ISSN : 0974 - 7435 Volume 10 Issue 15

2014

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(15), 2014 [8462-8467]

# Study of the economic order quantity-partial back ordering model : Implementing rationing strategies under incentive discounts

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## ABSTRACT

The disclosure of government financial information has always been the weak line in the disclosure of financial information, thus have an impact on the quality of government financial information. It's urgent to accurately reflect the status of our government accounting information, establish a transparent government and enhance the transparency of government financial information. This article examines the problem of China local government financial information disclosures. According to the research, the results show that the government financial conditions, degree of punishment a series of other factors are significantly positively related with its disclosures. It is hoped that the research can provide an excellent practice guide to the online disclosure of government financial information.

## KEYWORDS

Government financial information; Disclosure quality; Factors; Consequences.

## INTRODUCTION

Advance selling is a method that has prevailed in the field of commercial housing sales. With the development of e-commerce, it has become even more popular, especially in the field of agricultural products and clothing. In the clothing industry, for example, an increasing number of consumer groups utilize fast fashion. To surpass logistics sales forecasts and promote the more efficient use of resources, combining spot sales and advance sales has become a new business model, allowing retailers to cope with changing markets and customer demands. In this model, customers are divided into two types: time-sensitive customers who require supply on-the-spot and price-sensitive customers who accept price discounts and delivery delays.

Research on advance sales has mainly focused on the forecast of pre-sale product demand (Moe & Fader, 2002)[1], advance sales and discounts (Weng & Parlar, 1999; Tang et al. 2004; Shugan & Xie, 2000; Shugan & Xie, 2001)[2-5], and advance selling strategy (McCardle, Rajaram, & Tang, 2004; Zhao & Stecke, 2010; Prasad, Stecke, & Zhao, 2010)[6-8]. A partial backlogging system, such as partial back ordering (PBO), suggests that some loyal customers will wait until the next replenishment cycle when the inventory system is out of stock. However, other customers are not willing to wait, demanding express delivery, which is often expensive.

The earliest study of the economic order quantity-partial back ordering (EOQ-PBO) model was conducted by Montgomery, Bazaraa, and Kesearch (1973)[9]. Related research in the subsequent 40 years has clarified initial assumptions, including the fact that partial backlogging rate is related to replenishment time and queue length. The potential for the inventory deterioration and damage of fresh products means that demand is often time-varying or relies such things as inventory availability (e.g., shopping malls' display number), quantity discount, etc. Padmanabhan and Vrat's (1995)[10] study was one of the earliest to establish the EOQ model between the stock out partial backlogging rate and the deteriorating items' stock quantity. Similarly, Chang and Dye (1999)[11]iwere the first to propose that partial backlogging rate and customer waiting time are related, and Erlenkotter (1990)[12]was the first to establish an EOQ model to indicate that backlogging rate is related to the period of customers' patience.

For customers, waiting to access stock can have a silent cost in the form of anxiety and unmet expectations. In such cases, customers may obtain the stock elsewhere, causing the enterprise to suffer a loss of profit. To remedy this, enterprises can assume the perspective of demand management, using discount incentives to guide demand (e.g., giving a price discount to compensate for the customers' waiting cost to improve their enthusiasm despite backlogging demand). Luo, Jing-jing, and Xin-he (2012)[13]considered that the index of the product purchase price increased with time and that two-stage inventory can affect sales; further, Xin-he's research established an EOQ model to illustrate that price growth and two-stage inventory affects selling rate. Huang et al. (2007)[14]also put forward an EOQ model. This model indicates that demand is an arbitrary distribution, while lead time and setup cost are controlled. They all found that offering price discounts can reduce inventory cost and increase profit.

The present study approaches EOQ-PBO from the perspective of supply management, using the rationing policy, controlling delivery to customers who can accept the delay of goods with a lower inventory, and reserving inventory services for customer who can only accept spot transactions to achieve higher profits. The rationing strategy involves controlling goods delivery, identifying types of customers effectively, and allocating inventory (according to the different types of demand at a lower inventory level. For instance, Veinott (1965)[15]was the earliest researcher to investigate the problem of inventory allocation in light of various types of demand. Nahmias and Demmy (1981)[16] also contributed to the area of inventory control in light of various types of demand; undermining the continuous inspection system, they put forward the critical point (critical level) control strategy and calculated the approximate expressions of delivery rate. Finally, Fadiloğlu, Karaşan, and Pinar (2010)[17]concluded the bounds of the dynamic critical point strategy, comparing the performance differences between static and dynamic strategy, general strategy in the method of simulation study, to measure the potential value of dynamic policy.

From the current literature, it can be seen that most studies of rationing inventories under the discount-pricing of the partial backlogging system and various kinds of demand are separated into two parts: studies that focus on rationing inventories under various demand and those carried out using a single, fixed price without discount. Studies of discount-pricing seldom consider the attributes of customers when inventories are not rationed. This paper focuses on the implementation of rationing strategy under the discount incentive, coordinating a EOQ-PBO model of stock allocation and discount-pricing to save cost both in ordering and inventorying, maximize corporate profits, and improve the overall satisfaction of customers.

## MODEL ASSUMPTIONS AND SYMBOL DEFINITIONS

According to the relevant research literature, partial backlogging rate is related to replenishment time and queue length. When considering the inventory deterioration and damage of fresh products, the demand is time-varying and relies on inventory capacity (e.g., shopping malls' display number), quantity discount, etc. Unlike these studies, this paper investigates the inventory system and the implementation of an incentive discount system, if backlogging rate depends on the size of the incentive discount, and distinguishes two categories based on the voluntary choice of customers. The first category refuses discount, demanding spot transactions; the second category accepts discount, backlogging demand. Rationing strategies are different for these two kinds of customers when inventory is at a low level: spot transactions are offered to the first kind, while discount is offered to the second. This supposes that the inventory status is unknown before it was purchased by customers; the shortage of queuing is often unknown because the queue (order) enterprises' private information and often will not be disclosed. The method here proposed avoids the strategic customers making speculations due to differences of price.

It is first necessary to investigate the inventory system facing the constant demand rate. If out of stock, only some enterprises will keep demand; others will lose the sale opportunities. The demand rate is a fixed constant ($\lambda$). In the condition of limited resources, enterprises offer a discount rate in per-unit time ($Z$) to retain customers and stimulate them to keep their demand. Reserved demand rate is $\gamma$, which is an undiminished function for discount, and to determine this value, this article takes a simple formula, $\gamma = \alpha + \beta Z$. Within this function, $\alpha$ is the initial retention rate, and $\beta$ is the marginal efficiency of discount. The initial retention rate and the marginal efficiency can be obtained through statistical analysis of the questionnaire survey.

This hypothesis differs from that which was put forward by Huang et al. (2007), as in this paper, the customer demand is gained from a single sales channel instead of a plurality of channels (such as in-store and online shopping). Analyzing a single channel means that all the prices are the same and customers can choose to either accept the discount price or ask for the product on-the-spot. Although there have been rapid developments within electronic commerce in recent years, a single sales channel is still very common. Compared with the classical EOQ model, discount incentives offer the benefit of providing more choices to consumers. However, if all the customers prefer spot transactions, this model will degenerate to the classical EOQ model.

At every T time, the inventory system will restock, supposing that the purchase occurred at 0 time. The purchase volume remains at Q before the delivery service is provided. In accordance with FCFS, the enterprise should fix the business it owes. When the inventory declines to a relatively lower point (at a corresponding time, t1), discounts are provided and delivery is paused to those customers who accept a delivery delay in order to keep inventory service to spot-oriented customers. When the inventory is reduced, spot-trading customers still receive spot service, while the customers who accept the delivery delay will receive the return-discount, calculated based on the waiting time until the arrival of supplementary goods. It is also possible that the next batch of goods will still not obtained before inventory depletion. Therefore, the replenishment cycle can be divided into three stages: the period of adequate inventory (t1), the period of rationing (t2), and the period of out-of-stock. This is represented by the formula T = t1 + t2 + t3, where $t_1 \geq 0, t_2 \geq 0, t_3 \geq 0$. This is represented in the Figure 1, below.

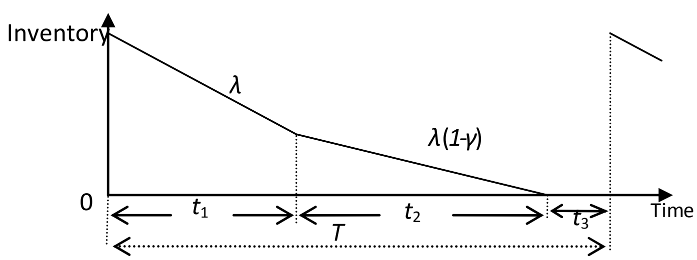


**Figure 1 : Changes in the inventory replenishment cycle.**

For the convenience of this study, the definition and description of relevant symbols are outlined in the following TABLE.

This paper adopts the linear hypothesis $\gamma = \alpha + \beta Z$, α as the initial partial backlogging rate and β as the marginal efficiency. Therefore, $1 > \alpha > 0, \beta > 0$ and $0 < \gamma \leq 1$, so $0 \leq Z \leq \frac{1-\alpha}{\beta}$.

Every order quantity is Q after the arrival of implements. The rest of the inventory is the initial inventory. In the period T, the beginning inventory is

$$S = \lambda t_1 + \lambda(1-\gamma)t_2 \quad (1)$$

Among them, $\gamma = \alpha + \beta Z$ and

$$B = \lambda\gamma(t_2 + t_3) \quad (2)$$

$$L = \lambda(1-\gamma)t_3 \quad (3)$$

Therefore, the profit during the T period is

$$\Pi_T = (S+B)(P-c) = (P-c)\lambda(t_1 + t_2 + \gamma t_3) \quad (4)$$

Because of the return discount provided by the delayed delivery is proportional to the time, the replenishment period discount cost is as follows:

$$C_B = \int_{t_1}^{T} \lambda\gamma Z(T-t)dt = \frac{1}{2}\lambda\gamma Z(t_2+t_3)^2 \quad (5)$$

$$C_L = Ld = d\lambda(1-\gamma)t_3 \quad (6)$$

$$C_h = \frac{1}{2}h\lambda t_1^2 + h\lambda(1-\gamma)(t_1t_2 + \frac{1}{2}t_2^2) \quad (7)$$

Based on K, therefore, the optimized model is

$$\max_{t1,t2,t3,Z} \pi = \frac{1}{T}(\Pi_T - C_B - C_L - C_h - K) \quad (8)$$

**TABLE 1 : Model symbols and their definitions**

| Model symbol | Symbol definition and description |
|---|---|
| T | The replenishment cycle. Every T time has a stock: T = t1 + t2 + t3. |
| t1 | The period of adequate inventory. Goods are to customers in accordance with FCFS. t1 is the control point of delivery time. When $t > t_1$, stop delivery to the first kind of customers. t1 is also the corresponding inventory control ring point for delivery (i.e. critical level). |
| t2 | The period of rationing. The inventory stays at a lower level; only deliver goods to those consumers who accept spot transactions. |
| t3 | Out of stock, as inventory has been depleted. At this point, consumers who require spot transactions could be lost. |
| S | The beginning inventory, after the arrival of implements. The rest of the inventory is the initial inventory. |
| d | All costs for the efforts did for maintain market due to the selling-lost. |
| $\gamma$ | The partial backlogging rate. Reservation demand rate while out of stock. The waiting demand accounts for the proportion of total demand and is affected by discounts. |
| Q | The purchase volume. The order quantity remains the same. Disposable instantaneous arrival. |
| τ | The customers' waiting time (until the supplementary goods arrive). |
| l | The cost of shortage (out of stock). |
| $\lambda$ | The demand rate. It is a known and continuous constant. |
| $Z$ | The unit time discount rate (based on yuan per day). |
| h | The holding unit goods' cost. |
| K | The single order cost. |
| P | The sales of a single cell. |
| c | The unit production cost. |
| B | The delivery delay. |
| L | The loss of sales. |
| $\pi$ | The profit per unit time |
| $\Pi_T$ | The sales profit (during the T time). |
| $C_B$ | The returned discounts from delayed delivery (during the T time). |
| $C_L$ | The sales lost (during the T time). |
| $C_h$ | The cost of goods held (during the T time). |

## OPTIMIZED SOLUTION

Assuming the optimized solution of the model, the partial derivative of equation (8) can be calculated, corresponding with t1, t2, and t3. By making these 0, we can get a new equation set:

$$\frac{\partial\pi}{\partial t_1}=\frac{1}{T}\{(P-c)\lambda-[h\lambda t_1+h\lambda(1-\gamma)t_2]-\pi\}=0 \tag{9}$$

$$\frac{\partial\pi}{\partial t_2}=\frac{1}{T}[(P-c)\lambda-\lambda\gamma Z(t_2+t_3)-h\lambda(1-\gamma)(t_1+t_2)-\pi]=0 \tag{10}$$

$$\frac{\partial\pi}{\partial t_3}=\frac{1}{T}[(P-c)\lambda\gamma-\lambda\gamma Z(t_2+t_3)-d\lambda(1-\gamma)-\pi]=0 \tag{11}$$

Comparing equations (9) through (11), we can then find the equations below:

$$\begin{aligned}&(P-c)-[ht_1+h(1-\gamma)t_2]\\&=(P-c)-\gamma Z(t_2+t_3)-h(1-\gamma)(t_1+t_2)\\&=(P-c)\gamma-\gamma Z(t_2+t_3)-d(1-\gamma)\end{aligned} \tag{12}$$

Equation (12) illustrates that during the period of rationing and out-of-stock, profit remains at the same level. In other words, they have the same "effective prices." This method expands the conclusion gained by investigating the supply suspension compensation strategy that was researched by Bhargava, Sun, and Xu (2006)[18].

According to equation (12), $ht_1=Z(t_2+t_3)$.

During $[0,\frac{1-\alpha}{\beta}]$, $\frac{\partial\pi}{\partial Z}<0$; therefore, $\pi$ is a decreasing function to Z and the discount strategies are luck of incentive, making $Z=Z_{\min}=0$ without any discounts. Therefore, t1=0, t3=0 and $t_2=\frac{2K}{h\lambda(1-\alpha)}$.

During $[0,\frac{1-\alpha}{\beta}]$, $\frac{\partial\pi}{\partial Z}>0$, the incentive of the discount strategy worked, as $\pi$ is an increasing function to Z. $Z=Z_{\max}=\frac{1-\alpha}{\beta}$, making $\gamma=1$, which, under the interval mentioned above, accordingly makes $\frac{\partial\pi}{\partial t_1}=\frac{1}{T}[(P-c)\lambda-\pi]>0$. Also note that $(P-c)\lambda$ represents the sales volume during the period of adequate inventory and $\pi$ represents the average profit at every turn after deducting various other costs. $\frac{\partial\pi}{\partial t_3}=\frac{1}{T}[(P-c)\lambda-\lambda Zt_3-\pi]>0$, in the same way, exists as the optimized solution t1 and t3 (which is not 0). According to equations (9) and (11), we can draw the following conclusion: $ht_1=Zt_3$. Simplifying function (8), $\max_{t1}\pi=(P-c)\lambda-\frac{1}{2}h\lambda t_1-\frac{ZK}{(Z+h)t_1}$. By making $\frac{d\pi}{dt_1}=-\frac{1}{2}h\lambda+\frac{ZK}{(Z+h)t_1^2}$ a value of 0, the solution then becomes $t_1^*=\sqrt{\frac{2ZK}{\lambda h(Z+h)}}$ and $t_3^*=\sqrt{\frac{2hK}{\lambda Z(Z+h)}}$. During $[0,\frac{1-\alpha}{\beta}]$, $\frac{\partial\pi}{\partial Z}=0$ because of $\frac{\partial^2\pi}{\partial Z^2}=-2\beta<0$. Therefore, Z has the only optimized solution. In this case, $Z=\frac{(d+P-c)t_3+h(t_1t_2+\frac{1}{2}t_2^2)}{(t_2+t_3)^2}-\frac{\alpha}{2\beta}$. Simulating equations (9) through (11) in a quaternary non-linear equation set, t2, t3, and Z can all be expressed by t1's function. Therefore, this model presents a viable solution.

## EXAMPLES AND MANAGEMENT IMPLICATIONS

If the cost rate of holding-stocks for a product is h = 1, the reservation demand parameters are $\alpha=0.2,\beta=0.1$, the price P = 1000, the cost c = 920, and the lost-sales cost rate $d=30$, then the ordering cost K is for 5000 yuan each time and the demand rate $\lambda=10$. According to the traditional EOQ model, we can find the order quantity Q=316. In this case, the profit is 484. Using the EOQ-PBO model, the solutions are t1* = 27, t2* = 6, t3 = 0, and Z* = 4.3. The target profit is 503 yuan per day, and the order quantity is 330. Compared with the traditional order model, the discount strategy saves cost in two aspects: it encourages more customers to accept the delivery delay, obtaining a greater order quantity and saving the ordering cost; and it reduces the inventory by offering discounts, saving inventory costs.

## CONCLUSIONS

Customers' purchasing patterns usually are usually highly distinguishable, as certain customers are sensitive to the time of arrival but not to the price, while other customers are just the opposite. To avoid the loss of any kind of customer, in this paper, we studied an EOQ-PBO model that can be implemented both in the inventory system and in the incentive discount system. Some customers are time-sensitive and price-insensitive, and in view of this type of consumer, spot trading and no discount strategy should be used. For customers who are price-sensitive and time-insensitive, a pre-sales approach should be implemented that uses the discount strategy. Mathematical analysis and simulation have shown that, compared to the traditional EOQ model, the EOQ-PBO model not only saves the ordering cost and inventory cost, it also gets rid of dependence on logistics sales forecasts, makes use of the resources more efficiently, achieves the maximum profits, and can improve customer satisfaction.

## ACKNOWLEDGEMENT

This research is supported by the National Social Science Fund planning project 2014 (Youth), the Education Department of Jiangxi province science and technology project (GJJ12266, GJJ14345), and the social science of Jiangxi province "1025" planning project (2013) (13YJ51)

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