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## Strategic research on the probability statistics of free kick direct score based on biomechanical principles

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### ABSTRACT

To study about the key of free kick direct score, this paper will find out the movement trail of the swirling free kick based on the models of physics and probability statistics. Through consider we get the conclusion that there will be a lot of factors that affect the goalkeeper and the shooter: ball speed, rotation, the man wall, the way the ball get through the man wall, the goalkeeper's psychological state, the way the goalkeeper saves, difficulty of shooting, goalkeeper's physical condition and experience, shooter's power, the deviation of the real trail and his accuracy, and the theoretical trail. After analyzing the scope that the goalkeeper can reach in some time, we can get a constraint equation of score. And by considering the instability of goalkeeper, the probability he can save a ball and the difficulty of shooting with other factors, we will finally get the best strategy.

### KEYWORDS

Free kick; Probability statistics; Biomechanical principles; Mathematics model.



## INTRODUCTION

In June and July of 2005, the Holland FIFA U-20 World Cup and the Confederations Cup were in their full swing. Rulin's free kick direct score in the match between China and Panama must still remain new in the memory of Chinese football fans. This kick together with a score from a Dutch player, was rated as the best goal. So what makes it a superb strike? In addition to the player's exceptional footwork and luck, it was also related to physic and mathematics theories.

Football is not only a good fitness activity, but to some degree, the carrier of a nation's glory. But China national football team was too disappointing. It can't enter the finals of the 2006 Germany World Cup, which made the Chinese football fans heartbroken. Not long ago, Real Madrid's arrogant attitude in Beijing Workers' Gymnasium gave us some moments of thinking. When will Chinese football get a standing room among the world? How to improve our football? Obviously, we can't solve the problem overnight. In 2005, Wu Junwei, Zhang Sheping, Ouyang Lin and Han Wei, after analyze the trail theory of free kick, have calculated the parameters of the trail and have quantified the shooting power of a free kick, the striking spot on the ball (eccentric distance), shooting angle, and the final placement the ball is at through the man wall. Their research has given out theoretical evidence about free kick can directly bypass a man wall. In 2011, Chen Xiaoke and Hou Zhitao have studied the attack of the 30-meter free kick and corner kick of the 16-power phase matches in the 19<sup>th</sup> South Africa World Cup. And after analyzing the free kick penalty spot area, placement area, attacking ways, corner kick, and organization form, they provide evidence to the future training and matches. And in 2012, Li Menglun, Shan Zhao, and Liu Zhibin, mainly with the help of information technology, have made comparison of the techniques of the two direct free kicks C Ronaldo and Messi had used, which has provided techniques support in training.

### Constraint conditions to shooting

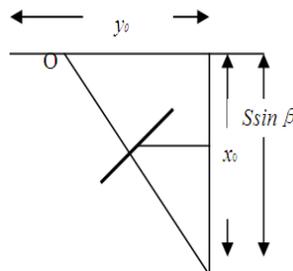
To make a free kick a goal, the main resistance is from the man wall and the goalkeeper. Now we make analysis for it.

First is about the man wall:

According to acceleration equation, we can easily get that the movement formula of the top of the man wall in the vertical direction is as following:

$$z = h + v_2 t - \frac{1}{2} g t^2$$

Assuming that the center of the wall is in the same line with the goalkeeper and the shooter and according to the rules, the man wall must be at least 9.15m away from the shooter. The closer the distance is, the bigger the blockade angle is. Therefore, in real matches, the distance will be as approximate as possible to or less than 9.15m. In this paper, we just regard it as a constant value. So we can get Figure 1 as following :( $\beta$  is the angle between the line of the shooter and the center of the goal and the line of the goal).



**Figure 1 : Diagram of the Free Kick Shooting**

$$x = x_0 - s \sin \beta$$

According to similar triangles principle, and with Figure 1 we can get the coordinates of the center of the man wall in the y-axis is:  $y_0 - \frac{y_0}{x_0} s \sin \beta$ .

The location of the man wall on the horizontal plane is the line linking two points, and the two points are:

$$(x_0 - s \sin \beta - \frac{n}{2} e \cos \beta, y_0 - \frac{y_0}{x_0} s + \frac{n}{2} e \sin \beta)$$

$$(x_0 - s \sin \beta + \frac{n}{2} e \cos \beta, y_0 - \frac{y_0}{x_0} s - \frac{n}{2} e \sin \beta)$$

The linear equation of the plane is:  $(y - y_0 + \frac{y_0}{x_0} s \sin \beta) = -tg\beta(x - x_0 + s \sin \beta)$

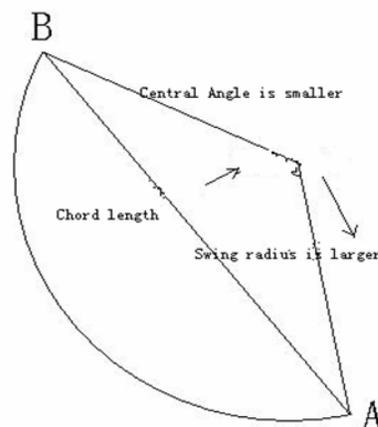
Finally, from the above we the constraint equation that the ball can enter the goal through the man wall is:  $f(t_0) = 0, f(t_1) = x'$

$x'$  is the range that the abscissa of the crossing point of the formula of the plane circle and the linear equation of the man wall. It has to meet the requirement of:  $|g(t_0)| \leq \frac{b}{2} - r, r \leq h(t_0) \leq a - r$

If the ball pass by the man wall from the top, then:  $h(t_1) \geq r + h + v_2 t_1 - \frac{1}{2} g t_1^2$ , and  $y$  can be any direction.

If from the sides, then:  $g(t_1) \leq y_0 - \frac{y_0}{x_0} s - (\frac{n}{2} e + r) \sin \beta$ , or  $g(t_1) \geq y_0 - \frac{y_0}{x_0} s + (\frac{n}{2} e + r) \sin \beta$ , and  $z$  can be any direction.

After conforming to the equations above, the probability of a goal exists. However, the equations haven consider about the goalkeeper. And as this factor is quite complicated, we will analyze it afterwards.



### Figure 2 : Horizontal Diagram

Now, we study about the different solutions the shooter will apply to the man wall.

According to the movement trail of the football in the horizontal plane, we know that when the ball is rising, there will be no differences whether it swirls or not. So, before it reaches the man wall, we can regard the trail as a parabola to simplify the model.

As the convoluted angular speed  $\omega''$  is not connected to  $\theta$ , in other words, if  $\omega_{xoy}$  is confirmed, the time for shooting is  $t_0 = \frac{\gamma}{\omega'}$ .  $\gamma$  is the central angle. Obviously,  $\gamma$  is the key point to shorten the shooting time.

Looking at Figure 2, if the shooter shoots at point A and wants the ball to reach (0, y) (point B), so chord length AB is set. And to make the central angle small, the radius of gyration should be as short as possible. From the equation  $R_{xoy} = \frac{mv_0 \cos \theta}{\pi \rho \omega_{xoy} r^3}$ , we can find that the radius of gyration is in proportion to the cos value of the angle between the initial velocity and the horizontal plane. Based on the character of cos, we should make  $\theta$  the least.

If the shooter let the ball pass from the top:

When  $\theta$  is the least, the value of  $\theta$  is exactly it is that from the top. At the same time, when the ball passes from the top, the horizontal placement of the ball and the shooting time is confirmed, but not that the higher the goal point is, the shorter the time is, as we have imagined. (actually, the point depends on the angular speed.)

To calculate the value of  $\theta$ , we can use the analysis above and make "the ball pass just from the top" an equation. Take the placement of the ball into it, and we can get the answer.

Giving the man wall, we use the parabola equation: ( $t_1$  is the time the ball reaches the man wall)

$$z = r + v_0 \sin \theta t_1 - \frac{1}{2} g t_1^2 = h + v_2 t_1 - \frac{1}{2} g t_1^2 + r$$

The goal placement is (0, y, z), so:

We can an equation in the direction of the y axis:  $g(t_0) = y$

The value of  $t_0$  can be gotten from  $f(t_0) = 0$ .

After simplification we can get two equations about  $\theta, \varphi$ . According to the principle of equations, there must be an answer. And we can get  $t_0$ .

If the shooter let the ball pass from the sides:

Assuming that the goal point is (0, y, z) (given that the ball is falling, the parabola can be used to simplify the equation.)

The same as the above, to make the ball reaches the goal as soon as possible, the radius of gyration should be as long as possible. However, the problem is that the key point is  $\theta$  when the ball passes from the top. Now the least value of  $\theta$  points to the below. Looking at the Figure, the below is not the most difficult point the goalkeeper can save a goal. Thus, it's not the best strategy. We have to set off from other ways. Considering that we can set a circle with three point in the same horizontal plane, when the ball moves clockwise, we have point A, B, and another is:

$$(x_0 - s \sin \beta + (\frac{n}{2} e + r) \cos \beta, y_0 - \frac{y_0}{x_0} s - (\frac{n}{2} e + r) \sin \beta)$$

Easily, to make the radius longest, point  $B$  should be  $\left(0, -\frac{b}{2} + r\right)$ . If the ball moves counterclockwise, the third point should be:

$$\left(x_0 - s \sin \beta - \left(\frac{n}{2}e + r\right) \cos \beta, y_0 - \frac{y_0}{x_0} s + \left(\frac{n}{2}e + r\right) \sin \beta\right)$$

Finding the three points, we can get the radius of gyration  $R_{xoy}$ . And according to the equations:

$$\begin{cases} f(t_0) = 0 \\ g(t_0) = \frac{b}{2} - r(\text{anti-clockwise}) \text{ or } \left(-\frac{b}{2} + r\right) (\text{clockwise rotation}) \end{cases}$$

We can get the time  $t_0$  the ball needs to reaches the goal line.

Next, we introduce the factor of the goalkeeper:

We first analyze the range that the goalkeeper can reach in a certain time.

Just ignoring the height of the goalkeeper, as the start posture is fish dive, no touching with the ground, the only affecting factor is the gravity. So the range is as following.

To any time  $t$ , it exists:

$$\vec{v}_{3t} = \vec{v}_3 + \vec{g}t$$

To integral  $dt$ :

$$\int_0^t \vec{v}_{3t} dt = \int_0^t \vec{v}_3 dt + \int_0^t \vec{g}t dt$$

We can get the position vector of  $t$ :  $\vec{s} = \vec{v}_3 t + \frac{1}{2} \vec{g}t^2$

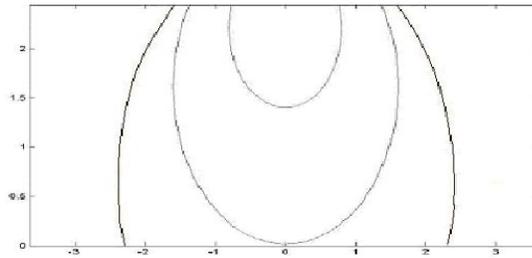
The first item on the left is a series of concentric circles. As the direction of the gravitational acceleration is always downward, the position vector comes from the translation of the concentric circles.

Now we consider the height. We can just count the scope of the goalkeeper's hand can reach for the posture of saving won't change greatly. We can think that the length of the translation is equal to the length of the goalkeeper's arm. But of course, we just want the part over the plane of  $xoy$ .

$$y^2 + \left(z + \frac{1}{2}gt^2 - l\right)^2 = (v_3 t)^2$$

We can get the scope equation is:

To be clear, we draw Figure 3 about  $t=0.2$ ,  $t=0.4$  and  $t=0.6$ (from within):



**Figure 3 : Mimic Diagram of the Scope the Goalkeeper can Reach**

The diagram (the outline border of it is the scope of the goal) shows that first we can confirm that as time goes by, the scope the goalkeeper can reach is bigger. The shooter has to make the ball reach the goal as soon as he can. Besides, we explain the definition of dead angle. The angles on the top left and top right is the farthest point the goalkeeper can reach. It's wrong that the low the ball is, the easier it can be saved.

Now we can understand why the shooter like to shoot from the base angle.

Now we can calculate the probability of a goal:

We set the time the ball reaches the goal is  $t_0$ , and the time the goalkeeper is moving in th air is  $t_0 - \Delta t$ .  $\Delta t$  will change as the goalkeeper's performance. We can easily know that the better his performance is, the shorter his reflecting time is and the higher he can jumps. We roughly think the product of the highest height and the reflecting time is a set value. Then,  $(v_{30}^2 / 2g)\Delta t_0 = (v_3^2 / 2g)\Delta t_3$ .

The scope equation is: 
$$y^2 + \left( z + \frac{1}{2}g(t_0 - \Delta t)^2 - l \right)^2 = \frac{\Delta t_0}{\Delta t} [v_{30}(t_0 - \Delta t)]^2$$

When it is  $t_0$ , the coordinate of the center of ball is  $(0, y, z)$ . The goal can just save the ball in time.

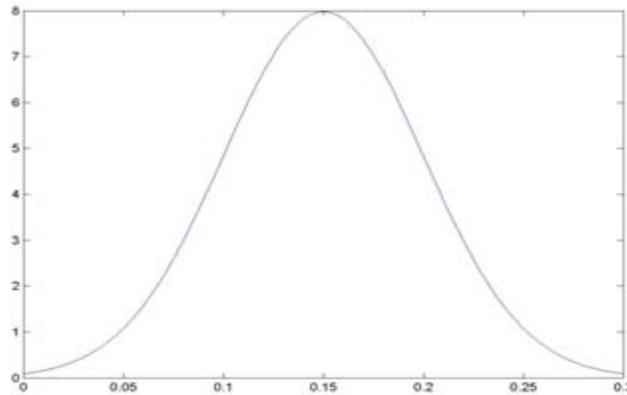
As there is only one unknown number, the must equation must have a solution. We make the solution is  $\Delta t(k)$ . When  $\Delta t \leq \Delta t(k)$ , the ball can be saved, or it will be a goal.

Now we will calculate the probability of a goal. With assumption, the index is a normal

distribution: 
$$y = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\Delta t - \Delta t_0)^2}{2\sigma^2}}$$

$\sigma$  is the standard deviation of the reflecting time of the goal keeper.

Referring to books,  $\sigma$  can be confirmed by "Three  $\sigma$  Principle". That means 99.7% of the area inside the circle made by the function and the x axis is contained in the interval  $[\Delta t_0 - 3\sigma, \Delta t_0 + 3\sigma]$ .  $\Delta t_0 - 3\sigma$  and  $\Delta t_0 + 3\sigma$  are the maximum and minimum in normal situation. We think the min is zero, so:  $\sigma = 0.05$ . The normal distribution diagram is just like Figure 4.



**Figure 4 : Normal Distribution Diagram of the Probability of a Goal**

It would be meaningless if the response time is a minus and actually it wouldn't happen. If the time for the ball to reach the goal and its placement are known, with the equation:

$$y^2 + \left( z + \frac{1}{2} g (t_0 - \Delta t)^2 - l \right)^2 = \frac{\Delta t_0}{\Delta t} [v_{30} (t_0 - \Delta t)]^2$$

We can know the value of  $\Delta t$ . And thus the probability of a goal is  $p = \xi(\Delta t)$ . The value of  $\xi(\Delta t)$  is  $\tau\left(\frac{\Delta t - \Delta t_0}{\sigma}\right)$ . As for  $\tau(u)$ , we can look it up in the Standard normal distribution table.

**Choice of strategy**

According to the analysis and our daily experience, there will be four best points of incidence-the four dead corners. We will focus on the advantages and disadvantages of them and work out whether the ball should pass from the top or by the sides.

As there is a lack of the textual materials, we can be certain about the rotation angular speed of the ball. So in the following study we will try to avoid it. If provided, the decision can be more precise.

The following is just in the condition that there have been angular speed on the direction of single vertical and single horizon. That means if the ball pass from the top, there will be an angular speed only on the vertical plane. If the sides, there will be an angular speed only in the horizontal plane. And as we don't know the difficulty relation between the angular speeds of the vertical and horizontal planes, the two situation should be calculated separately.

We define the excellence index:  $\psi_0 = \Omega\psi_1 + (1-\Omega)\psi_2$

$\psi_1$  is torture goalkeeper index, and its value is to standardize the four probabilitirs.

The expression is: 
$$\psi_{1i} = \frac{p_i - \frac{\sum_{i=1}^4 p_i}{4}}{\max\{p_j\} - \min\{p_j\}}$$

$\psi_2$  is the shooter's relaxation index. Its definition is the opposite of the standardized values of the four angular speeds. (because it will be more easier to kick the ball if the angular speed is smaller.)

$$\psi_{2i} = \frac{\sum_{i=1}^4 \omega_i}{4} - \omega_i$$

Its expression is:  $\frac{\sum_{i=1}^4 \omega_i}{4} - \omega_i$

The one with the biggest  $\psi_0$  will be chosen.

## CONCLUSION

Although this paper has no direct effect on improving the football players, it has an instructive meaning to free kick. It can help the shooter to choose the best way to shoot and at the same time can give reference to the goalkeeper. If a team can handle free kick well, improvement in its performance is consequent.

The model aims at football, but it can be strongly recommended. First, all those sports with rotation, like golf and baseball, can apply the model. As to baseball, the pitcher is just like the shooter while the batter is like the man wall and the goalkeeper. And the strike zone the goal.

What should be changed is the statistics. Make some adjustments to the restraint conditions and details according to the real situation, and we can use this model to choose the best strategy for it.

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