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Research on billiards stroke technique affects cue ball movement trajectory

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ABSTRACT

Billiards is a kind of international widely popular elegant indoors sports event, is a kind of using cue to stroke on the table, relying on calculation ace to define competition result indoors recreational sports event. Billiards not only can be regarded as competition event, but also it can also be regarded as rigid body in mechanical researches, billiards as a rigid body exercise typical example, cue ball and object ball collision process conform to rigid body exercise basic rules. Billiards collision process conforms to rigid body collision perfect elastic collision process. In case that normally strokes cue ball, cue ball not collides with billiards table side after it colliding with object ball, applies theoretical mechanical knowledge and physics collision rules, it can make anticipation on cue ball sports trajectory. This paper, after analyzing cue ball force status after colliding, it makes qualitative and quantitative analysis of cue ball sports trajectory. Results show mass center speed gets larger, rotational angular speed gets smaller, and then cue ball trajectory will get closer to cue ball and object ball common tangent.
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KEYWORDS

Theoretical mechanics;
Rigid body movement;
Perfect elastic collision;
Friction forced;
Billiards technique.

INTRODUCTION

Billiards as a kind of indoors leisure sports event, it has already 500 to 600 years history until now, and well-received by people. Now, billiards has already developed into diversities, it has Chinese eight-ball, Russian pocket, British pocket, carom pool, American pocket and snooker, from which snooker is most popular, which has already become a kind of competition item. China billiards team has ranked in the Asian leading level, and achieved champion in Asian Games. Well known players are Ding Jun-Hui, Pan Xiao-Ting and so on.

Billiards includes lots of physical knowledge and mathematical knowledge. In stroking process, stroking cue ball cue playing and strength have greater influence on movement trajectory after cue ball colliding with object ball. After stroking, cue ball movement trajectory and stop position directly affect next stroking. In order to make cue ball better position and beneficial to next stroking, correct use cue ball stroking strength and cue playing are getting relative important. Therefore, it should make more detailed, precise theoretical analysis of movement trajectory after cue ball colliding with object ball. This paper will based on previous research, assume that before cue ball colliding with object ball

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movement state is known, and cue ball and object ball make perfect elastic collision, put emphasis analysis on movement rules after cue ball colliding with object ball, before it touching table edge.

When playing billiards, it not only needs to judge object ball movement direction after cue ball colliding with object ball and let it enter into net, but also should judge cue ball movement direction so as to helpful for next stroking, especially it should prevent cue ball entering into net hole caused stroking foul after collision. Therefore, cue ball position is very important; sometimes one simple cue ball position will decide a frame result. A good billiards player not only should have good stroking technique, meanwhile but also should have correct judge cue ball movement trajectory ability; only make accurate anticipation on cue ball movement trajectory; it can analyze him how to stroke in next step. For the subject research, it mainly analyzes movement trajectory after cue ball colliding with object ball. Regard billiards as rigid body movement, billiards movement trajectory after colliding is not easier to master, which needs us to analyze billiards movement trajectory according to physics collision rules, theoretical mechanics, rigid body knowledge and other knowledge.

BILLIARDS COLLISION BASIC THEORIES

Given both cue ball and object ball mass are m, instantaneous speed before cue ball colliding with object ball is $\vec{v}$, when cue ball colliding with object ball, cue ball instantaneous speed changes into $\vec{v}_1$, object ball instantaneous speed changes into $\vec{v}_2$, and it happens perfect elastic collision, and then it has momentum relation(1):

$$m \cdot \vec{v} = m \cdot \vec{v}_1 + m \cdot \vec{v}_2 \quad \textbf{(1)}$$

Energy relation:

$$\frac{1}{2} m \cdot \vec{v}^2 = \frac{1}{2} m \cdot \vec{v}_1^{\,2} + \frac{1}{2} m \cdot \vec{v}_2^{\,2} \quad \textbf{(2)}$$

Simultaneous formula(1) and formula (2), it can get:

$$\vec{v}_1^{\,2} + \vec{v}_2^{\,2} + 2\vec{v}_1 \cdot \vec{v}_2 = \vec{v}_1^{\,2} + \vec{v}_2^{\,2} \quad \textbf{(3)}$$

Thereupon:

$$2\vec{v}_1 \cdot \vec{v}_2 = 0 \quad \textbf{(4)}$$

Formula (4) is vector formula, from which $\vec{v}_1$ and $\vec{v}_2$ numerical relation that is :

$$|\vec{v}_1| \cdot |\vec{v}_2| \cos\theta = 0 \quad \textbf{(5)}$$

Thereupon, we can know:

(1) When it occurs central collision (as Figure 1), cue ball and object ball included angle in plane $\theta = 0^\circ$, therefore formula(5) changes into $|\vec{v}_1| \cdot |\vec{v}_2| = 0$

From physical collision rule, it is clear that cue ball cannot go beyond object ball after collision, therefore it can get:

$$\vec{v}_1 = 0$$

Input into formula (1), it can get:

$$\vec{v}_2 = \vec{v}$$

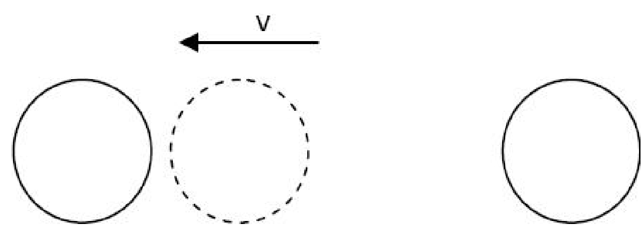

Figure 1 : Central collision

(2) When it happens non central collision (as Figure 2), in the plane, cue ball and object ball after collision, there is no zero angle included angle θ, from physical collision rules, it is known after cue ball colliding with object ball, respective speed cannot change into 0, according to formula(5) it can know that $\cos\theta = 0$, so included angle $\theta = 90^\circ$.

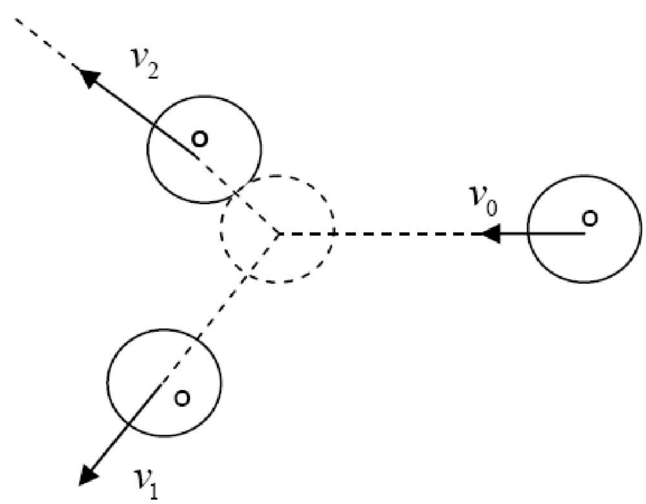

Figure 2 : Non central collision

That cue ball and object ball after collision, their movement directions are mutual vertical. Because that cue ball and object ball in collision process, they follow momentum conservation law, it established coordinate system as Figure 3, then it has horizontal:

$$m \cdot \vec{v} = m \cdot \vec{v}_1 \sin\alpha + m \cdot \vec{v}_2 \cos\alpha \quad \textbf{(6)}$$

Vertical:

$$0 = m \cdot \vec{v}_1 \cos\alpha - m \cdot \vec{v}_2 \sin\alpha \quad \textbf{(7)}$$

Make simultaneous formula (6) and formula (7), it gets:

$\vec{v}_1 = \vec{v}\sin\alpha$, $\vec{v}_2 = \vec{v}\cos\alpha$

Analyze from above status, it can make preliminary judgment on cue ball movement trajectory.

MOVEMENT TRAJECTORY MODEL ESTABLISHMENT AND SOLUTION

When researching on billiards, it can regard billiards as smooth sphere rigid body. Therefore, cue ball and object ball collision can be regarded as perfect elastic collision; cue ball and object ball can be regarded as equal mass spheres. If not considering energy loss, after collision cue ball will transfer normal momentum to object ball, cue ball tangential direction momentum doesn't change. Therefore, after colliding with object ball, cue ball two balls' tangential direction momentum and rotation will decide cue ball movement trajectory.

In case not considering ball blocking and cue ball not colliding with table edge, it will not analyze left and right rotation influences on cue ball movement. Therefore it only considers cue ball top spin and back spin status after cue ball colliding with object ball here.

Establish coordinate system as Figure 3 show, $\vec{v}_0$ is cue ball mass center speed, it regards along x axis positive direction as positive; ω_0 is cue ball surrounding it mass center rotational speed, it regards clockwise as positive; f is billiards table to cue ball sliding friction force, it takes along x axis positive direction as positive. Given billiards radius to be r ,mass to be m ;table and cue ball sliding friction coefficient to be μ ;gravity accelerated speed to be g .From theoretical mechanical knowledge, we can know that when cue ball making rotations, sphere plane linear speed is $r \cdot \omega_0$, when cue ball sphere plane linear speed value is above cue ball mass center sports speed value that $r \cdot \omega_0 > v_0$, cue ball and billiards table contact point movement direction is x axis negative direction, at this time, billiards table provided cue ball friction force is positive, therefore it can know $f = \mu mg$; when cue ball sphere plane linear speed value and cue ball mass center speed values are equal that $r \cdot \omega_0 = v_0$, cue ball is making pure rolling. At this time, cue ball and billiards table contact point is relative static, billiards table provided cue ball friction force is 0; when cue ball sphere plane linear speed value is small than cue ball mass center speed value that $r \cdot \omega_0 < v_0$, cue ball and billiards table contact point movement direction is x axis positive direction, at this time, billiards table provided cue ball friction force is negative, therefore it can know $f = -\mu mg$. In the following, for above three cases, it will respectively make solution of kinematic equation on cue ball movement.

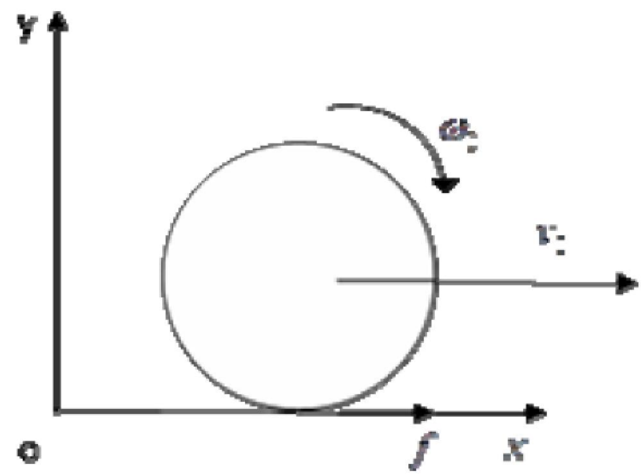

Figure 3 : Cue ball roll coordinate

When $r \cdot \omega_0 > v_0$, billiards table to cue ball sliding friction force is $f = \mu mg$, cue ball kinematic equation:

$$f \cdot t = m(v - v_0) \quad \textbf{(8)}$$

$$-f \cdot r \cdot t = J \cdot (\omega - \omega_0) \quad \textbf{(9)}$$

$$f = \mu mg \quad \textbf{(10)}$$

Among them, $\vec{v}$ is cue ball mass center instantaneous speed, ω is cue ball surrounding mass center instantaneous rotational angular speed, $J = \frac{2}{5} m \cdot r^2$ is ball to mass center passing horizontal axis rotational inertia.

From theoretical mechanics, it is known when it happens cue ball mass center instantaneous speed and cue ball sphere plane linear speed are the same that $v = \omega \cdot r$, cue ball starts to make pure rolling. At this time cue ball and billiards table have no relative movement tendency, thereupon, sliding friction force disappears that $f = 0$, cue ball will static after rolling some distance under resistance (air resistance, cue ball and billiards table rolling friction resistance) effects.

From formula (8), formula (9), formula (10) and $v = \omega \cdot r$, it can solve sliding friction force acting time that:

$$t_1 = \frac{2}{7} \cdot \frac{(r \cdot \omega_0 - v_0)}{\mu g} \quad \textbf{(11)}$$

Input formula (11) into formula (8), it can get cue ball pure rolling mass center speed that:

$$v' = \frac{5}{7} v_0 + \frac{2}{7} r \cdot \omega_0 \quad \textbf{(12)}$$

When $t < t_1$, from formula (8) and formula (10), it can get:

$$v = v_0 + \mu g t \quad \textbf{(13)}$$

According to above formulas analysis result, it deducts cue ball displacement:

$$s_1 = \begin{cases} \int_0^t v dt = v_0 + \frac{1}{2}\mu g t^2 \\ \int_0^{t_1} v dt + \int_{t_1}^t v' dt = -\frac{2v_0^2}{49\mu g} + \frac{4v_0 r\omega_0}{49\mu g} - \frac{2(r\omega_0)^2}{49\mu g} + \left[\frac{5}{7}v_0 + \frac{2}{7}r\omega_0\right]t \end{cases}$$

(if: $t \le t_2$) (if: $t > t_2$) **(14)**

From formula (14), it can deduce that cue ball movement distance decreases with sliding friction coefficient μ increasing, it increases with initial speed v_0 increasing.

When $r \cdot \omega_0 = v_0$, billiards table to cue ball sliding friction force $f = 0$. Cue ball will gradually static under resistance (air résistance, rolling friction resistance etc.) effects, at this time, cue ball displacement is:

$$s_2 = \int_0^t v dt = v_0 t \quad \textbf{(15)}$$

When $r \cdot \omega_0 < v_0$, billiards table to cue ball sliding friction force is $f = -\mu m g$, cue ball kinematic equation, from formula(8)0formula(9)0 $f = -\mu m g$ and $v = \omega \cdot r$, it can solve sliding friction force acting time that:

$$t_2 = \frac{2}{7} \cdot \frac{(v_0 - r \cdot \omega_0)}{\mu g} \quad \textbf{(16)}$$

Input formula (16) into formula (8), it can get cue ball pure rolling mass center speed that:

$$v_2 = \frac{9}{7} v_0 - \frac{2}{7} r \cdot \omega_0 \quad \textbf{(17)}$$

When $t < t_1$, from formula(8)and $f = -\mu m g$, it can get:

$$v = v_0 - \mu g t \quad \textbf{(18)}$$

According to above formula analysis result, it deduces cue ball displacementÿ

$$s_3 = \begin{cases} \int_0^t v dt = v_0 - \frac{1}{2}\mu g t^2 \\ \int_0^{t_2} v dt + \int_{t_2}^t v_2 dt = \frac{2v_0^2}{49\mu g} - \frac{4v_0 r\omega_0}{49\mu g} + \frac{2(r\omega_0)^2}{49\mu g} + \left[\frac{9}{7}v_0 - \frac{2}{7}r\omega_0\right]t \end{cases}$$

(if: $t \le t_2$) **(18)**

After collision cue ball movement trajectory

Cue ball collides with object ball at speed v and angular speed ω, direction and two balls' center connection line included angle is α. Take cue ball center as origin, it takes two balls center line connection line as y axis; it establishes coordinate system as Figure show. Given after object ball colliding, cue ball displacement projections in x and y axis are respectively s_x and s_y. According to vector compound theorem, before colliding with object ball, in x axis direction, cue ball mass center speed is $v_x = v\sin\alpha$, surrounding mass center rotational speed is $\omega_x = \omega \sin\alpha$ (rotational axis is y axis); in y axis direction, cue ball mass center speed $v_y = v\cos\alpha$, surrounding mass center rotational speed $\omega_y = \omega\cos\alpha$ (rotational axis is x axis). In the following, it respectively makes analysis of non-spinning stroke, high cue (top spin) stroke and low cue (back spin) stroke to make analysis.

Non-spinning stroke cue ballÿ in that case $\omega = 0$, after cue ball colliding with object ball, cue ball kinematic speed: $v_x = v\sin\alpha$, $v_y = 0$, $\omega_x = \omega_y = 0$.

From $\omega_x = 0$, it can get $r \cdot \omega_x < v_x$, input $v_0 = v_x = v\sin\alpha$, $\omega_x = \omega_0 = 0$ into formula (16)and formula (18), it gets:

$$t_2 = \frac{2}{7} \cdot \frac{v\sin\alpha}{\mu g} \text{ (if: } t \le t_2) \quad \textbf{(19)}$$

$$s_x = \begin{cases} v\sin\alpha \cdot t - \frac{1}{2}\mu g t^2 \\ \frac{2v^2\sin^2\alpha}{49\mu g} + \frac{9}{7}v\sin\alpha \cdot t \end{cases} \text{ (if: } t > t_2) \quad \textbf{(20)}$$

Solve s_y, $v_y = 0$ and $\omega_y = 0$, obviously it hasÿ $s_y = 0$

To sum up, when cue ball in non-spinning and collides with object ball with angle

α, received trajectory equations are (19) and (20). From trajectory equation, it can know cue ball moves along two balls' common tangent direction at this time.

High cue (top spin)strokes cue ballÿ in that case $\omega > 0$, after cue ball colliding with object ball, cue ball in y axis direction mass center instantaneous speed is zero that $v_y = 0$, movement state in 在 x axis will not change.

Solve s_x, because $r \cdot \omega_x$ and v_x sizes cannot define at this time, so it needs to discuss, cases have three kinds.

(1) When $r \cdot \omega_x < v_x$ that $r \cdot \omega\sin\alpha < v\sin\alpha$, it simplifies into $r \cdot \omega < v$. Input $v_0 = v_x = v\sin\alpha$, $\omega_0 = \omega_x = \omega\sin\alpha$ into formula(16)and formula(18), it solves:

$$t_2 = \frac{2}{7} \cdot \frac{(v - r\cdot\omega)\sin\alpha}{\mu g} \quad \textbf{(21)}$$

$$s_x = \begin{cases} v\sin\alpha \cdot t - \frac{1}{2}\mu g t^2 \\ \frac{2v^2\sin^2\alpha}{49\mu g} - \frac{4vr\cdot\omega\sin^2\alpha}{49\mu g} + \frac{2(r\cdot\omega)^2\sin^2\alpha}{49\mu g} + \left(\frac{9}{7}v - \frac{2}{7}r\cdot\omega\right)\sin\alpha \cdot t \end{cases}$$

$$\text{(if : } t \le t_2) \quad \textbf{(22)}$$

(2) When $r \cdot \omega_x = v_x$ that $r \cdot \omega\sin\alpha = v\sin\alpha$, it simplifies into $r \cdot \omega = v$. The current status is pure rolling, it can input data into formula(15)and get:

$$s_x = v\sin\alpha \cdot t \quad \textbf{(23)}$$

(3) When $r \cdot \omega_x > v_x$ that $r \cdot \omega\sin\alpha > v\sin\alpha$, it simplifies into $r \cdot \omega > v$. Input $v_0 = v_x = v\sin\alpha$, $\omega_0 = \omega_x = \omega\sin\alpha$ into formula(11)and(14), it solvesÿ

$$t_1 = \frac{2}{7} \cdot \frac{(r\cdot\omega - v)\sin\alpha}{\mu g} \quad \textbf{(24)}$$

$$s_x = \begin{cases} v\sin\alpha \cdot t + \frac{1}{2}\mu g t^2 \\ -\frac{2v^2\sin^2\alpha}{49\mu g} + \frac{4vr\cdot\omega\sin^2\alpha}{49\mu g} - \frac{2(r\cdot\omega)^2\cdot\sin^2\alpha}{49\mu g} + \left(\frac{5}{7}v + \frac{2}{7}r\cdot\omega\right)\sin\alpha \cdot t \end{cases}$$

$$\text{(if : } t \le t_1) \text{ (if : } t > t_1) \quad \textbf{(25)}$$

Solve s_y, from $\omega > 0$, $v_y = 0$ it can know $r \cdot \omega_y > v_y$, therefore input $v_0 = v_y = 0$, $\omega_0 = \omega_y = \omega\cos\alpha$ into formula (11) and formula (14)

$$t_1' = \frac{2}{7} \cdot \frac{r\cdot\omega\cos\alpha}{\mu g} \quad \textbf{(26)}$$

$$s_y = \begin{cases} \frac{1}{2}\mu g t^2 \\ -\frac{2(r\cdot\omega)^2\cdot\cos^2\alpha}{49\mu g} + \frac{2}{7}r\cdot\omega\cos\alpha \cdot t \end{cases}$$

$$\text{(if : } t \le t_1) \text{ (if : } t \le t_1) \quad \textbf{(27)}$$

To sum up, when cue ball top spin speed is ω and linear speed is v, and when colliding with object ball at angle α, if $r \cdot \omega < v$, its kinematic trajectory equations are (22) and (27); if $r \cdot \omega = v$, its kinematic trajectory equations are(23) and (27); If $r \cdot \omega > v$, their kinematic trajectory equations are (25) and (27). From the perspective of equations, when $\alpha = 0$ that cue ball collides with object ball, it has $s_x = 0$ and $s_y > 0$. Therefore, it forms into cue ball forward following status that

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follow shot.

Low cue (back spin) stroking cue ball: in that case $\omega < 0$, after cue ball colliding with object ball, cue ball in y axis direction mass center instantaneous speed is zero that $v_y = 0$, sports state in x axis will not change.

Solve s_x, known $\omega < 0$, $v_y > 0$, it can know $r \cdot \omega_x < v_x$, input $v_0 = v_x = v \sin\alpha$, $\omega_0 = \omega_x = \omega \sin\alpha$ into formula (16) and formula (18), it gets:

$$t_2 = \frac{2}{7} \cdot \frac{(v - r \cdot \omega)\sin\alpha}{\mu g} \quad \textbf{(28)}$$

$$s_x = \begin{cases} v\sin\alpha \cdot t - \frac{1}{2}\mu g t^2 \\ \frac{2v^2\sin^2\alpha}{49\mu g} - \frac{4vr\cdot\omega\sin^2\alpha}{49\mu g} + \frac{2(r\cdot\omega)^2\sin^2\alpha}{49\mu g} + \left(\frac{9}{7}v - \frac{2}{7}r\cdot\omega\right)\sin\alpha\cdot t \end{cases}$$

$(\text{if}: t \le t_2)$ $(\text{if}: t > t_2)$ **(29)**

Solve s_y, know $\omega < 0$, $v_y = 0$, it can know $r \cdot \omega_y < v_y$, input $v_0 = v_x = 0$, $\omega_0 = \omega_x = \omega \cos\alpha$ into formula (16) and formula (18), it gets:

$$t_2' = -\frac{2}{7} \cdot \frac{r \cdot \omega \cos\alpha}{\mu g} \quad \textbf{(30)}$$

$$s_y = \begin{cases} -\frac{1}{2}\mu g t^2 \\ \frac{2(r\cdot\omega)^2 \cdot \cos^2\alpha}{49\mu g} + \frac{2}{7} r \cdot \omega \cos\alpha \cdot t \end{cases}$$

$(t \le t_1)$ $(t > t_1)$ **(31)**

To sum up, when cue ball back spin rotational speed is ω and collides with object ball at angle α, its kinematic trajectory equations are(29)and(31). Thereupon, it is clear when $\alpha = 0$ that cue ball collides with object ball, it will have $s_x = 0$ and $s_y < 0$, at this time it appears cue ball retreat phenomenon that is backspin ball.

CONCLUSIONS

Based on above research, it found that when cue ball collided with object ball, its mass center speed and rotational angular speed had great influences on its trajectory, on a whole, the larger mass speed was, the smaller rotational angular speed was, the cue ball trajectory would get closer to cue ball and object ball common tangent. Due to research on the subject was ideal, for other forces such as (rolling friction force, air resistance) and so on, it should take them into consideration. In order to make more precise control cue ball position, it suggested to research on kinematic equation after cue ball and table edge collision.

This paper established program had mature theoretical basis and corresponding software supports, the reliability was high; Model principle was simple and easy to understand, solution was simple and feasible; model considering and actual status connections were relative close. But calculation quantity was big, calculation time was long, constraint conditions were too strict; model hypothesis was relative ideal, which had larger gap with actual status.

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