2014

ISSN : 0974 - 7435 Volume 10 Issue 23

BioTechnology

An Indian Journal

FULL PAPER

BTAIJ, 10(23), 2014 [14382-14386]

Structural reliability analysis based on optimization kriging model

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ABSTRACT

In structural reliability In reliability analysis of structure, the limit state function is often implicit and nonlinear influenced by many factors, usually using the response surface simulation. And the structure of the small failure probability is difficult to get a large number of reliability data to verify the correctness of the response model. The form of polynomial limits the accuracy of reliability calculation.Therefore the optimization kriging model method is put forward to improve the precision, optimize deviation and reduce the sample. A function of variables and the response is obtained by the approximate model, which replace the limit state function in reliability analysis. But the kriging model requires a lot of sample verification, global optimization(GO) is used to determine the sample points near the limit state and relatively high uncertainty as the new sample. And so a less sample is used to build kriging surrogate model meeting precision, and then the structural reliability is computed gradually with H-L(Hasofer-Lind) method. The numerical simulation results show this method is the higher accuracy. Comparing with the classical kriging, MCS and FOSM, the method is less computation error and less sample. It can meet the engineering demand and greatly reduce the amount of calculation..

KEYWORDS

Kriging model; Global optimization; Reliability analysis.

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INTRODUCTION

Structural reliability calculation are mostly under the condition of the known limit state function structure. But the practical engineering structure is complex, and moreover the affecting factors are stochastic and uncertainty. So the simulation of the structural limit state function become an important problem in the calculation of the reliability. The common method is Monte Carlo Simulation(MCS)and Response Surface Method(RSM). MCS is often combined with finite element method[1]. It avoid the mathematical difficulty in structural reliability analysis. The response of the structure is just needed , without considering the complexity of the structural limit state surface. But the calculation is large, so it is often used in testing of structural reliability precision and checking of the calculation results. RSM is using a simple explicit function to approximate the actual implicit limit state function[2,3]. But as a parametric design, it often have a bigger different reliability with the different expressions.

Kriging method is a method of fitting, prediction and approximation[4]. It is an estimate technology called by South Africa's mining engineer D.G.K krige name. This method does not rely on complete information samples, just needing a little information near a smaller region. So it can do a linear unbiased and minimum variance estimation to the same information features in the region, has a strong ability of forecasting information. In 2004, Romero, etc apply the method to the structural reliability analysis. since then, some scholars began to study Kriging method applied to the field of reliability and optimization[5]. But the kriging method needs a lot of samples to verify the approximate model. Kriging method based on the global optimization(GO) is proposed. The points near the limit state and relatively high uncertainty can be added to the sample. So Kriging prediction accuracy is raised with the less sample. Usually in combination with First-Order Second Moment method(FOSM) to calculate the reliability of the structure, the high-order item is omitted leading to a large calculation error. This paper will combine the optimization kriging model with the H-L(Hasofer-Lind) method to computer the structure reliability, the efficiency and feasibility of the method is verified by numerical simulation with less samples, less amount of calculation.

BASIC PRINCIPLE OF KRIGING

Kriging is an improved linear regression technology, which includes linear regression and non-parametric part[6]. The non-parametric part is seen as the random distribution, so the composition of Kriging Model is as below[7]

$$G(x)=F(\beta,x)+z(x)=f^T(x)\beta+z(x) \qquad (1)$$

Where, β is the regression coefficient; $f(x)$ is a polynomial function for the variable x, providing the global approximate simulation.

$z(x)$ is the local approximation for the simulation, it is a random process obeyed to the normal distribution $N(0,\sigma^2)$ with its mean for 0 and its co-variance for nonzero. It can be expressed as

$$E[z(w)z(x)]=\sigma^2R(\theta,w,x) = \sigma^2\prod_{j=1}^{n}R_j(\theta,w_j-x_j) \qquad (2)$$

Where, σ is the standard deviation for the response; $R(\theta,w,x)$ is the correlation function of w, x and θ, it match the kernel function playing a decisive role on the simulation precise. w and x is the two random sample points. Gaussian function is considered to be is the best for the calculation effect, it is suitable for the simulation of the high nonlinear function. So the Gaussian function is selected as the kernel function in this paper.

After Kriging model is set up, other sample points can be selected to verify the precision of the model. Kriging method provides the response value and variance for the each point. To identify a specific response value, the linear combination of the response value $(G=[g_1,\cdots,g_m])$ can be used to build, namely

$$\hat{g}(x)=c^TG \qquad (3)$$

The deviation between the prediction value and true value is expressed by below

$$=c^TZ-z+(F^Tc-f(x))\beta \quad \hat{G}(x)-g(x)=c^TG-g(x) \qquad (4)$$

In order to ensure the no-deviation for the prediction process, the mean of the deviation must be zero. Namely $F^Tc-f(x)=0$,the formula can be obtained by the maximum likelihood estimation (MLE)below

$$\beta^*=\left(F^TR^{-1}F\right)^{-1}F^TR^{-1}G \qquad (5)$$

The prediction value can be got through the Formula (4) and Formula (5)

$$\hat{g}(x)=rR^{-1}G-(F^TR^{-1}r-f)^T\beta^* \ = f^T\beta^*+r^TR^{-1}\left(G-F\beta^*\right) \qquad (6)$$

$\sigma^2=(G-F^T\beta^*)^TR^{-1}(G-F^T\beta^*)$ The relevant parameters θ can be determined by the maximum likelihood estimation. It is equal to solve the nonlinear unconstrained optimization problem.

$$\max_{\theta>0}\left[-\frac{m\ln(\sigma^2)+\ln|R|}{2}\right] \qquad (7)$$

OPTIMIZATION KRIGING MODEL METHOD

Kriging model can predict the response value and its variance. The greater variance indicates the greater uncertainty of the prediction. So when the number of the sample is the bigger, the kriging approximation model of implicit function function is the more accurate. But the amount of calculation will increase accordingly. The optimal sample is determined with EFF criterion, and then the points of the larger uncertainty and near limit state are chose as the initial sample. The optimization can ensure the kriging global accurate approximation of the implicit function function, and also makes the approximation of the failure range more accurate. The more accurate kriging model of implicit $z=G(x)$ function can be ε established with $\sigma_G(x)$ the sample as less as possible.

The function of EFF can be expressed by below

$$EF[\hat{G}(x)]=\int_{\bar{z}-\varepsilon}^{\bar{z}+\varepsilon}[\varepsilon-|\bar{z}-\hat{G}(x)|]f_{\hat{G}}d\hat{G} \qquad (8)$$

Where, is the constrain function, is proportional to Suppose ,the integral the formula(8) is computed as below

$$\begin{aligned}EF[\hat{G}(x)]=&(\mu_G-\bar{z})[2\Phi(\frac{\bar{z}-\mu_G(x)}{\sigma_G(x)})-\Phi(\frac{z^+-\mu_G(x)}{\sigma_G(x)})-\Phi(\frac{z^--\mu_G(x)}{\sigma_G(x)})]\\&-\sigma_G(x)[2\varphi(\frac{\bar{z}-\mu_G(x)}{\sigma_G(x)})-\varphi(\frac{z^+-\mu_G(x)}{\sigma_G(x)})-\varphi(\frac{z^--\mu_G(x)}{\sigma_G(x)})]+\varepsilon[\Phi(\frac{z^+-\mu_G(x)}{\sigma_G(x)})-\Phi(\frac{z^--\mu_G(x)}{\sigma_G(x)})]\end{aligned} \qquad (9)$$

Kriging model based on the EFF rule is shown in Figure 1.

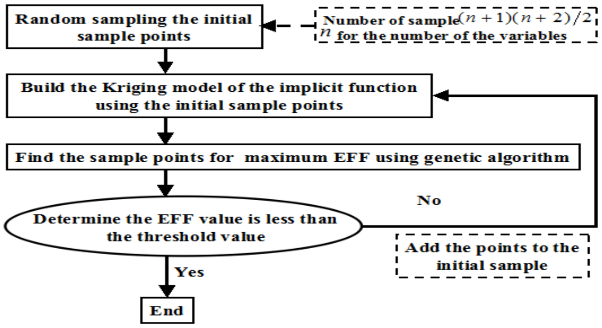

FIGURE 1 Kriging model computation process based on the EFF rule

COMPUTION OF RELIABILITY INDEX

The calculation of the structural reliability is essential to seek the most short distance between the coordinate origin and the limit state curve in the standard normal space. Namely, the optimization problem is as blow

$$\min\|G\| \quad s.t\ G(X)=0 \qquad (10)$$

Where, X is the standardization random variable.

The computation process is showed in Figure 2.

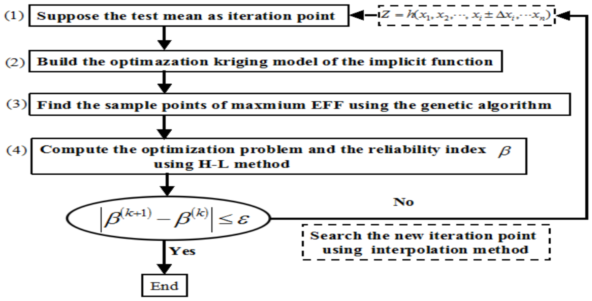

FIGURE 2 Computation procedure of the reliability index

SIMULATION EXAMPLE

Rectangular cantilever beam is subjected to the uniform distributed load. The maximum vertical displacement of the beam free end is no more than $L/8$ times of allow deformation. The structure function can be expressed as $g(x) = L/8 - f(E, I, L)$, including elastic modulus $E = 4 \times 10^7 Mpa$, beam length $L = 1m$, the inertia moment $I = 8m^4$ and the variation coefficient for 0.2.

Sample points are selected in the design space. If the range of sampling space is too large, the simulation accuracy of the whole structural response will be reduced; Instead, the points of the greatest influence on reliability index is excluded from the sample space, and then affecting the calculation accuracy. To obtain the mean and variance of the load q, moment of inertia I ,elastic modulus E, the random number is formed by MATLAB[8]. Respectively three variables is randomly sampled 1000 times, as shown in Figure 3.

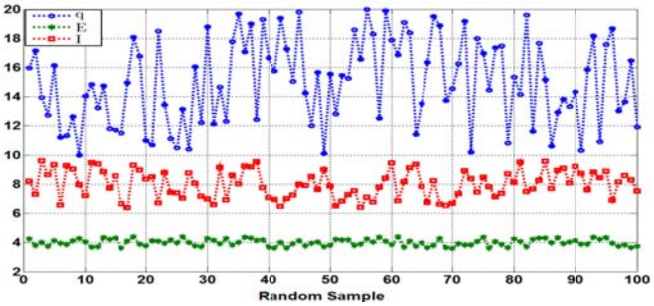

FIGURE 3 Roman Sampling of the q, E, I

TABLE 1 Mean and Variance of q, E, I

Random variable	Variable range	Mean μ	Standard variance σ	Distribution
q	10~20	15	2.88	Normal
E	3.6~4.4	4	0.23	Normal
I	6.4~9.6	8	0.92	Normal

The mean and variance of the three variables are obtained as shown in Table 1.

10 sets of data are selected to predict the value of the response, the number of each group of data is 5, 10, 30, 60, 90, 150, 180, 300, 500, 800 respectively. The reliability of the structure is obtained with L-S method as shown in Figure 4. When the sample is 180, the accurate and stable reliability index is gained, namely β=3.2870.The computation results shown in Table.2 N is the number of samples; ε_β is the percentage Error for reliability index β. The results of the classical Kriging method and the optimization kriging method is almost the same to MCS, which indicate the method in this paper is applicability and efficiency. From Table 2, the optimization kriging method need less sample points to get higher precision. The relative error keeping about 0.09% is smaller than the krigng method and FOSM. The computation precision is the higher, meeting the engineering requirement.

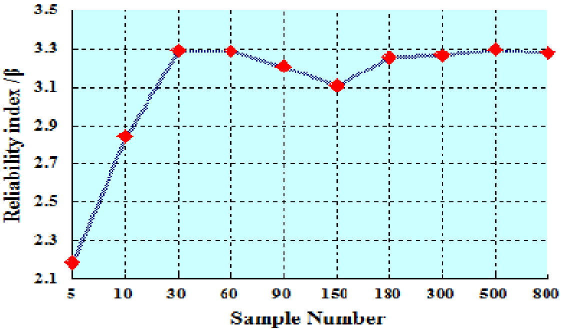

FIGURE 4 Reliability index of different sample number

TABLE 2 Results comparative of different methods

method	N	β	ε_β(%)
GO-Kriging	30	3.290	0.0913
	60	3.287	0
	90	3.284	0.0912
Kriging	90	2.975	9.4919
	180	3.109	5.4153
	300	3.284	0.0912
FOSM	__	3.279	0.2434
MCS	10^6	3.284	0.0913

CONCLUSIONS

In this paper, the kriging model based on optimization method is put forward, and the structural reliability is solved combining with H-L method. The surrogate model of the variables and response is set up using the kriging model based on EFF rule.

Kriging approximate model is more accurately established using the points near limit state and larger uncertainty as the initial sample.

This method need less sample points than classical kriging method. The error is obviously smaller than a second order moment method commonly. Through engineering application, it shows that the method is high efficiency and high precision, offering an effective computation method for engineering structural reliability.

ACKNOWLEDGEMENT

Authors thank for the sponsor of the National Science and Technology Support Project.(2011BAK06B05)

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