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Study Of B (E2) Values Of Even-Even $\frac{72-78}{32}\mathrm{Ge}$ Isotopes Using Interacting Boson Model-1

Hasan M, Tabassum M, Roy PK, Tasnim MM, Sakib KN and Islam J*

Department of Physics, Mawlana Bhashani Science and Technology University, Satosh,Tangail-1902, Bangladesh.

***Corresponding author:** Islam J, Department of Physics, Mawlana Bhashani Science and Technology University, Satosh,Tangail-1902, Bangladesh, Tel: 0542 236 8558; E-mail: ijobaidul@mbstu.ac.bd

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Abstract

The Electric Reduced Transition Probabilities $B(E2)\downarrow$ have been estimated for the even neutron numbers of $^{72-38}_{32}Ge$ isotopes using Interacting Boson Model-1 (IBM-1). The U(5) symmetry, $B(E2)$ values, intrinsic quadrupole moments and deformation parameters of even neutron $N=40-46$ of $\frac{72-78}{32}Ge$ isotopes have been studied. The $R_{4/2}$ values of $\frac{72-38}{32}Ge$ isotopes have been calculated for the first 4^+ and 2^+ energy states and thus U(5) limit is identified.

***Keywords:** Geometrical; Rotational; Microscopic; Neutron*

Introduction

Iachello and Arima developed the interacting boson model (IBM-1) [1, 2]. For many-body systems, thevibrational and rotational frequencies characterize the nuclear collective motion of comparable order of magnitude, to prevent a clear-cut distinction between the two types of motion. Moreover, other intermediate situations can occur in nuclei, for example asymmetricrotations and spectra which are neither vibrational nor rotational. A unified description has been proposed for the collective nuclear motion in order to accomodate these facts in terms of a system of interacting bosons [3].

To provide a phenomenological description of spectroscopic data over a wide range of nuclei demonstrating collective features including those customarily interpreted in terms of anharmonic vibrators or deformed rotors the interacting-boson model (IBM) was found. However, the connection between the parameters of the IBM and the geometrical description has

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not been clarified, and this has reduced from the method of the IBM. In this research work has been proposed an intrinsic state for the IBM which may provide the desired connection. States in the IBM are constructed from s bosons, with intrinsic spin 0 and d bosons, with intrinsic spin 2, the corresponding boson creation operators being denoted $s^\dagger$ and $d_\mu^\dagger$ respectively, where μ is the projection quantum number of the d boson [4].

The IBM-1 model has been employed theoretically to study the intermediate configuration and configuration mixing around the shell closure Z=28. In IBM-1 and IBM-associated models such as the configuration mixing model in strong connection with the shell model, the conventional collective Hamiltonian approach and the microscopic energy-density function calculated the empirical spectroscopic study within the configuration mixing around the shell. The evolution properties of even-even $^{104\text{-}112}$Cd [5], $^{102\text{-}106}$Pd [6], $^{108\text{-}112}$Pd [7], $^{100\text{-}102}$Ru [8] and $^{120\text{-}126}$Te [9] have been studied very recently.

The models IBM1 and IBM2 are restricted to nuclei with even numbers of protons and neutrons. In order to fix the number of bosons both types of nucleons constitute closed shells with particle numbers 2, 8, 20, 28, 50, 82 and 126 (magic numbers) can be taken. Provided that the protons fill less than half of the furthest shell the number of the corresponding active protons has to be divided by two for the boson number N_π attributed to protons. If more than half of the shell is occupied the boson number reads N_π= (number of holes for protons)/2. By treating the neutrons in an analogous way, one obtains their number of bosons N_ν. In the IBM1 the boson number N is calculated by adding the partial numbers i.e. $N = N_\pi + N_V$. For example the nucleus shows the numbers N_π = (54-50)/2=2, N_V = (64-50)/2=7 and for $\frac{128}{54}Xe_{74}$ the values $N\pi$ = (54-50)/2=2, N_V = (82-74)/2=4 hold. Electromagnetic transitions don't alter the boson number but transfers of two identical nucleons lift or lower it by one [10].

IBM-1 Method

It is assumed that in the IBM the low-lying collective states are filled with the valance protons and valance neutrons only (i.e. particles outside the major closed shells at 2, 8, 20, 28, 50, 82 and 126), while the closed shell core is inert. Furthermore, it is assumed that the particle configurations are coupled together, forming pairs of angular momentum 0 and 2. These proton (neutron) pairs are treated as bosons. Proton (neutron) bosons with angular momentum $L=0$ are denoted by $s_\pi(\mathrm{s}_v)$ and are called s-bosons, while proton (neutron) bosons with angular momentum $l=2$ are denoted by $d_\pi(\mathrm{d}_v)$ and are called d-bosons. The six dimensional unitary groups U(6) of the model gives a simple Hamiltonian, which describes three specific types of collective structure with classical geometrical analogs, these are vibrational U(5), rotational SU(3) and γ-unstable O(6). In IBM-1 the Hamiltonian of the interacting boson is given by [11, 12]:

$$H = \sum_{(i=1)}^{N} \varepsilon_i + \sum_{(i<j)}^{N} V_{ij} . \qquad (1)$$

Where ε is the intrinsic boson energy and V_{ij} is the interaction between i and j.

Hamiltonian H can be written explicitly in terms of boson creation $(s^\dagger, d^\dagger)$ and annihilation $(\hat{s}, \hat{d})$ operators such that

$$H=\varepsilon_s(s^\dagger.\hat{s})+\varepsilon_d(d^\dagger.\hat{d})+\sum_{l=0,2,4}\frac{1}{2}(2L+1)^{\frac{1}{2}}C_L[(d^\dagger\times d^\dagger)^{(L)}\times(\hat{d}\times\hat{d})^{(L)}]^{(0)}$$
$$+\frac{1}{\sqrt{2}}v_2[(d^\dagger\times d^\dagger)^{(2)}\times(\hat{d}\times\hat{s})^{(2)}+(d^\dagger\times s^\dagger)^2\times(\hat{d}\times\hat{d})^{(2)}]^{(0)} \quad (2)$$
$$+\frac{1}{2}u_o[(s^\dagger\times s^\dagger)^{(0)}\times(\hat{s}\times\hat{s})^{(0)}]^{(0)}+u_2[(d^\dagger\times s^\dagger)^{(2)}\times(\hat{d}\times\hat{s})^{(2)}]^{(0)}$$

Where it can be written in the general form as

$$H=\varepsilon\hat{n}_d+a_\circ\hat{p}.\hat{p}+a_1\hat{L}.\hat{L}+a_2\hat{Q}.\hat{Q}+a_3\hat{T}_3\hat{T}_3+a_4\hat{T}_4\hat{T}_4 \quad (3)$$

Where $\hat{n}_d=(\mathrm{d}^\dagger.\hat{\mathrm{d}})^{(1)}$, the total number of d_{boson} operator; $\hat{\mathrm{p}}=\frac{1}{2}[(\hat{\mathrm{d}}.\hat{\mathrm{d}})]\_(\hat{\mathrm{s}}.\hat{\mathrm{s}})]$, the pairing operator; $\hat{L}=\sqrt{10}(\mathrm{d}^\dagger\times\hat{\mathrm{d}})^{(1)}$, the angular momentum operator; $\hat{T}_r=(\mathrm{d}^\dagger\times\hat{\mathrm{d}})^r$, the octupole and hexadecapole operator; $\varepsilon=\varepsilon_d-\varepsilon_s$, the boson energy; and $\hat{Q}=(\mathrm{d}^\dagger\times\hat{\mathrm{s}}+\mathrm{s}^\dagger\times\hat{\mathrm{d}})^{(2)}+x(\mathrm{d}^\dagger-\hat{\mathrm{d}})^{(2)}$ (χ=CHQ), the quadrupole operator. The parameters $a_0,a_1,a_2,a_3\ and\ a_4$ are the strength of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction respectively between the bosons. The reduced transition probabilities in IBM-1 for even-even nuclei of low-lying levels (L_i= 2,4,6,8,……..) are given for anharmonic vibration limit U (5) by [9]

$$B(E2;L+2\rightarrow L)\downarrow=\frac{1}{4}\alpha_2^2(L+2)(2N-L)=\frac{1}{4}\frac{(L+2)(2N-L)}{N}B(E2;2\rightarrow 0) \quad (4)$$

Where L is the angular momentum and N is the boson number. From the given experimental value of $B=(\mathrm{E}\,2)\uparrow$ of transition $(0^+\rightarrow 2^+)$ the $B=(\mathrm{E}\,2)\uparrow$ values have been calculated for the transition $(2^+\rightarrow 0^+)$ with the values of $g=(2L_f+1)(2L_i+1)^{(-1)}$. Thus the value of parameters α_2^2 (square of effective charge) has been calculated.

Quadrupole Moments

The intrinsic quadrupole moments (Q_0) of the nuclei can be written as [9]

$$Q_0=\left[\frac{16\pi}{5}\frac{B(\mathrm{E}\,2)\uparrow}{e^2}\right]^{\frac{1}{2}} \quad (5)$$

Here $B(E2)\uparrow$ is the upward electromagnetic quadrupole transition probability of the nuclei and e is the electric charge. The upward electromagnetic quadrupole transition probability $(\hat{s},\hat{d})$ is related by

$$B\left(E2;L_i\rightarrow L_f\right)\downarrow=B\left(E2;L_f\rightarrow L_i\right)\uparrow\times g$$

and the deformation parameter β can be written as

$$\beta=[B(E2)\uparrow]^{(1/2)}\left[\frac{3ZeR_0^2}{4\Pi}\right]^{(-1)} \quad (6)$$

Here Z is the atomic number, e is the electric charge and $R_0^2 = 0.0144A^{2/3}b$ is the square of the average radius of the nuclei.

$R_{4/2}$ Classification

The $R_{4/2}$ is defined by the ratio of the first 4^+ energy state and the first 2^+ energy state. We can also express it by

$$R_{(4/2)} = E(4_1^+)/E(2_1^+) \tag{7}$$

There are three types of dynamic symmetries. [TABLEs 1 and 2]. They are classified into $U(5)$, $SU(3)$ and $0(6)$ groups. The $\mathrm{U(5)}$, $\mathrm{SU(3)}$ and $0(6)$ symmetries are called harmonic vibrational, rotational and Y^- unstable respectively. We can identify the symmetries by using $R_{4/2}$. If $R_{4/2} \leq 2$ then the symmetries will be vibrator symmetries, if the $2 \leq R_{4/2} \leq 3$ then the symmetries will be γ-unstable symmetries and if $R_{4/2} > 3$ then the symmetries will be rotational symmetries [9].

TABLE 1. **The $R_{4/2}$ values for $\frac{72-78}{32}Ge$ isotopes.**

Neutron number	**Energy(KeV) [13, 14]**		$R_{(4/2)} = \frac{E(4_1^+)}{E(2_1^+)}$
	2_1^+ **state**	4_1^+ **state**	
40	1464.094	2464.043	1.68
42	1204.21	2165.26	1.80
44	2503.6	2733.4	1.09
46	1305.3	1498.7	1.15

Results and Discussion

TABLE 2**. The deformation parameter and the quadrupole moment from the IBM-1 of 2^+ isotopes.**

Nuclide	$B(E2)\uparrow(w.u.)$ [15]	Transition level $L_i \rightarrow L_f$	$B(E2)\downarrow(w.u.)$	$R_o^2(b)$	$\beta\times10^{19}$	Q$_o$(b)
Ge_{32}^{72}	23.46	$2^+\rightarrow0^+$	4.69	0.25	1.58	9.60×10^{19}
		$4^+\rightarrow2^+$	8.04			
		$6^+\rightarrow4^+$	10.05			
		$8^+\rightarrow6^+$	10.72			
		$10^+\rightarrow8^+$	10.05			
Ge_{32}^{74}	33.1	$2^+\rightarrow0^+$	6.62	0.254	1.85	1.14×10^{20}
		$4^+\rightarrow2^+$	11			
		$6^+\rightarrow4^+$	13.2			
		$8^+\rightarrow6^+$	13.2			
		$10^+\rightarrow8^+$	11			
		$12^+\rightarrow10^+$	6.6			
Ge_{32}^{76}	28.61	$2^+\rightarrow0^+$	5.72	0.258	1.70	1.06×10^{20}
		$4^+\rightarrow2^+$	9.12			
		$6^+\rightarrow4^+$	10.26			
		$8^+\rightarrow6^+$	9.12			
		$10^+\rightarrow8^+$	5.7			
Ge_{32}^{78}	22.4	$2^+\rightarrow0^+$	4.48	0.263	1.47	9.38×10^{19}
		$4^+\rightarrow2^+$	6.72			
		$6^+\rightarrow4^+$	6.72			
		$8^+\rightarrow6^+$	4.48			

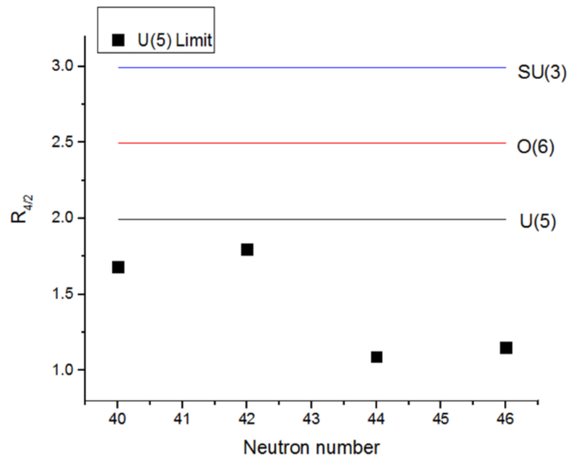

FIG. 1. **B (E2) ↓ values in W.u against the transition levels for $^{72-78}_{32}Ge$ isotope.**

FIG. 1 and 2 shows the values of B (E2) ↓ in Weisskopf unit (W.u.) as a function of transition yrast level for $^{72-78}_{32}Ge$ isotopes in IBM-1. Using equation (1) the B (E2) ↓ values in W.u. have been calculated and used for $^{72-78}_{32}Ge$ isotopes for the transition levels 2^+-0^+, 4^+-2^+, 6^+-4^+ and 8^+-6^+.

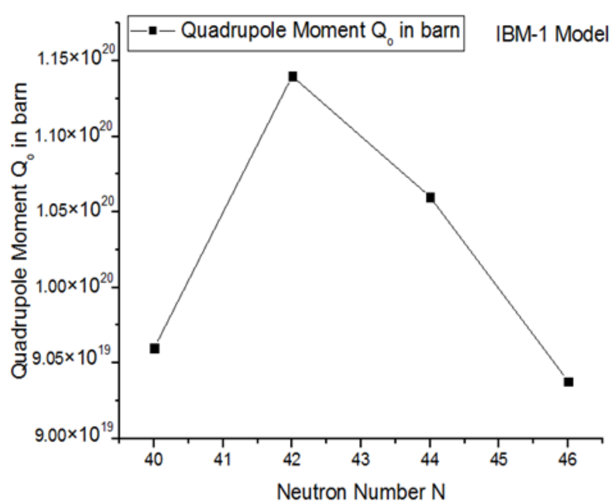

FIG. 2. **The intrinsic quadrupole moment Q_o in barnvs even neutron number of $^{72-78}_{32}Ge$ isotopes.**

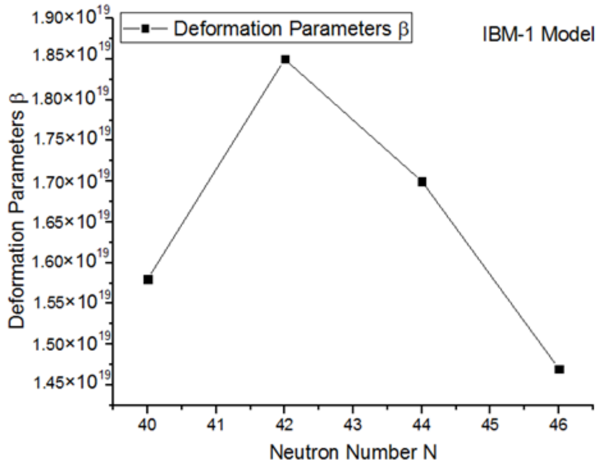

FIG. 3. **The deformation parameter against neutron number.**

FIG. 3 shows that the intrinsic quadrupole moment Q_o in barn as a function of even neutron number of $\frac{72-78}{32}Ge$ isotopes in IBM-1. The intrinsic quadrupole moment Q_o in barn have been plotted for the $\frac{72-78}{32}Ge$ isotopes.

Figure 3 shows the deformation parameter β as a function of even neutron number of $\frac{72-78}{32}Ge$ isotopes in IBM-1. The deformation parameter β have been plotted for the $\frac{72-78}{32}Ge$ isotopes.

Conclusion

The Reduced Transition Probabilities $B(E2)\downarrow$ for the $\frac{72-78}{32}Ge$ isotopes from $8^+ \rightarrow 6^+$, $6^+ \rightarrow 4^+$, $4^+ \rightarrow 2^+$ and $2^+ \rightarrow 0^+$ energy states have been estimated by using IBM-1. Using the IBM-1 quadrupole moments and deformation parameters are also calculated. The $R_{4/2}$ values have been calculated for the $\frac{72-78}{32}Ge$ isotopes for the first 4^+ and 2^+ energy states and from the values of $R_{4/2}$ it is found that these nuclei satisfied the U(5) limit.

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