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Table tennis diameter changes research based on analytic hierarchy process and kinetic energy comprehensive optimization control

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ABSTRACT

The paper carries out analysis from athletes' experience qualities and audiences' appreciation qualities two aspects, applies analytic hierarchy process and table tennis dynamical simulation model researching table tennis diameter increasing from 38mm to 40mm influences on competition appreciation. After that, establish falling time and kinetic energy comprehensive optimization controlling mathematical planning model, it gets best table tennis diameter. Research process takes athlete experience quality as first class indicator, takes technical difficulty, tactical thinking, forehand using times, backhand using times, receiving rate as second class indicators, applies analytic hierarchy process method getting each influence factors weights and athlete experience quality and each influence factor relationship, it can know that table tennis diameter increasing improves athlete experience quality. And then consider table tennis diameter changes to audience appreciation quality influence, audience appreciation quality mainly reflects on table tennis speed and rotational speed; to table tennis speed, it applies table tennis dynamical simulation model getting table tennis speed and diameters relationship, which can learn that table tennis diameter increasing let speed reduce and audience appreciation quality improve. To table tennis rotational speed, according to moment of momentum theorem, it solves different rotational speeds on condition table tennis has different diameters, and finally gets rotation reduction will affect ball aggressiveness, increases table tennis competition round numbers, and let audience appreciation quality improve.

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KEYWORDS

Table tennis;
Analytic hierarchy process;
Kinematic equation;
Simulation model;
Optimization.

INTRODUCTION

Table tennis is a kind of world popular ball type sports event that derived from Britain. In the end of 19th century, table tennis was prevail in Europe, but due to field and weather constraints, some college students in Britain transferred table tennis into indoors. In the

beginning of 20th century, table tennis has been flourishing in Europe and Asia. In 1926, Germany Berlin organized international table tennis tournament, subsequently it was posthumously accepted as the first session world table tennis championship, meanwhile it founded international table tennis league. World table tennis, if it counts from the first session world table ten-

nis competition, until now roughly can be divided into four periods: European table tennis heyday; Japanese team dominates world table tennis period; Chinese table tennis rising; European table tennis reviving and fighting between Europe and Asia.

Since 2000, international table tennis federation increased international table tennis professional competition official ball diameter from 38mm to 40mm. In this way it further increases table tennis air resistance during air running, slow down competition's ball running speed, so that achieve the purpose of further increasing and enriching table tennis professional athletes hitting techniques and skills, and finally increase table tennis competitions' overall appreciation. However, since incoming of table tennis "big ball era" up to now, it leads to professional athletes themselves factors difference to ball diameter changes sensitivities differences, and causes ball diameters disputes never cease. For the problem, the paper firstly takes athlete experience quality as first class indicator, takes technical difficulty, tactical thinking, forehand using times, backhand using times and receiving rate as second class indicators, applies analytic hierarchy process judging athlete experience quality main influence factors, after that it gets each influence factor weight as well as first class and second class indicators relationships, and finally gets different table tennis to athlete experience quality influences. The next applies table tennis dynamical simulation model researching table tennis speed and diameter relationship, and then according to moment of momentum theorem, it solves different rotational speeds under different diameters, finally it gets table tennis rotational speed and diameter relations. Finally on the basis of audience and athlete satisfaction, it establishes table tennis falling time minimum planning model and table tennis falling kinetic energy maximum mathematical planning model.

ATHLETE EXPERIENCE EVALUATION MODEL BASED ON AHP

The model mainly considers athlete experience quality main influence factors that are: technical difficulty, tactical thinking, forehand using times, backhand using times, receiving rate^[1]. In order to explain athlete experience quality and technical difficulty, tactical thinking, forehand using times, backhand using times, as well as

receiving rate relations, apply analytic hierarchy process getting each influence factor weight as well as experience quality and each influence factor formula, it further gets different table tennis diameters to athlete experience quality influence level^[2].

In order to consider athlete experience quality and technical difficulty, tactical thinking, forehand using times, backhand using times, and receiving rate influence degree. The paper considers to define athlete experience quality as first class indicator, and technical difficulty, tactical thinking, forehand using times, backhand using times^[3,4], as well as receiving rate as second class indicators, indicator system is as following Figure 1.

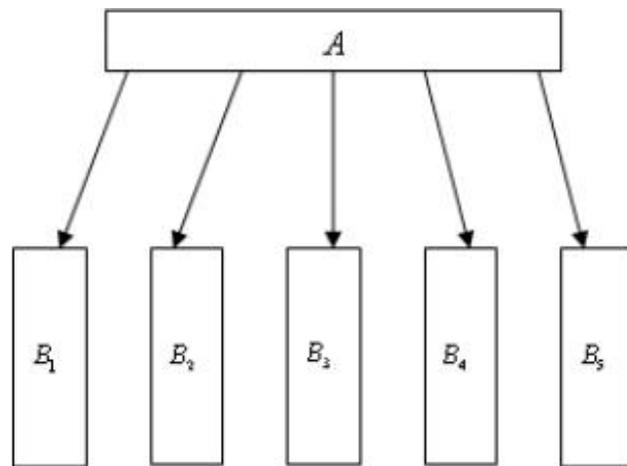


Figure 1: Indicator system schematic

Among them, A athlete experience quality, B₁ technical difficulty, B₂ tactical thinking, B₃ forehand using times, B₄ backhand using times, B₅ receiving rate.

According to analytic hierarchy process nine-scale method, it gets small ball criterion layer to target layer judgment matrix as TABLE 1 show.

It solves random consistency rate is $CR_1 = 0.0408 < 0.1$, therefore, criterion layer to target layer consistency test passes.

Then according to analytic hierarchy process nine-scale method, it gets big ball criterion layer to target layer judgment matrix is as TABLE 2 show.

It solves random consistency rate is $CR_1 = 0.0579 < 0.1$, therefore, criterion layer to target layer consistency test passes.

Set judgment matrix is a_{ij} , then every line element

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TABLE 1 : Small ball criterion layer to target layer judgment matrix

A	B ₁	B ₂	B ₃	B ₄	B ₅
B ₁	1	2	4	3	9
B ₂	1/2	1	2	4	3
B ₃	1/4	1/2	1	2	4
B ₄	1/3	1/4	1/2	1	2
B ₅	1/9	1/3	1/4	1/2	1

TABLE 2 : Big ball criterion layer to target layer judgment matrix

A	B ₂	B ₁	B ₃	B ₄	B ₅
B ₂	1	3	5	6	2
B ₁	1/3	1	3	5	6
B ₃	1/5	1/3	1	3	5
B ₄	1/6	1/5	1/3	1	3
B ₅	1/2	1/6	1/5	1/3	1

product:

$$M_i = \prod_{j=1}^m a_{ij} \tag{1}$$

$$a_i = \sqrt[m]{M_i}, i = (1,2,3,4, \dots, m) \tag{2}$$

To vector $\alpha = (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_m)^T$ it proceeds with normalization, and gets maximum feature value λ_m feature vector:

$$W = (W_1, W_2, W_3, \dots, W_m)^T \tag{3}$$

$$\lambda_m = \frac{1}{m} \sum_{i=1}^m \frac{(AW)_i}{w_i}, i = (1,2, \dots, m) \tag{4}$$

$$C.I = \frac{\lambda_m - m}{m - 1}, CR = \frac{C.I}{RI} \tag{5}$$

TABLE 3 is all factors to target relative important weight vectors; it calculates judgment matrix consistency indicator CI value. When random consistency ratio $CR < 0.1$, then it is thought that hierarchical single arrangement result has satisfaction consistency, otherwise it needs to adjust matrix elements values.

To small ball, use MATLAB software solving: $\lambda_{max} = 5.1829, CI = 0.0457, RI = 0.0408$

Therefore it is thought that it meets consistency indicator, judgment matrix is feasible. Maximum feature values weight vector is $W = (0.4473, 0.2539, 0.1522, 0.0941, 0.0525)$. According to W , it is clear that technical difficulty has maximum influence on athlete experience quality.

To big ball, use MATLAB solving: $\lambda_{max} = 5.2595, CI = 0.0649, RI = 0.0579$

Therefore it is thought that it meets consistency indicator, judgment matrix is feasible. Maximum feature values weight vector is $W = (0.5061, 0.2602, 0.1321, 0.0669, 0.0348)$. According to W , it is clear that tactical thinking has maximum influence on athlete experience quality.

Define athlete experience quality as y , and respectively define technical difficulty, tactical thinking, forehand using times, backhand using times, receiving rate as x_1, x_2, x_3, x_4, x_5 , it gets computing relationship is as following:

To small ball:

$$y = 0.4473x_2 + 0.2539x_1 + 0.1522x_3 + 0.0941x_4 + 0.0525x_5 \tag{6}$$

$$y = \frac{1}{19} (1 \times 0.4473 + 2 \times 0.2539 + 4 \times 0.1522 + 3 \times 0.0941 + 9 \times 0.0525) = 0.12203 \tag{7}$$

To big ball:

$$y = 0.5061x_1 + 0.2602x_2 + 0.1321x_3 + 0.0669x_4 + 0.0348x_5 \tag{8}$$

$$y = \frac{1}{17} (1 \times 0.5061 + 3 \times 0.2602 + 5 \times 0.1321 + 6 \times 0.0669 + 2 \times 0.0348) = 0.142247 \tag{9}$$

TABLE 3 : Random consistency indicator *RI* value

n	1	2	3	4	5	6	7	8	9	10	11
<i>RI</i>	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51

Because $y = 0.12203 < y = 0.142247$, therefore athlete experience quality improves after changing from small ball to big ball.

TABLE TENNIS DYNAMICAL SIMULATION MODEL

In small ball era, athlete tends to win by service; it greatly reduces mass appreciation quality. And in big ball era, table tennis ball speed and rotational speed are main factors affect competition appreciation, accordingly establish table tennis dynamical simulation model, by which it looks for big ball and small ball's ball speed and rotational speed changes status after diameter changing so as to verify its relation with audience appreciation quality. In the following, it respectively researches on audience appreciation quality influence from table tennis ball speed and table tennis rotational speed two aspects^[6].

Table tennis diameter increasing to speed influence

Rotating table tennis in running process mainly suffers gravity, buoyancy, additional mass force, air resistance and Magnus force effects^[7], from which, gravity force and buoyancy directions are opposite, air resistance and table tennis moving directions are opposite.

Gravity F_g expression is as following:

$$F_g = mg = \frac{1}{6} \pi \rho_a d^3 g \tag{10}$$

Among them, ρ_d is table tennis density, d is table tennis diameter, buoyancy F_b is equal to table tennis sphere displaced same volume air mass force, its computing formula is:

$$F_b = m_a g = \frac{1}{6} \pi \rho_a d^3 g \tag{11}$$

Among them, ρ_a is air density.

To magnet induction caused additional mass force, its size is:

$$F_{m'} = \frac{1}{12} \pi \rho_a d^3 \frac{dv}{dt} = \frac{1}{2} m_a \frac{dv}{dt} \tag{12}$$

$\frac{1}{2} m_a$ is commonly recorded as m' , which is called as additional mass. Additional mass force can be ignored when table tennis changes are not big.

Spherical object suffered resistance in fluid is equal to the spherical object radius, speed, fluid viscosity and 6π product. The law is called Stokes law. Suffered resistance is called Stokes force. Its F_s computing formula is:

$$F_s = 6\pi r v \mu = 3\pi d v \mu \tag{13}$$

When a rotational object rotational angular speed vector and object flight speed vector don't overlap, it will produce a horizontal force in rotational angular speed vector and translational speed vector composed plane vertical direction. Under the horizontal force effects, object flight trajectory occurred deviation phenomenon is called Magnus effect. To Magnus effect, by scholars researching, main mechanisms have: 1) asymmetric displacement thickness, 2) asymmetric centrifugal force, 3) asymmetric wall friction stress, 4) asymmetric transition, 5) asymmetric separate and vortex, 6) asymmetric secondary flow. To sphere, Magnus force can

TABLE 4 : Model initial parameter

Parameter	Value
Big ball rotational speed	116.5r / s
Small ball rotational speed	133.5r / s
Big ball	$m = 2.7g, d = 40mm$
Small ball	$m = 2.5g, d = 38mm$
Environment	$T = 20\text{ }^\circ\text{C}, P = 1\text{ atm}$
Air density	$\rho_a = 1.293kg / m^3$
Viscosity	$\mu = 1.86 \times 10^{-5} Ns / m^3$

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be expressed by following formula:

$$F_M = \frac{1}{8} \pi \rho_a d^3 V \omega \quad (14)$$

Among them, ω is rotational speed. According to Newton the second law, it can establish rotating table tennis horizontal and vertical dynamical equations:

Horizontal direction:

$$(m + m') \frac{d^2 x}{dt^2} = -3\pi \mu d \frac{dx}{dt} + \frac{1}{8} \pi \rho_a d^3 \frac{dy}{dt} \omega \quad (15)$$

Vertical direction:

$$(m + m') \frac{d^2 y}{dt^2} = -F_g + F_b - 3\pi \mu d \frac{dy}{dt} - \frac{1}{8} \pi \rho_a d^3 \frac{dx}{dt} \omega \quad (16)$$

Among them, $F_g = mg$, $F_b = m_a g$, t is time.

Due to additional mass force can be neglected when table tennis speed changes are not big, to small ball and big ball, all can neglect m' .

To small ball:

Known that small ball horizontal direction acceler-

ated speed $a_x = \frac{d^2 x}{dt^2} = 14.5581 m/s^2$, vertical direc-

tion accelerated speed $a_y = \frac{d^2 y}{dt^2} = -50.6362 m/s^2$.

Big ball rotational speed $\omega = 116 r/s$, big ball mass $m = 2.7 g$,

diameter $d = 40 mm$,

$\mu = 1.86 \times 10^{-5} Ns/m^3$, $\rho_a = 1.293 kg/m^3$.

To big ball:

Known that big ball horizontal direction acceler-

ated speed $a_x = \frac{d^2 x}{dt^2} = 15.416 m/s^2$, vertical direction ac-

celerated speed $a_y = \frac{d^2 y}{dt^2} = -44.254 m/s^2$. Small ball ro-

tational speed $\omega = 133.5 r/s$, small ball mass $m = 2.5 g$,

diameter $d = 38 mm$,

$\mu = 1.86 \times 10^{-5} Ns/m^3$, $\rho_a = 1.293 kg/m^3$. Calcula-

tion adopted initial condition is as Table show.

Input small ball known data into rotating table tennis horizontal direction and vertical direction dynamical

equations, and get $v_x = \frac{dx}{dt} = 12 m/s$,

$v_y = \frac{dy}{dt} = 13 m/s$ and $v = 17.6918 m/s$

Input big ball known data into rotating table tennis horizontal direction and vertical direction dynamical

equations, and get $v_x = \frac{dx}{dt} = 13 m/s$,

$v_y = \frac{dy}{dt} = 11 m/s$ and $v = 17.0294 m/s$.

Table tennis diameter increasing influences on rotational speed

If athlete hits different sizes two balls with same way and equal size force, because big ball and small ball rotational inertia are different, then ball movement state changes will have obvious differences, two balls rotational inertia computing formula is:

$$I_2 = 2/3 m_2 r_2^2 = 7.9312 (g \cdot cm^2) \quad (17)$$

$$I_1 = 2/3 m_1 R_2^2 = 6 (g \cdot cm^2) \quad (18)$$

Calculate according to small ball rotational speed is 50 turn/second, according to moment of momentum theorem, it can calculate big ball rotational speed ω_2 is:

$$\text{because: } M \cdot t = I_1 \omega_1 \quad M \cdot t = I_2 \omega_2$$

then: $I_1 \omega_1 = I_2 \omega_2$ $\omega_2 = I_1 \omega_1 / I_2 = 40.59 (\text{turn} / s)$ two balls angular speed difference is:

$$\Delta \omega = \omega_1 - \omega_2 = 9.4112 (\text{turn} / s) \quad (19)$$

By calculation, it is clear when hitting different sizes two balls with same way, big ball rotational speed reduces 9.4112 turn/s (near to 1/5) to small ball rotational speed.

Conclusion analysis

Big ball speed $v = 17.0294 m/s$ and small ball speed $v = 17.6918 m/s$, big ball speed is smaller than small ball speed. Comparing with using small ball, every round competition time will be extended, competition intense degree will be increased, and audience appreciation quality is greatly improved.

Small ball rotational speed is 50 turn/s, and big ball rotational speed is 40.59 turn/s, according to Bernoulli's

theorem, high speed rotating ball's flight trajectory in the air is a curve not a straight line, curve crooked level is up to eccentric force leads to ball produced rotational speed and seed compound vector size and direction. Therefore, rotational speed reduction will affect ball assaulting, increase table tennis competition round numbers, and let audience appreciation quality improve.

TABLE TENNIS DIAMETER COMPREHENSIVE EVALUATION MODEL

On the condition that athlete is satisfied, establish table tennis maximum falling kinetic energy mathematical planning model, and get maximum falling kinetic energy ω , input obtained ω into table tennis dynamical simulation model, and get maximum table tennis diameter. After that, on the condition that audience is satisfied, establish minimum table tennis falling time planning model, and get minimum falling time ω , input obtained ω into table tennis dynamical simulation model, and get minimum table tennis diameter. To get best table tennis diameter, it should consider conditions that both audience and athlete are satisfied, therefore establish falling time and kinetic energy comprehensive optimization control mathematical planning model, and get best ω , input it into table tennis dynamical simulation model, and get best table tennis diameter.

Minimum table tennis falling time mathematical planning model

Establish as Figure 2 showed space coordinate system, table midpoint is coordinate origin o , from perspective of hitter, right hand direction is positive direc-

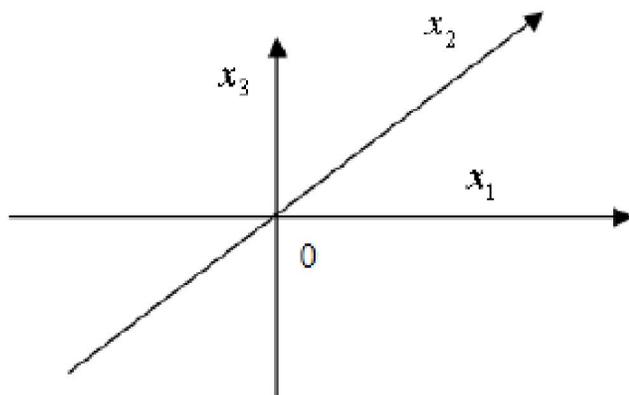


Figure 2: Space coordinate system

tion of x_1 axis, the forward is positive direction of x_2 axis, the upward is positive direction of x_3 axis.

Table tennis initial speed value is: v_{10}, v_{20}, v_{30} , table tennis initial rotational speed value is: $\omega_{10}, \omega_{20}, \omega_{30}$.

Let: $x_{10}(t) = \omega_{10}; x_{11}(t) = \omega_{20}; x_{12}(t) = \omega_{30}$

Model establishment

$\min f = t_f(v_{10}, v_{20}, v_{30}, \omega_{10}, \omega_{20}, \omega_{30})$ (20)

$g_1 = v_{\max} - (\sqrt{v_0^2} + \frac{2}{3} r \frac{\omega_0^2}{\sqrt{v_0^2}}) \geq 0$ (21)

$g_2 = 1 - \frac{2}{3} r \sqrt{\frac{\omega_0^2}{v_0^2}} \geq 0$ (22)

$h_1 = v_0 \omega_0 = 0$ (23)

In formula,

$v_0 = (v_{10}, v_{20}, v_{30}), \omega_{10} = (\omega_{10}, \omega_{20}, \omega_{30})$, g_1 represents maximum swinging speed constraint, g_2 represents hitting point cannot be out of table tennis: h_1 represents orthogonal relation between table tennis initial mass center speed and angular speed.

Model solution

When table tennis initial position is:

$x_1(0) = 0, x_2(0) = -10, x_3(0) = -1, x_4(0) = 0, x_5(0) = 0, x_6(0) = 0$, and $v_{\max} = 200$,

It solves:

$t_f = 0.09443$ (24)

$v_0 = \{0, 158.444, 29.355\}$ (25)

$\omega_0 = \{-738.984, 0, 0\}$ (26)

$\omega = \frac{738.984}{2\pi} = 117.613r/s$ (27)

Input $\omega = \frac{738.984}{2\pi} = 117.613r/s$ into table tennis dynamical simulation model, and can get maximum diameter is: $d = 39.8734mm$.

Maximum table tennis falling kinetic energy mathematical planning model

Model establishment

$\min f = \frac{1}{T(t_f)}$ (28)

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$$g_1 = v_{\max} - \left(\sqrt{v_0^2} + \frac{2}{3} r \frac{\omega_0^2}{\sqrt{v_0^2}} \right) \geq 0 \quad (29)$$

$$g_2 = 1 - \frac{2}{3} r \sqrt{\frac{\omega_0^2}{v_0^2}} \geq 0 \quad (30)$$

$$h_1 = v_0 \omega_0 = 0 \quad (31)$$

Model solution

When initial position coordinate is:

$$x_1(0) = 0, x_2(0) = -10, x_3(0) = -1, \\ x_4(0) = 0, x_5(0) = 0, x_6(0) = 0, \quad , \text{ and } v_{\max} = 200,$$

it solves:

$$t_f = 0.15681s \quad (32)$$

$$v_0 = \{0, 94.233, 78.271\} \quad (33)$$

$$\omega_0 = \{-909.924, 0, 0\} \quad (34)$$

$$\omega = \frac{909.924}{2\pi} = 144.819r/s \quad (35)$$

$$\text{Input } \omega = \frac{909.924}{2\pi} = 144.819r/s \text{ into table tennis dynamical simulation model, and can get maximum diameter is:}$$

$d = 37.2015mm$.

Falling time and kinetic energy comprehensive control mathematical planning model

Model establishment

$$\min f = k \frac{t_f}{0.09443} + (1-k) \frac{46.653}{T(t_f)} \quad (36)$$

$$g_1 = v_{\max} - \left(\sqrt{v_0^2} + \frac{2}{3} r \frac{\omega_0^2}{\sqrt{v_0^2}} \right) \geq 0 \quad (37)$$

$$g_2 = 1 - \frac{2}{3} r \sqrt{\frac{\omega_0^2}{v_0^2}} \geq 0 \quad (38)$$

$$h_1 = v_0 \omega_0 = 0 \quad (39)$$

Model solution

k value range is $0 \sim 1$, when $k = 1$, it is time optimal control, when $k = 0$ it is kinetic energy optimal control, during model calculating $k = 10\%$.

When initial coordinate position is:

$$x_1(0) = 0, x_2(0) = -10, x_3(0) = -1, \\ x_4(0) = 0, x_5(0) = 0, x_6(0) = 0, \quad , \text{ and } v_{\max} = 200,$$

it solves:

$$t_f = 0.11964s \quad (40)$$

$$v_0 = \{0, 153.013, 44.049\} \quad (41)$$

$$\omega_0 = \{-752.455, 0, 0\} \quad (42)$$

$$\omega = \frac{752.455}{2\pi} = 119.757r/s \quad (43)$$

$$\text{Input } \omega = \frac{752.455}{2\pi} = 119.757r/s \text{ into table tennis dynamical simulation model, and can get best diameter is: } d = 39.634mm.$$

$d = 39.634mm$.

Result analysis

Apply table tennis minimum falling time mathematical planning model, and it can get table tennis maximum diameter $d = 39.8734mm$, apply table tennis maximum falling kinetic energy mathematical planning model and it can get table tennis minimum diameter $d = 37.2015mm$, therefore it is clear that table tennis diameter range is $37.2015mm \sim 39.8734mm$. And because table tennis minimum diameter taking $d = 38mm$ is relative reasonable, finally it gets table tennis diameter range is $38mm \sim 39.8734mm$. After that apply falling time and kinetic energy comprehensive optimization control mathematical planning model, it gets table tennis best diameter is: $d = 39.634mm$.

CONCLUSIONS

Apply analytic hierarchy process analyzing athlete experience quality influences factors, decompose a complicated qualitative analysis problem into several factors that can make quantitative analysis and solve, obtained result is relative intuitional. Utilize table tennis dynamical simulation model more vividly combining ideal state mechanical research with practical mechanical trajectory, it describes table tennis mechanical state in specific running, obtained conclusion is correct and conforms to practice. Apply falling time and kinetic energy comprehensive optimization control mathematical planning model, combine table tennis speed with rotational

speed, on the premise audience appreciation quality is ensured, let athlete experience quality arrive at maximum, establish minimum table tennis falling time mathematical planning model, it gets ω that lets falling time to be minimum. After that in case athlete is satisfied, establish maximum table tennis falling kinetic energy mathematical planning model, it gets ω that lets falling kinetic energy to be maximum. In case that both audience and athlete are satisfied, establish falling time and kinetic energy comprehensive optimization control mathematical planning model, and get best $\omega = 119.757r/s$, input obtained ω into table tennis dynamical simulation model, and solve best table tennis diameter is: $d = 39.634mm$.

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