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Research of B-value of China Earthquake based on weibull distribution

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ABSTRACT

China Earthquake time intervals have been calculated and passed the hypothesis test based on Weibull distribution. Comparative analyses have been carried out with China Earthquake Catalogue according to different time intervals and different zones. From the results, we can see that the shape parameters are enlarged with the year's extrapolation.

KEYWORDS

Weibull distribution; B value; Earthquake; Shape parameter; Scale parameter.

INTRODUCTION

Weibull Distribution possesses rich predictions contents which can be applied in the analysis of the reliability and risks of the systems. Weibull Distribution contains three parameters such as shape parameter, scale parameter and position parameter. Exponential distribution and normal distribution can be looked as the special cases of Weibull Distribution which is applicable of all types of tests and is used widely in all areas[1-3].

The cumulative fault distribution function is given as below:

$$F(t{:}k,b,a)=1-e^{-(\frac{t-a}{b})^{k}} \quad (1)$$

The fault density formula of three-parameter Weibull distribution is given as below:

$$f(t{:}k,b,a)=\frac{k}{b}\left(\frac{t-a}{b}\right)^{k-1}e^{-(\frac{t-a}{b})^{k}} \quad (2)$$

CALCULATION AND HYPOTHESIS TESTING

According to the earthquake time in China earthquake catalog which comes from USGS, we can judge whether the earthquake time interval obeys Weibull distribution or not and whether the data can pass Weibull distribution hypothesis testing or not. The time of the catalog is from January 1, 1973 to June 30, 2010 and the earthquake magnitude is above 4.0 and the depth of focus is from 0 to 70 km.

We group the earthquake catalog of China according to the areas and study the rules followed of the different parts. These are Taiwan areas, Kunlun-Qilian Mountains-Qinling areas, Tibet and Xinjiang areas, Tianshan areas, Sichuan and Yunnan areas, Erdos-Qilian Mountains-Qinling areas, North of China and Northeast of China. Then we analyze the laws followed by the whole earthquake catalog.

Group the earthquakes occurred in the areas according to intervals of each year and get thirty-eight groups.

We study Taiwan areas (longitude: 115-125, latitude: 28-20) and carry out the calculation and hypothesis testing.

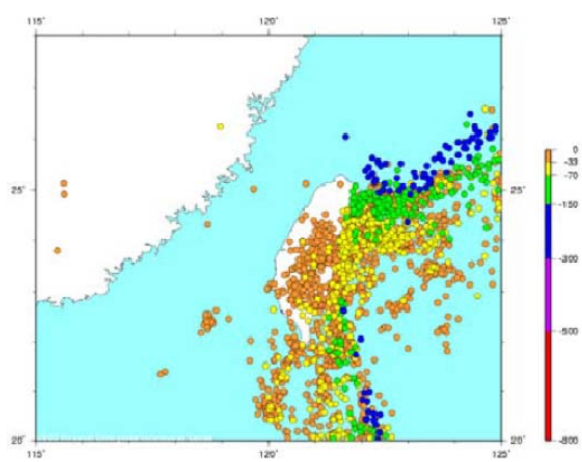

Figure 1: the earthquake map of taiwan

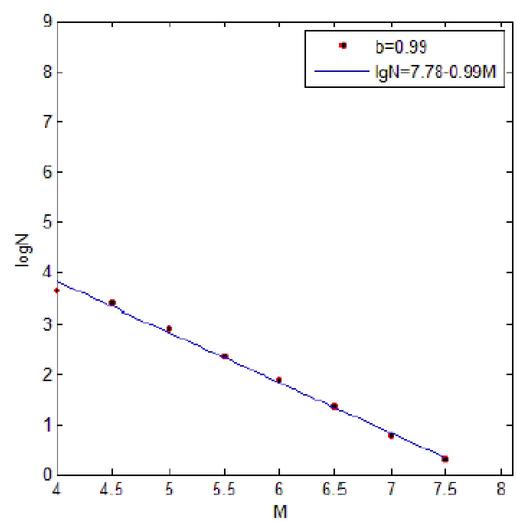

Figure 2: B-value curve of taiwan

Calculation and hypothesis testing of weibull distribution

We draw the scattered diagram of density function f(t) and distributed function F(t) of Taiwan areas as shown in Fig.3 and Fig.4. From the diagram we can see that the density function curve decreases monotonically during some intervals which is similar to Weibull distribution and exponential distribution. We carry out the parameters estimation and hypothesis testing[4-5].

Parameters estimation

According to the hypothesis of the scattered diagram and the type of distributed function, we think the time interval of the earthquake follows Weibull distribution and then we carry out regression analysis.

Regression analysis is a method to solve the relations of variables in mathematical statistics. In the statistical analysis of reliability data, regression analysis method not only can estimate the parameters, calculate the reliability index, but also can test the distributed type.

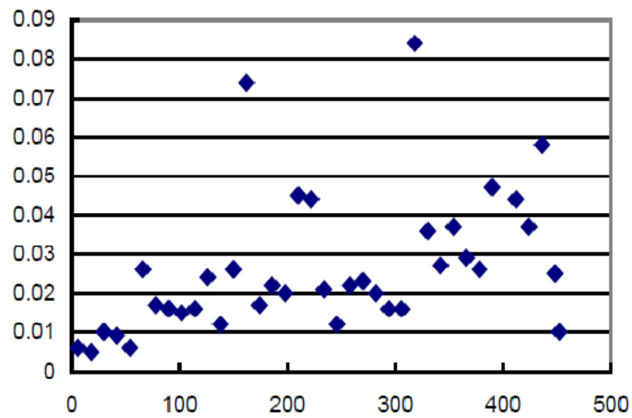

Figure 3: The scattered diagram of density function f(t)

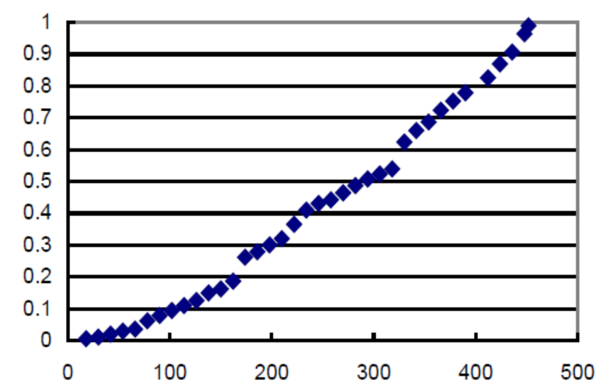

Figure 4: The scattered diagram of distributed function f(t)

We get n groups of data as (x_1, y_1)、(x_2, y_2)、…、(x_n, y_n) and the variables presented a linear relationship generally. We model the linear regression equation and get the transformed formula to test the linear correlations.

Weibull distributed function is given as below:

$$F(t) = 1 - e^{-(\frac{t-a}{b})^k} \quad (3)$$

Transformed as:

$$\ln\ln\frac{1}{1-F(t)} = k\ln(t-a) - k\ln b \quad (4)$$

Then:

$$X_i = \ln(t-a) \quad (5)$$

$$Y_i = \ln\ln\frac{1}{1-F(t)} \quad (6)$$

$$A = -k\ln b \quad (7)$$

$$B = k \quad (8)$$

We obtain a linear relationship as:

$$y = A + Bx \tag{9}$$

The linear correlations test is given as below:

We build the liner regression equation for q group of test data and test the liner relationships between x and y. We call the process the linear correlations test.

The correlation parameter $|\hat{\rho}|$ is less than or equal 1. When $|\hat{\rho}|$ is 1, we say that the observed values y_i are all on the regression line. $|\hat{\rho}|$ is closer to 1, the relationships of the liner correlations are stronger. From the table, we can refer that x is correlated with y when $|\rho|$ is greater than ρ_α under significant level α .

$$\hat{\rho} = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n\overline{x}^2)(\sum_{i=1}^{n} y_i^2 - n\overline{y}^2)}} = 0.9906$$

When α is 0.1, the correlation parameter is given as below:

$\rho_\alpha = 1.645/(n-1)^{1/2} = 0.2705$, n=38。

The liner regression equation is given as below:

$$\hat{B} = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sum_{i=1}^{n} x_i^2 - n\overline{x}^2} = 1.011 \tag{11}$$

$$\hat{A} = \overline{y} - \hat{B}\overline{x} = -4.80 \tag{12}$$

$$y = \hat{A} + \hat{B}x = -4.80 + 1.011x \tag{13}$$

The estimated parameters are given as below:

$$\hat{k} = \hat{B} = 1.011 \tag{14}$$

$$\hat{b} = \exp(-\hat{A}/\hat{B}) = 115.55 \tag{15}$$

The results show that $\hat{\rho} = 0.9906 > \rho_\alpha = 0.2705$. We can refer that x is correlated with y .

The hypothesis test is given as below:

We carry out *d* test about the supposed distribution function of the earthquake intervals.

The accepted domains are given as below:

$$D_n = \sup_{-\infty < t < \infty} \left| F_n(t) - F_0(t) \right| = \max\{d_i\} \le D_{n,\alpha} \tag{16}$$

Where: $F_0(t)$—the original distribution function;

$F_n(t)$—the empirical distribution function with the sample size of n;

$D_{n,\alpha}$—thresholds.

$$F_n(t) = \begin{cases} 0 & t < t_i \\ \frac{i}{n} & t_i \le t < t_{i+1} (t_1 \le t_2 \le \cdots \le t_n) \\ 1 & t \ge t_n \end{cases} \tag{17}$$

$$d_i = \max\left\{F_0(t_i) - \frac{i-1}{n}, \frac{i}{n} - F_0(t_i)\right\} \qquad (18)$$

$D_{\max} = 0.1438 < D_{38,0.1} = 0.217$。 When $n = 38, \alpha = 0.1, D_{n,\alpha} = 0.217$. From the results we can draw that the distribution of the earthquake intervals accord to Weibull Distribution.

The parameters of Weibull Distribution in Taiwan areas are given as below:

The shape parameter: $\hat{k}$=1.011 , the scale parameter: $\hat{b}$=115.55 . We draw the distribution function fitting figure of Taiwan areas.

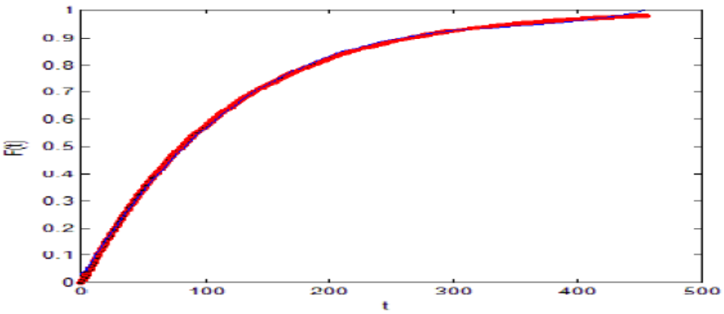

Figure 5: The distribution function fitting figure of Taiwan areas

THE ESTIMATED PARAMETERS OF OTHER AREAS

The earthquake intervals of other areas also accord to Weibull Distribution and pass through hypothesis tests. We calculate the parameters of Weibull Distribution of other areas which are given as below:

TABLE 1: The parameters of weibull distribution of other areas

the areas	$\hat{\rho}$	ρ_α	b	the estimated parameters (1973-2010) (t=456m)
Taiwan areas	0.9906	0.2705	0.99	$\hat{k}$=1.011 $\hat{b}$=115.55
Kunlun-Qlianshan-Qinling areas	0.9910	0.2705	0.87	$\hat{k}$=1.233 $\hat{b}$=136.65
Tibet and Xinjiang areas	0.9905	0.2705	0.99	$\hat{k}$=1.172 $\hat{b}$=140.63
Tianshan areas	0.9825	0.2705	0.98	$\hat{k}$=1.077 $\hat{b}$=135.52
Chuandian areas	0.9782	0.2705	0.90	$\hat{k}$=1.125 $\hat{b}$=130.19
Erdos-Qilianshan-Qinling areas	0.9725	0.2705	1.18	$\hat{k}$=1.109 $\hat{b}$=126.65
North of China	0.9809	0.2705	0.68	$\hat{k}$=1.299 $\hat{b}$=160.08
The total of the eight areas	0.9925	0.2705	0.95	$\hat{k}$=1.149 $\hat{b}$=135.55
Mainland and Taiwan areas	0.9975	0.2705	0.99	$\hat{k}$=1.009 $\hat{b}$=107.26

From TABLE 1 we can see that the estimated parameters $\hat{b}$ and $\hat{k}$ of different areas are different and the correlation parameters are also different. There are always some states showing a single peak in the curve of density function and the shape parameter changes about 1.0.

From the above analysis, we can think that the earthquake intervals obey Weibull distribution. We can discuss the changes of b and its laws when the shape parameter is about 1.0.

PREDICTION

We analyze the earthquake data in China earthquake catalog from Jan.1, 1973 to Nov.30, 2003 with the magnitude lower limit of 4.0. The parameters and b values are given in TABLE 2. We contrast the results with the data from Jan.1, 1973 to June 30, 2010.

TABLE 2 : The contrast analysis of China Earthquake catalog

the areas	the estimated parameters (1973-2010) (t=456m)	b(the magnitude lower limit of 4.0)	b(the magnitude lower limit 5.0)	the estimated parameters (1973-2005) (t=396m)	b(the magnitude lower limit 4.0)	b(the magnitude lower limit 5.0)
Taiwan areas	$\hat{k}$=1.011 $\hat{b}$=115.55	b=0.99	b=1.01	$\hat{k}$=1.009 $\hat{b}$=100.47	b=0.98	b=1.02
Kunlun-Qlianshan-Qinling areas	$\hat{k}$=1.233 $\hat{b}$=136.65	b=0.87	b=0.82	$\hat{k}$=1.212 $\hat{b}$=118.80	b=0.85	b=0.80
Tibet and Xinjiang areas	$\hat{k}$=1.172 $\hat{b}$=140.63	b=0.99	b=0.91	$\hat{k}$=1.169 $\hat{b}$=122.28	b=0.99	b=1.00
Tianshan areas	$\hat{k}$=1.077 $\hat{b}$=135.52	b=0.98	b=0.98	$\hat{k}$=1.056 $\hat{b}$=117.84	b=0.96	b=0.94
Chuandian areas	$\hat{k}$=1.125 $\hat{b}$=130.19	b=0.90	b=0.87	$\hat{k}$=1.092 $\hat{b}$=113.20	b=0.87	b=0.89
Erdos-Qilianshan-Qinling areas	$\hat{k}$=1.109 $\hat{b}$=126.65	b=1.18	b=1.18	$\hat{k}$=1.088 $\hat{b}$=110.13	b=1.22	b=1.86
North of China	$\hat{k}$=1.299 $\hat{b}$=160.08	b=0.68	b=0.50	$\hat{k}$=1.287 $\hat{b}$=139.20	b=0.67	b=0.57
The total of the eight areas	$\hat{k}$=1.149 $\hat{b}$=135.55	b=0.95	b=0.93	$\hat{k}$=1.117 $\hat{b}$=117.87	b=0.92	b=0.92
Mainland and Taiwan areas	$\hat{k}$=1.009 $\hat{b}$=107.26	b=0.99	b=0.99	$\hat{k}$=1.001 $\hat{b}$=93.27	b=0.98	b=1.03

From TABLE 2 we can see that b values from 1973 to 2005 are lower than those from 1973 to 2010 (except Erdos-Qilianshan-Qinling) and the shape parameters correspondingly deduce.

CONCLUSIONS

We contrast the data in China earthquake catalog according to different time intervals and areas. From the results we can see that the shape parameters increase with the years increased (only with the magnitude lower limit of 4.0). The conclusions are correct or not needs to be proven with a lot of calculations and facts. We will develop this part in the near future.

During the process of the hypothesis testing, we equally divide the time intervals for the convenience of calculation which are considerable discrepancy between the fault intervals of the engineering. So we think it is necessary to divide the earthquake time randomly and then carry out the hypothesis testing of Weibull distribution. Thus the conclusions will own actual meaning and implications of science.

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