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# Spin Superfuidity in the Frustrated Two-Dimensional Anisotropic XY Model

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**Abstract**

**We use the SU(3) Schwinger's boson theory to study the spin transport properties in the two dimensional anisotropic frustrated Heisenberg model in the triangular lattice at T = 0. We have investigated the behavior of the spin conductivity for this model which presents an single-ion anisotropy. We study the spin transport in the Bose-Einstein condensation regime where we have that the tz bosons are condensed and the following condition is valid: ⟨tz⟩ = $\langle t_z^\dagger \rangle$ = t. Our results show a metallic spin transport for $\omega$ > 0 and a superfluid spin transport in the limit of DC conductivity, $\omega \to 0$, where $\sigma(\omega)$ tends to infinity in this limit of $\omega$ .**

***Keywords: Superfuidity; Two-dimensional anisotropic; Antiferromagnet***

## Introduction

The antiferromagnet in the triangular lattice has played a fundamental role in the understanding of frustrated quantum systems. It is one of the most fundamental systems of geometrically frustrated magnets and has been studied using several techniques such as quantum Monte Carlo, and so on [1]. The main effects of frustrating interactions, in the neighborhood of a Neel state, are the increase of the coupling and the decrease of the spinwave velocity. It is believed that for S=1/2, the system displays classical Neel-ordered ground state with a 120$^o$ spiral order [2]. It was argued in Ref. [3] that the triangular lattice has a magnetization of the lattice more reduced from its classical value than the square lattice and its Neel order may be estabilized more easily. The progress in the investigations of Spin Supercurrent and magnon BEC was recently described in the review [4]. Particularly there was overviewed the spin supercurrent Josephson Effect which is the response of the current tothe phase between two weakly connected regions of coherent quantum states. For quasiparticles such as magnons and excitons in Bose-Einstein condensation (BEC), it demonstrates the interference between two quasiparticles condensates. Spin current as a function of the phase difference across the junction, $\alpha_2-\alpha_1$, where $\alpha_1$ and $\alpha_2$ are phase's precession in two

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coherently precessing domains. It is the response of the current to the phase between two weakly connected regions of coherent quantum states [4]. It was described by Josephson in [5].

The Bose-Einstein condensation of quasiparticles whose number is not conserved is an important phenomenon of condensed matter physics. In thermal equilibrium the chemical potential of excitations vanishes and, as a result, their condensate does not form [4]. The only way to overcome this situation is to create a non-equilibrium but dynamically steady state, in which the number of excitations is conserved, since the loss of quasiparticles owing to their decay is compensated by pumping of energy. Thus the Bose-condensation of quasiparticles belongs to the phenomena of second class, when the emerging steady state of the system is not in a full thermodynamic equilibrium [4]. The spin superfluid transport and Bose condensation (BEC) are related but a different phenomenon. The spin superfluid transport is well known from 1984 when it was discovered in superfluid $^3$He [6,7]. The superfluid $^3$He is an antiferromagnet. All magnetic properties, including magnon BEC and spin superfluid in superfluid $^3$He are indeed the properties of magnetically ordered system. $^3$He has just small relaxation rate. This simplified the discovery of BEC and spin supercurrent. Later these phenomena were found in many other systems, like antiferromagnets with Suhl-Nakamura interaction ($CsMnF_3$, $MnCO_3$ and so on) [8]. It was also found in YIG films [9]. The gradients of excited magnons wave function lead to spin superfluidity, which is the quantum transport of magnetization by magnons. The properties of spin superfluidity and magnon BEC has been recently well discussed in the Ref. [4]. The spin transport properties of materials are the corner stones for many applications. Once having these properties determined, it is possible to calculate the parameters for devices which can operate in the basis of these structures. Recently, the transport phenomenon by magnetic excitations such as magnons, excitons and so on, has been much studied due to its connection with spintronics [10-13]. The injection of the spin current into a magnetic film can generate a spin-transfer torque that acts on the magnetization collinearly to the damping torque [11]. The spin transport properties in the spin systems has been studied theoretically by Sentef et al. [14] who has analyzed the spin transport in the easy-axis Heisenberg antiferromagnetic model in two and three dimensions, at T = 0. Damle and Sachdev15 have treated the two-dimensional case using the non-linear sigma model in the gapped phase. Pires and Lima [16-18] treated the two-dimensional easy plane Heisenberg antiferromagnetic model. Lima and Pires [19] studied the spin transport in the two-dimensional anisotropic XY model using the SU(3) Schwinger boson theory in the absence of impurities, Lima [20] has studied the case of the Heisenberg antiferromagnetic model in two dimensions with Dzyaloshinskii-Moriya interaction. Zewei Chen et al. [21] analyzed the effect of spatial and spin anisotropy on spin conductivity for the S=1/2 Heisenberg model on a square lattice and more recently, Kubo et al. [22] studied the spin conductivity in two-dimensional non-collinear antiferromagnets at T=0 using spin wave theory and Lima at.al [23] have studied the spin transport in the site diluted two-dimensional anisotropic Heisenberg model in the easy plane, using the self-consistent harmonic approximation. The aim of this paper is to study the spin transport in the two-dimensional anisotropic frustrated Heisenberg model on a honeycomb lattice using the SU(3) Schwinger's boson approximation. Recently, this formalism has been used to study the spin transport [24-27]. The critical properties of this model were studied using this method in [2]. This work is divided in the following way. In section II, we discuss about the method used, in section III is dedicated to our conclusions and final remarks.

## The Model

The model that we are interested is represented in the FIG. 1 and is defined by the following Hamiltonian.

$$\mathrm{H}=\sum_{\langle i,j\rangle} J_{ij}\left(S_i^x S_j^x + S_i^y S_j^y\right) + D\sum_i \left(S_i^z\right)^2 \qquad (1)$$

Where $J_{ij}$=J on all horizontal bonds, $J_{ij}$=J on all the other bonds, $J_{ij}$= J2 on the next-nearest neighbors. We consider the value of spin S=1 [28,29]. The frustration here is due to the triangular lattice. This model is interesting in view of its connections to the insulating phase of some layered organic superconductors [30]. The isotropic model with D=0 has an ordered ground state. In the limit of large D, the model will be in a disordered ground state with total magnetization null separated by a gap from the first excited states, where there is a critical Dc denoting a quantum phase transition, described by the condensation of magnons, from the large D phase to the ordered phase2.

## Methodology and spin transport

The SU(3) Schwinger boson formalism has been derived to treat systems with single ion anisotropy by Papanicolau [31] being a generalization of the SU(2) formalism. In this formalism we choose the basis:

$$|x\rangle = \frac{i}{\sqrt{2}}\left(|1\rangle - |-1\rangle\right), |y\rangle = \frac{i}{\sqrt{2}}\left(|1\rangle + |-1\rangle\right), |z\rangle = -i|0\rangle$$

Where |n⟩ are eigenstates of $S_z$. The spin operators are written via a set of three boson operators $t_\alpha$ (α=x, y, z) defined as [29]

$$|x\rangle = t_x^\dagger |\upsilon\rangle, |y\rangle = t_y^\dagger |\upsilon\rangle, |z\rangle = t_z^\dagger |\upsilon\rangle \qquad (2)$$

Where |v⟩ is the vacuum state. We also have the constraint condition $t_x^\dagger t_x + t_y^\dagger t_y + t_z^\dagger t_z = 1$

In terms of the t operators we can write

$$S^x = -i\left(t_y^\dagger t_z - t_z^\dagger t_y\right), S^y = -i\left(t_z^\dagger t_x - t_x^\dagger t_z\right), S^z = -i\left(t_x^\dagger t_y - t_y^\dagger t_x\right) \qquad (3)$$

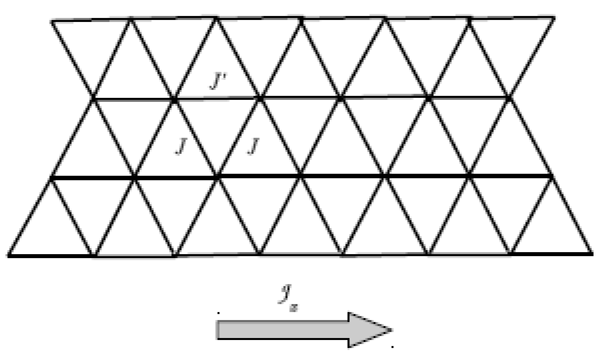


FIG. 1. Representation of the flow of spin current in the horizontal direction on a triangular lattice. The nearest-neighbor exchange J′ on all horizontal bonds and J are the other bonds. The next-nearest neighbor bonds $J_2$ are not showed in the figure.

The states $t_x^\dagger|\upsilon\rangle$ and $t_y^\dagger|\upsilon\rangle$, both consist of eigenstates $S^z$ = ±1 and have the average $\langle S^z\rangle$ = 0. This property will preserve the disorder of the ground state. To study disordered phases, it is convenient to introduce other two bosonic operators $u^\dagger$ and $d^\dagger$ [31].

$$u^t = -\frac{1}{\sqrt{2}}\left(t_x^\dagger + it_y^\dagger\right), d^t = \frac{1}{\sqrt{2}}\left(t_x^\dagger + it_y^\dagger\right) \qquad (4)$$

And so

$$\left|1\right\rangle = u^t\left|\upsilon\right\rangle, \left|0\right\rangle = t_z^\dagger\left|\upsilon\right\rangle, \left|-1\right\rangle = d^\dagger\left|\upsilon\right\rangle \qquad (5)$$

With the constraint $u^\dagger u + d^\dagger d + t_z^\dagger t_z = 1$. The spin operators can be also written as [29]

$$S^+ = \sqrt{2}\left(t_z^\dagger d + u^\dagger t_z\right), S^- = \sqrt{2}\left(d^\dagger t_z + t_z^\dagger u\right), S^z = u^{\dagger u} + d^\dagger d \qquad (6)$$

We use the SU(3) Schwinger's boson approximation [31] with objective to determine the regular part of the spin conductivity (AC conductivity) or continuum conductivity at T=0. A spin current appears if there is a gradient of magnetic field $\nabla\vec{B}$, through the system. It plays the role of a chemical potential for spins. One connects a low dimensional magnet with two bulk ferromagnets. They act as reservoirs of spins [12,13]. One flow of spin current appears if there is a difference, $\nabla\vec{B}$, between the magnetic fields at the two ends of the sample. As we are interested in calculating the longitudinal spin conductivity, we will add an external space, one dependent on time, magnetic field $\vec{B}(x,t)$, applied along of the axis $\hat{Z}$, direction of the Hamiltonian Eq. (1). In the Kubo formalism [14,16,32-36] the spin conductivity is given by the following formula:

$$\sigma(\omega) = \lim_{\vec{q}\to 0}\frac{\langle K\rangle + \Lambda\left(\vec{q},\omega\right)}{i\left(\omega + i0+\right)} \qquad (7)$$

Where ⟨K⟩ is the kinetic energy and $\Lambda\left(\vec{q},\omega\right)$ is the current-current correlation function defined by

$$\Lambda\left(\vec{q},\omega\right) = \frac{i}{\hbar N}\int_0^\infty dte^{i\omega t}\left\langle\left[J\left(\vec{q},t\right), J\left(-\vec{q},0\right)\right]\right\rangle \qquad (8)$$

$\Lambda\left(\vec{q},\omega + i0+\right)$ Is analytic in the upper half of the complex plane and extrapolation along the imaginary axis can be reliably done.

The continuity equation for the lattice allows us to write the discrete version of the current as

$$J_{n+x} - J_n = -\frac{\partial^2 S_n^z}{\partial t} \qquad (9)$$

Where n+x is the nearest neighbor site to the site n in the positive x direction. The Heisenberg equation of motion $S_n^z = i\left[H, S_n^z\right]$, can be used with equation (9) to obtain the spin current operator

$$J = -J\frac{t^2}{2}\sum_k \frac{\left[(1+\alpha)\sin k_x + \eta\sin\sqrt{3}k_x\right]}{\omega_k} \times \left(\alpha_k + \beta_k\right)\left(\alpha_k^\dagger + \beta_k^\dagger\right) \qquad (10)$$

The real part of σ(ω), σ′(ω) can be written in a standard form as [36]

$$\sigma^{'}(\omega)=\sigma_0(\omega)+\sigma^{reg}(\omega) \qquad (11)$$

Where $\sigma_0(\omega)$ gives a measure of the ballistic transport and is given by $\sigma_0(\omega)$ = DSδ(ω), where DS is the Drude's weight

$$Ds=\pi\left[\langle K\rangle+\Lambda^{'}\left(\vec{q}=0,\omega\rightarrow\vec{0}\right)\right] \qquad (12)$$

And $\sigma^{reg}(\omega)$, the regular part of σ′(ω), is given by [36]

$$\sigma^{reg}(\omega)=\Lambda^{''}\frac{\left(\vec{q}=0,\omega\right)}{\omega} \qquad (13)$$

That represents the continuum contribution to the conductivity. In equations (12) and (13), Λ′ and Λ" stand for the real and imaginary part of Λ. Using the Matsubara's Green's function, we obtain $\sigma^{reg}(\omega)$ at zero temperature as

$$\sigma^{reg}(\omega)=\frac{\Lambda(k=0,\omega)}{\omega}=(g\mu B)^2\frac{\pi}{\hbar}\int_0^{\pi}\int_0^{\pi}\frac{d^2k}{(2\pi)^2}\times\frac{\left[(1+\alpha)\sin k_x+\eta\sin\sqrt{3}k_x\right]^2}{\omega_k^3}\delta(\omega-\omega_k) \qquad (14)$$

We can integrate analytically the Eq.(14) to obtain in, we present the behavior of $\sigma^{reg}(\omega)$ with ω. we must have a spin current flowing in the horizontal as depicted in the FIG. 1 over the material. Since the system is gapless we have obtained a behavior of the AC conductivity tending to infinity when ω→0, what correspond to the DC limit. Consequently we obtain a superconductor behavior to spin transport for the DC spin current. This behavior is similar one recently obtained for the honeycomb lattice and for the two-dimensional ferroquadrupolar model [24,25]. This is a characteristic of spin systems without gap in the excitation spectrum. As this system does not present gap in the spectrum, we have exactions to form the spin current in all the ω values of the spectrum. Besides, if there is no scattering mechanism, as in this model, it is expected that the conductivity is divergent; the reason is that spin-spin scattering is not treated properly in a mean field approach.

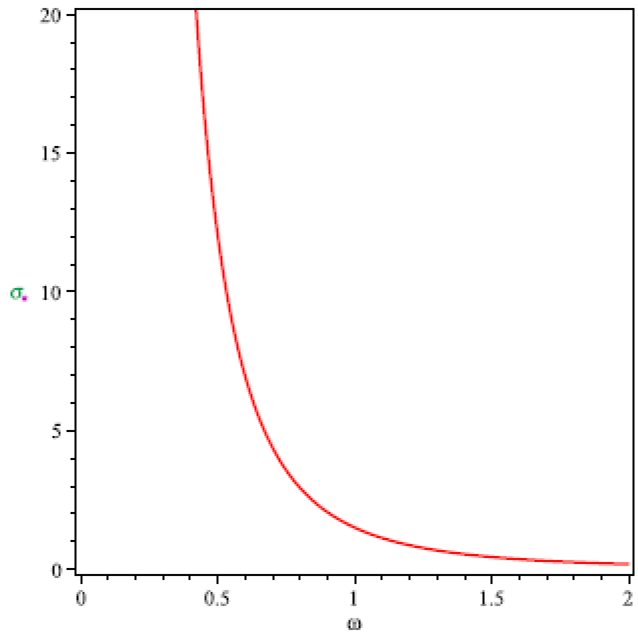


FIG. 2. Behavior of $\sigma_*(\omega)$ where $\sigma^{reg}(\omega) = \frac{(g\mu b)^2}{\hbar}\sigma_*(\omega)$ for α=0:3. In the range of $D_c$=0 and η= 0:15 at the T=0, the behavior of the spin conductivity does not vary a lot in the region of the quantum phase transition. As the AC conductivity tends to infinity in the limit ω → 0, we have an ideal spin conductor in that limit.

## Hexagonal Lattice

The honeycomb lattice is a frustrated spin system where the frustration arises from the competing interactions rather than geometric constraints. The frustration is enhanced by strong quantum fluctuations due to the low coordination number [37,38]. Moreover, experimental data are available for several compounds such as the family of compounds $BaM_2(XO_4)_2$, where M=Co, Ni and X=P. The magnetic ions M are arranged in weakly coupled frustrated hexagonal lattice. However, most of the compounds lie in the Neel ordered region. A model relevant for these materials is the J1−J2−J3 model with first, second and third interactions. This model was studied recently by D. C. Cabra et al. [39,40] for S=1/2 using the linear spin wave theory (LSWT) and SU(2) mean field Schwinger boson approximation. J. B. Fouet [41] studied the same model using exact diagonalization and LSWT for selected values of J2 and J3. Li et al. [42] has treated the same system using the coupled cluster method and the J1−J2−J3 model with S=1 was studied by S. S. Gong et al. [43] and Pires [38,44]. The model is depicted in the FIG. 3 and is defined by the following Hamiltonian

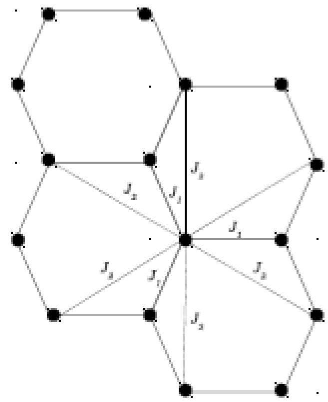

FIG. 3. Representation of the model.

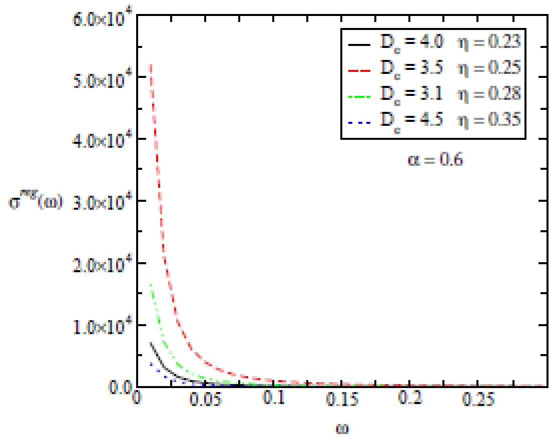


FIG. 4. Behavior of $\sigma^{reg}(\omega)$ for α= 0:6 and for different values of $D_c$ and η. For Dc and η points before of the cusp of the graphic of Dc vs. η as Dc=4:0, η=0:23 (solid line), Dc=3:5, η=0:25 (dashed line). On the cusp, Dc=1:5, η=0:28 (dot-dashed-line) and for large value of Dc=4:5, η=0:35 (dotted line). The quantum phase transition point occurs on the cusp point (Dc=0:35 and η=0:25). Thus, we obtain an influence of the QPT point on the spin conductivity. The AC conductivity tends to infinity in the limit ω → 0. Therefore, we have an ideal spin conductor in the DC limit.

In FIG. 4, we present the behavior of $\sigma^{reg}(\omega)$ with ω. Due to the honeycomb crystal lattice, we must have a spin current in zig-zag, as depicted in FIG. 1. We obtain an influence of the quantum phase transition point of the graphic of Dc vs. η of the Ref. [2] on the spin conductivity. We get a bit influence of the variation of Dc and η on the curve of the spin conductivity, as showed in the FIG.4. The curve suffers a small increases until the cuspidpoint where after this the curve suffers a sudden decrease (the solid-line changes to dashed line and then it changes abruptly to dot-dashed line, decreasing further to dotted line).

## Conclusion

We studied the spin transport properties in the two dimensional anisotropic frustrated Heisenberg model in the triangular and hexagonal lattices using the SU(3) Schwinger's boson theory. Because of the triangular symmetry of the lattice, we must have a spin current flowing in the J′ (horizontal direction) through the material. How we obtain a spin conductivity tending to infinity when $\omega \rightarrow 0$, we must have, in this limit, ideal spin transport, independent of the value of the Drude's weight DS found. Recently, the critical properties of this model were studied using the SU(3) Schwinger boson method at [2]. From a general way, the two-dimensional Heisenberg model is a very important model due the intense tentative to understand the interplay of antiferromagnetism and superconductivity [45,46]. From an experimental point of view, recently there is an intense research about the quantum Hall effect for spins and magnon spintronics [10-13]. In the studies of these effects, often only the sign differences between related quantities like magnetic fields and generated spin and charge currents are determined.

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