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Table tennis changes to movement trajectory influence research based on fuzzy comprehensive evaluation and differential equation

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ABSTRACT

The paper firstly makes force analysis of table tennis when lifting the ball, and according to moment of momentum theorem, it gets rotational speed and diameter inverse ratio relations; then it makes force analysis of air movement table tennis, and gets kinematic equation. After diameter getting big, ball speed and rotational speed get small, and ball movement time gets long, let opponent probability of receiving the ball and fighting back increase. And establish analytic hierarchy process and fuzzy comprehensive evaluation test model to test. It concludes that big ball promotes competition appreciation relative to small ball, but players' competition experiences reduce. © 2014 Trade Science Inc. - INDIA

KEYWORDS

Table tennis diameter;
Differential equation;
Analytic hierarchy process;
Fuzzy comprehensive
evaluation.

INTRODUCTION

In 2000, international table tennis federation increased international table tennis professional competition official ball diameter from 38mm to 40mm. The aim is to further increase ball's air resistance during air running, slow down competition's ball running speed, so that achieve the purpose of further increasing and enriching table tennis professional athletes hitting techniques and skills, and finally increase table tennis competitions' overall appreciation. Due to sphere increasing, it causes sphericity, arc, ball speed, rotation, rebound angle and others have a series of changes, which will put forward higher requirements on athlete's psychological quality, physical quality and technical level so on. It is worth noting that due to professional athletes' height, playing habit, gripping habit differences, their sensitivities to ball diameter changes are also dif-

ferent.

Table tennis as a kind of sport that extremely exercises participants reactions, coordination and operational thinking abilities, it is well received by Chinese mass. As participants themselves, whether can easier play higher offensive ball is the key to participating experiences, therefore, to athlete's experience quality, the paper mainly from ball speed, strength, rotational speed three aspects and diameter relations, by consulting corresponding data, it further gets before and after ball diameter changes, ball's changes in speed, strength, and rotational speed the three aspects sizes, according to final values comparison, it objectively reflects whether athletes can more easily hit high offensive ball and that is closely related to their experience qualities.

As audience, competition intense degree no doubt is the main factor that affects competition appreciation. An intense competition, it is surely that participating two

parties' strength difference is not big so that can stimulate players' highest levels. Players reflected abilities are stronger and then that reflects on competition each round number of hitting rackets increasement as well as each game number of rounds increasement. If every round number of hitting rackets increase, and then athletes surely have higher receiving accuracy, in case ignoring players themselves abilities, only increase players' receiving reaction time that can extremely increase participating players' receiving accuracy, and finally improve competition appreciation quality.

COMPETITION EXPERIENCE QUALITY AND APPRECIATION RESEARCH

Table tennis dynamical analysis

As Figure 1 show, when athlete swings and lifts the ball, racket and horizontal line angle is α , make force analysis of ball: in the direction of racket movement, ball suffered friction force is f , in racket movement vertical direction, ball suffered force is F , racket to ball acting time is t_0 . Set table tennis quality is m , diameter is D , speed is 0 , set f in its own direction gives table tennis impulse $\int_0^{t_0} f dt$ is I_1 , set F in its own direction gives table tennis impulse $\int_0^{t_0} F dt$ is I_2 , meanwhile f to table tennis mass center one torque is $M = \frac{D \cdot f}{2}$, set torque M to table tennis impulsive moment $\int_0^{t_0} M dt$ is ΔL , table tennis rotational inertia

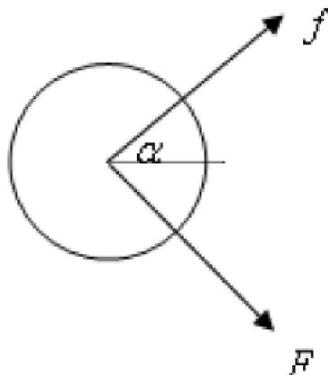


Figure 1 : Table tennis force analysis

$$\frac{mD^2}{6} \text{ is } J$$

$$I_1 = mv_1 - 0 \tag{1}$$

$$I_2 = mv_2 - 0 \tag{2}$$

$$\Delta L = J\omega - 0 \tag{3}$$

It solves:

$$v_1 = \frac{f \cdot t_0}{m} \tag{4}$$

$$v_2 = \frac{F \cdot t_0}{m} \tag{5}$$

$$\omega = \frac{3f \cdot t_0}{mD} \tag{6}$$

$$\text{And } n = \frac{\omega}{2\pi} \tag{7}$$

Conclusion one: By formula(6), it is clear that when impulsive moment is fixed, table tennis angular speed ω is inversely proportional to its diameter D , by formula (7) it is clear that angular speed ω is in direct proportion to rotational speed n , therefore when D changes from 38mm to 40mm, n will correspondingly reduce.

After athlete swinging and lifting the ball, analyze table tennis speed as Figure 2.

Because f and F are perpendicular, v_1 and v_2 included angle is 90 degree, and v_2 and y axis included angle is also α . Set v_1 horizontal direction component $v_1 \cos \alpha$ is v_{x1} , vertical direction component $v_1 \sin \alpha$ is v_{y1} ; v_2 horizontal direction com-

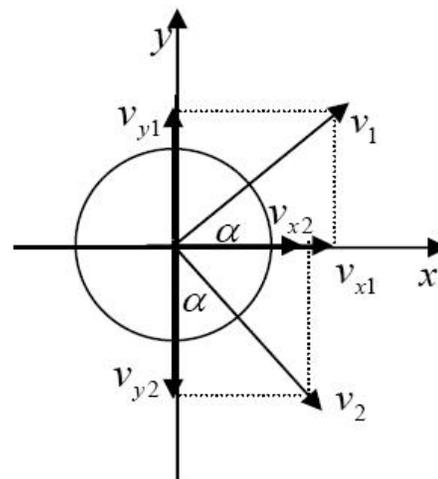


Figure 2 : Table tennis speed analysis

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ponent $v_2 \sin \alpha$ is v_{x2} , vertical direction component

$v_2 \cos \alpha$ is v_{y2} .

Then:

$$\begin{cases} v_x = v_{x1} + v_{x2} \\ v_y = v_{y1} - v_{y2} \end{cases} \quad (8)$$

It solves:

$$\begin{cases} v_x = \frac{(f \cos \alpha + F \sin \alpha)t_0}{m} \\ v_y = \frac{(f \sin \alpha + F \cos \alpha)t_0}{m} \end{cases} \quad (9)$$

Table tennis force analysis when moving in the air:

When no rotating table tennis moves in the air, it suffers gravity G , friction force F_m , buoyancy F_f , friction force and speed direction are opposite, buoyancy and gravity direction are opposite. When table tennis rotates at higher speed, it should also consider Magnus force. As Figure 3 show:.

Gravity expression:

$$G = mg \quad (g = 9.8m \cdot s^{-2});$$

Suffered buoyancy F_f is equal to table tennis displaced air mass, table tennis volume $V = \frac{1}{6}\pi D^3$, set air

density is ρ , so

$$F_f = \rho g V = \frac{\pi}{6} \rho g D^3 \quad (10)$$

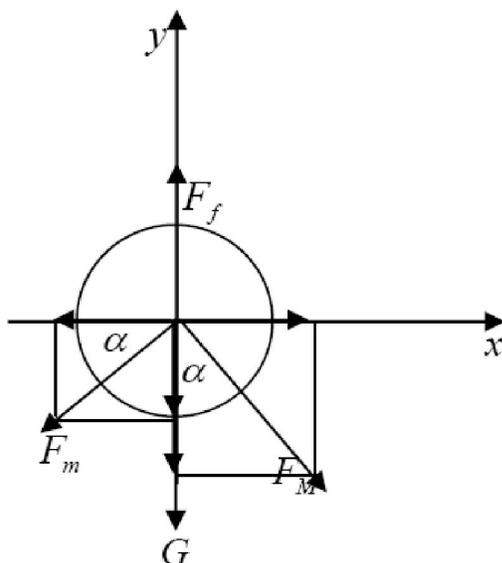


Figure 3 : Table tennis Magnus force analysis

In movement, friction force formula is :

$$F_m = \frac{1}{8} C_D \pi \rho D^2 v_1^2 \quad (11)$$

Among them, C_D is resistance coefficient, ρ is air density, D is ball diameter, v is table tennis speed when it is out of racket.

Magnus force is a force generated due to object two sides air flow speeds are different on condition that object rotational angular speed vector and object flight speed vector don't overlap, its direction is as Figure 3 show, its direction is vertical to speed direction, its expression is:

$$F_M = C_L \rho D^3 f v_1 \quad (12)$$

Among them, C_L is lift force coefficient, ρ is air density, D is table tennis diameter, f is table tennis rotational frequency, v_1 is table tennis speed when it is out of racket.

According to Figure 3, establish table tennis rotational movement dynamical equation^[2]:

$$\begin{cases} m a_x = C_L \rho D^3 f v_1 \cdot \sin \alpha - \\ \frac{1}{8} C_D \pi \rho D^2 v_1^2 \cdot \cos \alpha \\ m a_y = \frac{\pi}{6} \rho g D^3 - G - C_L \rho D^3 f v_1 \\ \cdot \cos \alpha - \frac{1}{8} C_D \pi \rho D^2 v_1^2 \cdot \sin \alpha \end{cases} \quad (13)$$

By Matlab programming, calculate when $D = 40mm$ and $D = 38mm$, assume that initial speed is $8m \cdot s^{-1}$, initial displacement is 0, speed v_1 relative to time t image comparison is as following Figure 4, displacement s relative to time t is as Figure 5, Figure 6.

When $D = 40mm$, speed v_1 with regard to time t function:

$$v_1 = 491./(360 - 2389/8.* \exp(-491/200000.* t)) \quad (14)$$

When $D = 38mm$, speed v_1 with regard to time t function:

$$v_1 = 4./(11 - 37/4.* \exp(-21/10000.* t)) \quad (15)$$

Assume that athlete swings in the external 1m, it drops on the table, length of table tennis table is 2.74m,

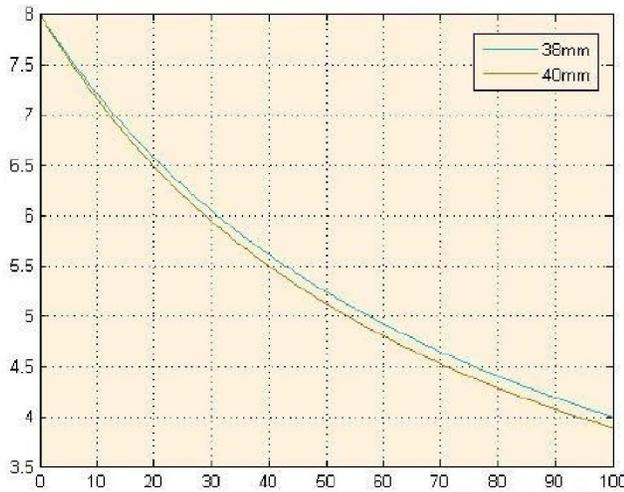


Figure 4 : Table tennis speed changes followed by time

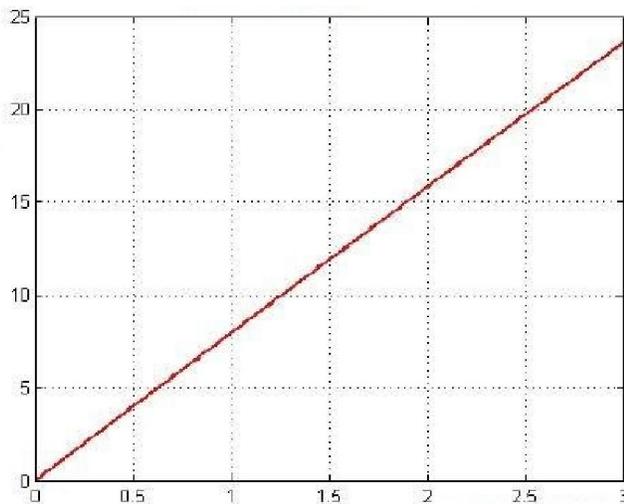


Figure 5 : 38mm table tennis horizontal displacement changes followed by time

so assume that two cases horizontal displacements are equal as 3.5m, from Figure 5 and Figure 7, it can get ball movement time are 0.45 s and 0.40 s . According to time, it finds out from Figure 4 that table tennis end speeds are $v_1 = 7.80 m \cdot s^{-1}$ and $v_2 = 7.97 m \cdot s^{-1}$.

Due to 40mm table tennis diameter increasing relative to that of 38mm, it let drop point speed reduce $0.02 m \cdot s^{-1}$. 40mm table tennis mass is $m_1 = 2.7g$, 38mm table tennis mass is $m_2 = 2.6g^{[3]}$, so the two end kinetic energies are respectively:

$$E_1 = \frac{1}{2} m_1 v_1^2 = 0.081 J$$

$$E_2 = \frac{1}{2} m_2 v_2^2 = 0.083 J$$

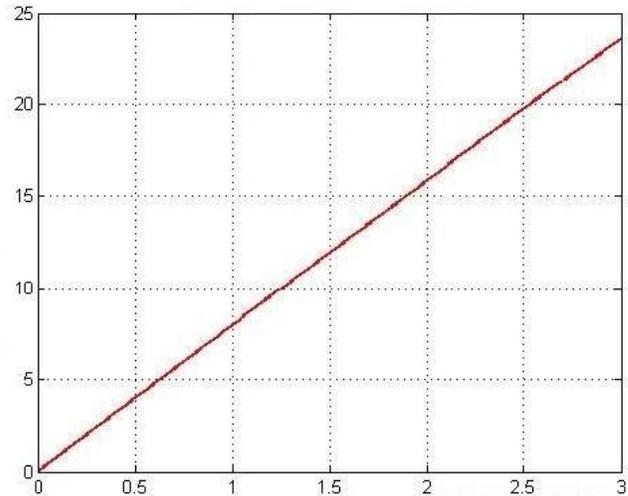


Figure 6 : 40mm table tennis horizontal displacement changes followed by time

Therefore after diameter increasing, its end kinetic energy reduces for instead.

By above, it is clear that after table tennis diameter increasing, arriving at drop point time is 1.12 times previous one, which provides opponent enough reaction time to judge ball trend and drop point, and receiving probability will greatly increase.

In order to check above whether above result conforms to practice or not, the paper investigates on partial audience opinions on competition exciting level after changing into big ball.

Audience’s evaluation on big ball competition intense level relative to small ball one has four measurement indicators: number of rounds, rotation, speed and strength^[4]. Different indicators relations are fuzzy, and impacts on competition intense level are different. Therefore, use fuzzy comprehensive evaluation model combining with analytic hierarchy process to solve the problem. First utilize analytic hierarchy process solving each indicator weight vector. And then make grading classification on impacts sizes, it constructs single item indicators to impacts grades membership function $F(x)$, weight vector combining with membership matrix to solve fuzzy comprehensive evaluation matrix, it can get audience overall evaluation.

Fuzzy comprehensive evaluation method

Set factors set as $X = \{x_1, x_2, \dots, x_m\}$, it represents object possessed m pieces of attributes; evaluation set is $Y = \{y_1, y_2, \dots, y_m\}$, it represents evaluation that may

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adopt to factors. Fuzzy evaluation is a fuzzy set on Y . In the paper $Y = \{A, B, C, D\}$, A grade represents: audience evaluation as very well; B grade represents: audience evaluation as good; C grade represents: audience evaluation as normal; D grade represents: audience evaluation as bad. When getting indicator x membership vector, in general, it adopts maximum membership principle to make grades classification on x , but the classification way generally is very fuzzy, it ignores x to other grades membership relations. Therefore, to comprehensive carry out grade evaluation on x , it scores the grades here that lets $A = 10, B = 7, C = 5, D = 1$. That is changing membership vector into weight relational vector, element x comprehensive grade score is:

$$M = 10A(x) + 7B(x) + 5C(x) + D(x)$$

Now assume that every factor x_i has a fuzzy evaluation $R_i = \{r_{i1}, r_{i2}, \dots, r_{in}\} \in F(X \times Y)$. Then, to m pieces of factors, it has m pieces of fuzzy evaluation R_1, R_2, \dots, R_m , they always can be expressed by matrix.

$$R = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_m \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & & \vdots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{pmatrix}$$

Call it as single factor evaluation matrix. If known fuzzy relation matrix R and factor weight distribution is

$$A = (a_1, a_2, \dots, a_m), \text{ from which } a_i > 0 \text{ and } \sum_{i=1}^m a_i = 1,$$

then it can solve fuzzy comprehensive evaluation B by A and R . The computing can be written as following form: $B = A \circ R$

Here "o" represents compositional operation.

Make use of analytic hierarchy process to solve each indicator weight

Analytic hierarchy process: analytic hierarchy process is a method that according to total target, each layer sub target, evaluation criterions till concreted switching programs orders, decompose decisive problems into different hierarchical structures, and then apply judgment matrix feature vectors solution solving every layer each element to previous layer one element priority weight, finally use weighting sum method hierarchical merging each alternative schemes to total target final weight. The maximum final weight one is optimal scheme.

According to nine scale method, it constructs indicator judgment matrix, nine-scale method is as TABLE 1.

Indicator judgment matrix is: $A = \begin{pmatrix} 1 & 3 & 6 & 9 \\ 1/3 & 1 & 3 & 5 \\ 1/6 & 1/3 & 1 & 2 \\ 1/9 & 1/5 & 1/2 & 1 \end{pmatrix}$

Matrix A maximum feature value is λ_{\max} , corresponding feature vector is $u = (u_1, u_2, \dots, u_n)^T$, normalize u , solve with the help of MATLAB software, it gets matrix A_4 maximum feature value is $\lambda_{\max} = 4.0340$, corresponding weight vector that normalized feature

vector is: $U = \begin{pmatrix} 0.5990 \\ 0.2505 \\ 0.0962 \\ 0.0543 \end{pmatrix}$.

Consistency test

Test principle: consistency test indicator is

$$CI = \frac{\lambda_{\max} - n}{n - 1} \text{ (n is matrix order, here n=3), consis-}$$

TABLE 1: Nine-scale method

Definition	u_i and u_j are equal	u_i is slightly	u_i is important	u_i is intensely	u_i is extremely
	important	important than u_j	than u_j	important than u_j	important than u_j
u_{ij} value	1	3	5	7	9

2, 4, 6, 8 between adjacent two judgment scale

tency ratio $CR = \frac{CI}{RI}$, when $CR < 0.1$, and then it calls the matrix has consistency. Consistency indicator RI value is as TABLE 2 show.

Consistency test result is as TABLE 3.

Establish comprehensive evaluation matrix

Membership function

To membership function, the paper makes comparison on relative data; it gets single indicators memberships TABLE 4;

TABLE 2 : Consistency indicator RI value

Matrix order number	2	3	4	5	6	7	8	9
RI value	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

TABLE 3 : Consistency test result

λ_{max}	CI	RI	CR	Whether meet consistency
4.0340	0.0113	0.90	0.0126	Yes

TABLE 4 : Single indicators memberships

Factor	A	B	C	D
Number of rounds	0.53	0.25	0.17	0.05
Rotation	0.48	0.43	0.09	0.00
Speed	0.33	0.42	0.16	0.09
Strength	0.29	0.54	0.08	0.09

$$V = \begin{pmatrix} 0.53 & 0.25 & 0.17 & 0.05 \\ 0.48 & 0.43 & 0.09 & 0.00 \\ 0.33 & 0.42 & 0.16 & 0.09 \\ 0.29 & 0.54 & 0.08 & 0.09 \end{pmatrix}$$

Fuzzy comprehensive evaluation matrix

According to each indicator weight vector U , combine with membership matrix V , and get fuzzy comprehensive evaluation matrix R . According to fuzzy matrix model that is $R = U \circ V$, from which to simplify algorithm, convert \circ into common matrix product computing that

$$R = U^T * V$$

It solves:

$$U^T * R = (0.4852 \quad 0.3272 \quad 0.1441 \quad 0.0435)$$

Qualitative analysis uses maximum membership principle and knows that 0.4852 is the maximum value, that is to say audience evaluation on competition is A grade, which shows audience evaluation on competition is very well, score is

$$M = 10 \times 0.4852 + 7 \times 0.3272 + 5 \times 0.1441 + 0.0435 = 7.9064$$

By testing, above computing result conforms to fact.

TABLE TENNIS BEST DIAMETER RE-SEARCH

By above, it is known that athlete experience quality and audience appreciation quality interact on each other. Table tennis movement speed has connections with diameter, when its displacement is fixed, the smaller diameter is, the shorter experienced time is, and smaller probability of athlete receiving would be. When athlete failed receiving ball, his reaction time is t , and reaction

time when receiving is T . Set $x = \frac{T}{t}$, when x gets

closer to most of players' two reaction times ratios, athlete receiving probability is bigger, player can more give his level into play, experience effect would also be better, meanwhile the number of rounds would be more, competition will also be more exciting, and audience appreciation quality will also improve. Therefore select best ratio x , and then it can get best table tennis diameter.

According to formula(13), it can solve:

$$\begin{cases} a_x = \frac{C_L \rho D^3 f v_1 \cdot \sin \alpha}{m} \\ - \frac{C_D \pi \rho D^2 v_1^2 \cdot \cos \alpha}{8m} \\ a_y = \frac{\pi \rho g D^3}{6m} - \frac{C_L \rho D^3 f v_1 \cdot \cos \alpha}{m} \\ - \frac{C_D \pi \rho D^2 v_1^2 \cdot \sin \alpha}{8m} - g \end{cases} \quad (16)$$

Set table tennis moves in the air at initial speed v_x , dropping to table moment horizontal end speed is v'_x , in horizontal direction, according to theorem of kinetic en-

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ergy, it gets:

$$ma_x = \frac{1}{2}mv_x'^2 - \frac{1}{2}mv_x^2 \tag{17}$$

By formula (14), it is clear that

$$a_x = \left(\frac{C_L \rho D f v_1 \cdot \sin \alpha}{m} - \frac{C_D \pi \rho v_1^2 \cdot \cos \alpha}{8m} \right) D^2,$$

and v_1 is a variable, in order to easier to calculate,

assu me $k = \max \left| \frac{C_L \rho D f v_1 \cdot \sin \alpha}{m} - \frac{C_D \pi \rho v_1^2 \cdot \cos \alpha}{8m} \right|,$

Therefore formula(17) can be converted as:

$$-mkD^2 = \frac{1}{2}mv_x'^2 - \frac{1}{2}mv_x^2 \tag{18}$$

That:

$$v_x^2 - v_x'^2 = kD^2 \tag{19}$$

Now accelerated speed is a fixed value,

so $\bar{v}_x = \frac{1}{2}(v_x + v_x')$, that $v_x' = 2\bar{v}_x - v_x$, therefore

$$v_x - v_x' = v_x - (2\bar{v}_x - v_x) = 2(v_x - \bar{v}_x)$$

$$\begin{aligned} v_x^2 - v_x'^2 &= (v_x - v_x')(v_x + v_x') \\ &= 2(v_x - \bar{v}_x) \times 2\bar{v}_x = kD^2 \end{aligned} \tag{20}$$

D_0 is 38mm ball, D_1 is an unknown quantity, D_1

average speed is \bar{v}_{x1} , due to hitting speed (that is also initial speed) basically do not change with ball diameter

changing, D_1 initial speed is also v_x . Assume same ex-

cellent athlete plays D_0, D_1 two different diameters

TABLE 5 : Receiving ball reaction time and missing ball reaction time ratios

1.18	1.18	1.20	1.16	1.21
1.22	1.20	1.22	1.22	1.19
1.17	1.18	1.18	1.19	1.19
1.16	1.21	1.17	1.20	1.18
1.18	1.18	1.18	1.18	1.18
1.17	1.22	1.20	1.17	1.17
1.18	1.20	1.28	1.16	1.21
1.20	1.28	1.23	1.18	1.27

placements are equal

Then:

$$\frac{(v_x - \bar{v}_{x0}) \times \bar{v}_x}{(v_x - \bar{v}_{x1}) \times \bar{v}_{x1}} = \frac{D_0^2}{D_1^2} \tag{21}$$

Set $\frac{\bar{v}_{x0}}{\bar{v}_{x1}} = x$, then $\bar{v}_{x0} = x \times \bar{v}_{x1}$

Therefore:

$$\frac{(v_x - \bar{v}_{x0}) \times \bar{v}_{x0}}{\left(v_x - \frac{\bar{v}_{x0}}{x} \right) \times \frac{\bar{v}_{x0}}{x}} = \frac{D_0^2}{D_1^2} \tag{22}$$

Assume same athlete plays D_0, D_1 two different diameters table tennis, and roughly thought that table tennis displacements are equal

Then: $t_0 \cdot \bar{v}_{x0} = t_1 \cdot \bar{v}_{x1}$, therefore $\frac{t_1}{t_0} = x$. t_0 is diameter as table tennis movement time, is diameter as table tennis movement time.

Table tennis air movement time t is athlete reaction judging time t . Watch 40 athletes competition video, and get their receiving ball's reaction time and missing ball reaction time ratios data is as TABLE 5 (reserves two decimal fractions).

By TABLE 5, it is clear that most of athletes data is equal or approximate to 1.18, so take $x = 1.18$

According to table tennis association investigation group provided information, when smashing 38mm table tennis, hitting speed is nearly 26.35m/s, ball average flight speed is nearly 17.8m/s. Due to hitting speed (that is also initial speed) basically will not change with ball diameter changing, diameter as D_1 table tennis hit-

ting speed can also approximately thought to be 26.35m/s.

Input data into(21)and solve: $D_1 = 43.6177mm$.

CONCLUSIONS

The paper gets 38mm and 40mm two table tennis influences on athlete experience quality and audience appreciation quality by analyzing different radius table tennis force and flight trajectory, and uses analytic hierarchy process carrying out weight measurement and fuzzy comprehensive evaluation method to make test, it eliminates subjective factors caused weight quantity not clear, it improves test result accuracy. It further solves best table tennis diameter on the condition that ensure athlete experience quality and audience appreciation quality, the result has higher reliability.

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