circulation is consistent to air flowing direction. Therefore left spin has right pinning postures in the air, right spin is opposite to left spin, as Figure 6.



**Figure 6 : Sidespin force during running in the air**

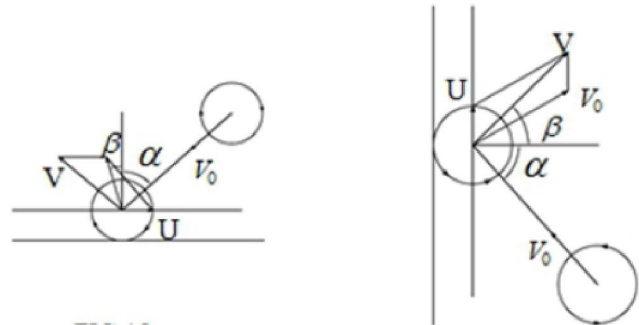
**Spinning ball falling research**

When spinning ball falls to the table, except for giving rebound force to table tennis, there is an another force acting on table tennis, table tennis table will give ball an equal size counter-acting force, and spinning force counter-acting force.

When topspin ball falls to table tennis table, ball spinning force direction is along horizontal direction backward acting on the table, but spinning force counter-acting force is forward. The force gives ball a forward speed  $U$ , therefore ball reflection speed is not  $V$  but  $V_0$ . Topspin incident angle is obviously smaller than reflection angle, ball speed after rebounding from table is still very big, and continue to maintain spinning state in pro-

ceeding.

Backspin falling is opposite to topspin, spinning force is acting on the front of table, and counter-acting force direction is backward, but reflection angle after ball falling to table is smaller than incident angle, forward speed after ball falling diminishes, impulse force is small, and ball turns to be higher after falling. If ball backspin extend is very high, ball will appear bounding phenomenon, as Figure 7.



**Figure 7 : Backspin rebounding direction after falling**

Sidespin, due to it is approximately vertical axis spinning; therefore it will not give table tennis table an acting force, sidespin will continue to turn laterally according to original direction.

**FORCE ANALYSIS WHEN TABLE TENNIS CONTACTS WITH TABLE**

When ball contacts with table, it will suffer table tennis acted friction force, friction force size has relations with inertial force and friction coefficient, friction force direction is vertical to ball positive reaction's direction, which is opposite to ball relative to table tennis table speed direction.

Ball will spring up after contacting with table, its incident angle is smaller than reflection angle, according to collision theory, it will have following relation:

$$\tan B = \left(\frac{1}{k}\right) \tan A \tag{1}$$

Among them,  $k$  is restitution coefficient, it always have the status that reflection angle is larger than incident angle, that is to say, after ball contacting with table, rebounding running routine always is lower than incident ball routine; But to ball, it still will intensely spin, therefore it will continue to bend down or bend laterally.

## FULL PAPER

As Figure 8 show,  $V$  and plumb line included angle is  $\alpha$ , in the instant that incident ball falls to table, ball surrounding horizontal axis angle is  $\omega_0$ , then it collides with table, due to friction force effects, horizontal speed is zero. Ball restitution coefficient is  $k$ , ball radius is  $r$ , table tennis mass is  $m$ , according to mass center collision movement theorem, it can get:



Figure 8 : Movement trajectory after ball contacting with table

$$mu - mv = s \quad (2)$$

Among them,  $s$  is impulse,  $u$  is ball rebounding instant sphere center speed, project formula (1) toward  $x$  axis:

$$mu_x - mv_x = s_x \quad (3)$$

$$mu_y - mv_y = s_y \quad (4)$$

Take sphere center as center of moment, by impulsive moment theorem, it gets:

$$J_C(\omega - \omega_0) = s_x r \quad (5)$$

$J_C$  is ball rotational inertia, ball is hollow, it has  $J_C = \frac{2}{3}mr^2$ , due to after colliding contacting point horizontal speed is equal to zero.

$$r\omega = -u_x \quad (6)$$

Input formula(6) into formula(5), by formula (2) eliminating  $s_x$ , it gets:

$$u_x = \frac{mr^2 v_x - J_C r \omega_0}{J_C + mr^2} = \frac{1}{5}(3v \sin \alpha - 2r\omega_0) \quad (7)$$

Due to after colliding ball vertical direction movement is related to restitution coefficient  $k$ , so it has:

$$u_y = -kv_y = kv \cos \alpha \quad (8)$$

Therefore when rebounding form table, ball rebound angle  $\beta$  is:

$$\tan \beta = \frac{u_x}{u_y} = \frac{1}{5k} \left( 3 \tan \alpha - \frac{2r\omega_0}{v \cos \alpha} \right) \quad (9)$$

### Momentum theorem and receiving

When ball contacts with face of racket, it let racket

face sponge or colloidal particle get deformed and absorb ball kinetic energy. When elastomeric restores and deforms, it let ball suffer elastic force. Elastic force direction has relations with incoming direction: when incoming ball doesn't vertical collide with racket face, ball contacted colloidal particle or sponge pressure distribution will different from vertical colliding racket face moment pressure distribution, so ball suffered elastic force direction will change with incoming ball directions as Figure 9. Set that it strokes in table height  $H$ , by principle of conservation of energy, it gets:

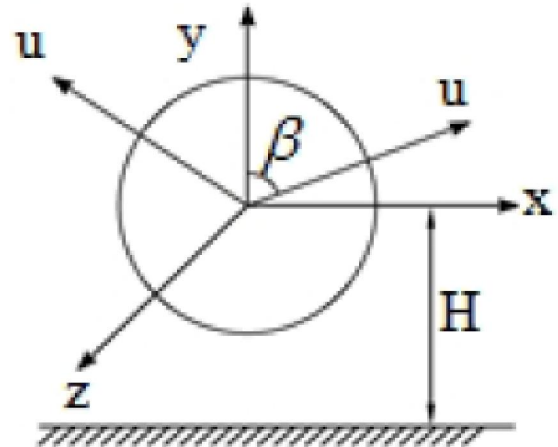


Figure 9 : Ball movement process schematic diagram

$$\frac{1}{2}mu^2 = \frac{1}{2}mu_1^2 + mgH \quad (10)$$

By formula (9), it gets:

$$u_1 = \sqrt{u^2 - 2gH} \quad u_{1x} = u_1 \sin \beta \quad u_{1y} = u_1 \cos \beta \quad (11)$$

Among them,  $u_1$  is ball instantaneous speed before ball contacting with racket. By mass center collision motion theorem, it gets:

$$mu_2 - mu_1 = s_1 \quad (12)$$

In formula  $u_2$ —— instantaneous speed after ball contacting with racket:

$s_1$ —— Ball suffered impulse

Project formula(11) towards  $x, y, z$  axis:

$$mu_{2x} - mu_{1x} = s_{1x} \quad (13)$$

$$mu_{2y} - mu_{1y} = s_{1y} \quad (14)$$

$$mu_{2z} = s_{1z} \quad (15)$$

By formula(11)—(14), it can get formula(16)(17):

$$u_2 = u_1 + \frac{s_1}{m} \quad u_{2x} = u_{1x} + \frac{s_{1x}}{m} \quad (16)$$

$$\mathbf{u}_{2y} = \mathbf{u}_{1y} + \frac{s_{1y}}{m} \quad \mathbf{u}_{2z} = \frac{s_{1z}}{m} \quad (17)$$

According to formula(16)(17), it can get ball motion differential equation after stroking as :

$$m \frac{d^2 \mathbf{x}}{dt^2} = \sum_{i=1}^n \mathbf{F}_{xi} = 0 \quad (18)$$

$$m \frac{d^2 \mathbf{y}}{dt^2} = \sum_{i=1}^n \mathbf{F}_{yi} = -mg \quad (19)$$

$$m \frac{d^2 \mathbf{z}}{dt^2} = \sum_{i=1}^n \mathbf{F}_{zi} = 0 \quad (20)$$

In formula,  $F_i$ —— ball suffered acting force in movement

By formula(18)——(20) simplify integral and then get:

$$\mathbf{x} = \mathbf{u}_{2x} \mathbf{t} \quad (21)$$

$$\mathbf{y} = -\frac{1}{2} \mathbf{g} \mathbf{t}^2 + \mathbf{u}_{2y} \mathbf{t} + \mathbf{H} \quad (22)$$

According to formula (22), it is clear that spinning ball and racket fact contact moment, it also suffers friction force, analyze friction force and direction is the same as above ball contacts with table, only due to sponge or colloidal particle friction coefficient is far bigger than that between table and ball, movement after ball is off racket face includes forward and sidespin, which will be more intense than movement after contacting with table.

## CONCLUSIONS

The paper makes force analysis of table tennis movement in the air, it establishes differential equation model, comprehensive detailed reveals table tennis movement rules in the air, which builds theoretical basis for future table tennis training. By mechanical analysis of table tennis and establishing differential equation, it finds that Magnus force plays crucial roles in table tennis movement trajectory, it should strengthen research on Magnus force in future table tennis scientific researches, so that better reveals table tennis movement rules. In researching process, it finds that table tennis regularities are very obvious, therefore in future research, it should focus on its combination with increasingly de-

veloped computer simulation technique, establish table tennis movement simulation trajectory, so that make contributions to Chinese table tennis progress.

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