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Research and improvement for one-dimensional optimization algorithm of structures non-probabilistic reliability

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ABSTRACT

The limit state equation of engineering structure will be divided into safety and failure of structure state. Currently one-dimensional optimization method of reliability analysis and geometric significance are analyzed. Based on structures non-probabilistic reliability, combining cubic interpolation function the limit state surface is fitted best, narrowing the range of structures reliability index, then in the standard space, the shortest distance of the structure reliability analysis is calculated from the origin to the fitted failure surface. The numerical example is given, illustrating the effectiveness and the feasibility of the proposed algorithm.

KEYWORDS

Non-probabilistic reliability index; limit State equation; One-dimensional optimization algorithm; Geometric method; Cubic interpolation function.

INTRODUCTION

When a structure is complicated, generally the variant parameters are not certain, but their boundary can be known. At this time, in essence, the non-probability reliability index[1] is the minimal norm of the coordinate vectors in the standardized space and its solving process is an equality constrained optimization problem. The reference[2] proposes the concept of the non-probability reliability based on the convex set theory. When the functionality function is not complicated, Guo Shuxiang[3] recommends to use definition method, conversion method and optimization method, which is only for special case and is an approximation method. Zhang Jianguo[1] realizes optimization by using quadratic programming method of matlab min and max function series. Jiang Tao[4] thinks that the interval variant reliability index at a crossing between the limited super rays and standardized limit state curve passing the original point and vertex of the convex domain in the convex formed by expanding vector in the standardized interval and its extension space, so he proposes the one-dimensional optimization method to solve non-probability reliability index. This method can better compute non-probability reliability index, but the equation computing workload is heavy with growth of variants. Chen Xuyong[5] proposes the limit state surface should intersect with the i-dimensional coordinate axis because the computing workload grows in a geometric progression scale with the dimension n of the interval vector in one-dimensional optimization algorithm and the value scope of the interval variant is locally reduces. This method reduces the solution number of the equation and optimizes one-dimensional algorithm for the explicit limit state equation. The reference[6] proposes the non-probability response surface method by combining the traditional response surface method and improved one-dimensional optimization algorithm and compares the computing result with the results of the traditional response surface method and parsing method. Jiang Chong[7] introduces the improved interval truncation method and computes the non-probability reliability index for the abutment foundation stabilization by using the improved one-dimensional optimization algorithm. From the analysis results, the researchers propose some methods to improve the capability of searching feasible solutions according to the specific conditions to reduce search scope. For the complicated functionality function, the values of the one-dimensional variants of the standardized functionality function are within the limitless-dimension space. It is difficult to solve the function equations. It is the key to find one extensively used, quick and effective method for solving the structure reliability index based on it. This paper proposes one structure reliability analysis method based on three-point cubic interpolation and iteration. This method can quickly find the optimal value in a convergence manner.

DEFINITION OF STRUCTURE RELIABILITY INDEX

Given that $X = (X_1, X_2, \cdots, X_n)^T$ ($i \in n$) is n basic random variants affecting the structure function. X can be geometric size of the structure, physical dynamics parameter of material and force on structure. The random function

$$Z = g(X) = g(X_1, X_2, \cdots, X_n) \quad \textbf{(1)}$$

is the structure functionality function (or invalidity function). $Z > 0$ indicates that the structure is under reliable state, $Z < 0$ indicates that the structure is under invalid state, and $Z = 0$ indicates that the structure is under the limit state. X_i is standardized in the standard space, namely

$$X_i = X_i^c(\alpha) + X_i^r(\alpha)\delta_i \quad \textbf{(2)}$$

Based on the structure interval non-probability reliability theory[8], the reliability index of the equation (1) functionality function is:

$$\beta = \min(\| \delta_i \|_\infty) = \min\{\max(|\delta_1|, |\delta_2|, \cdots, |\delta_n|)\} \quad \textbf{(3)}$$

It meets the condition:

$$Z = g(X) = g(\alpha, \delta) = 0 \quad \textbf{(4)}$$

From the geometry, β of the equation (3) is the shortest distance from the origin of the coordinate to the limit state curved surface measured by the infinite norm $\| \bullet \|_\infty$ within the extension space of the standardization interval variant.

IMPROVED ONE-DIMENSIONAL OPTIMIZATION ANALTSIS METHOD

Based on the above definition, the structure non-probability index computing can be converted to root solution of several single-variant algebraic equations, show as the following equation:

$$\begin{cases} \min\{\max(|\delta_1|,|\delta_2|,\cdots,|\delta_n|)\} \\ g(\alpha,\delta)=0 \\ -\infty<\delta_i<+\infty \end{cases} \quad \textbf{(5)}$$

If the parameters and variants of the functionality function are added, the computing burden for one-dimensional optimization is very huge. Some optimization methods such as space intelligent are easy to fall into the local minimal solution or the final extreme results can not converge due to improper selection method of initial points, so the feasible domain of the non-probability reliability index should reduce.

Method improvement and related proofs

Given that three-point cubic interpolation function is $f(\delta)$, we can get:

$$f(\delta)=f(\delta_{k_1})+c(\delta-\delta_{k_2})+b(\delta-\delta_{k_1})(\delta-\delta_{k_2})+a(\delta-\delta_{k_1})(\delta-\delta_{k_2})(\delta-\delta_{k_3}) \quad \textbf{(6)}$$

wherein $c=f(\delta_{k_1},\delta_{k_2})=\dfrac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}$, $b=f(\delta_{k_1},\delta_{k_2},\delta_{k_3})=\dfrac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}$ 。

$$\begin{aligned} f'(\delta)&=c+b(2\delta-\delta_{k_1}-\delta_{k_2})+a(\delta-\delta_{k_2})(\delta-\delta_{k_3}) \\ &+a(\delta-\delta_{k_1})\frac{d}{dx}[(\delta-\delta_{k_2})(\delta-\delta_{k_3})] \end{aligned} \quad \textbf{(7)}$$

given $\delta=\delta_{k_1}$,

$$f'(\delta_{k_1})=c+b(\delta_{k_1}-\delta_{k_2})+a(\delta_{k_1}-\delta_{k_2})(\delta_{k_1}-\delta_{k_3}) \quad \textbf{(8)}$$

We can get:

$$a=\frac{f'(\delta_{k_1})-c-(\delta_{k_1}-\delta_{k_2})b}{(\delta_{k_1}-\delta_{k_2})(\delta_{k_1}-\delta_{k_3})} \quad \textbf{(9)}$$

To substitute the coefficient a, b and c into the interpolation function $f(\delta)$, we can get:

$$\begin{aligned}
f(\delta)&=f(\delta_{k_1})+\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}(\delta-\delta_{k_2})+\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}(\delta-\delta_{k_1})(\delta-\delta_{k_2}) \\
&+\frac{f'(\delta_{k_1})-\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}-(\delta_{k_1}-\delta_{k_2})\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}}{(\delta_{k_1}-\delta_{k_2})(\delta_{k_1}-\delta_{k_3})}(\delta-\delta_{k_1})(\delta-\delta_{k_2})(\delta-\delta_{k_3}) \\
&=\{\frac{f'(\delta_{k_1})-\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}-(\delta_{k_1}-\delta_{k_2})\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}}{(\delta_{k_1}-\delta_{k_2})(\delta_{k_1}-\delta_{k_3})}\}\delta^3 \\
&+\{\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}-(\delta_{k_1}+\delta_{k_2}+\delta_{k_3})\frac{f'(\delta_{k_1})-\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}-(\delta_{k_1}-\delta_{k_2})\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}}{(\delta_{k_1}-\delta_{k_2})(\delta_{k_1}-\delta_{k_3})}\}\delta^2 \\
&+\{\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}-\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}(\delta_{k_1}+\delta_{k_2})+ \\
&+\frac{f'(\delta_{k_1})-\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}-(\delta_{k_1}-\delta_{k_2})\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}}{(\delta_{k_1}-\delta_{k_2})(\delta_{k_1}-\delta_{k_3})}(\delta_{k_1}\delta_{k_2}+\delta_{k_2}\delta_{k_3}+\delta_{k_1}\delta_{k_3})\}\delta \\
&+f(\delta_{k_1})-\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}\delta_{k_2}+\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}\delta_{k_1}\delta_{k_2} \\
&-\delta_{k_1}\delta_{k_2}\delta_{k_3}\frac{f'(\delta_{k_1})-\frac{f(\delta_{k_2})-f(\delta_{k_1})}{\delta_{k_2}-\delta_{k_1}}-(\delta_{k_1}-\delta_{k_2})\frac{f(\delta_{k_2},\delta_{k_3})-f(\delta_{k_1},\delta_{k_2})}{\delta_{k_3}-\delta_{k_1}}}{(\delta_{k_1}-\delta_{k_2})(\delta_{k_1}-\delta_{k_3})}
\end{aligned} \quad \textbf{(10)}$$

Three independent points can identify one surface. The extreme state curved surface of the original functionality function can be fitted in the standard coordinate system by using three-point cubic interpolation function. The shortest distance from the extreme state curved surface of the original functionality function to the origin is equivalent to the distance from the origin to interpolation fitting surface in the standard coordinate system and can be expressed as the shortest distance from the origin to the new interpolation extreme state curved surface $f(\delta)$ in the standardization space according to the structure non-probability reliability index β, shown as the Figure 1. β is expressed as:

$$\beta = \frac{|D|}{\sqrt{A^2 + B^2 + C^2}} \tag{11}$$

Wherein:

$$A = \frac{f'(\delta_{k_1}) - \dfrac{f(\delta_{k_2}) - f(\delta_{k_1})}{\delta_{k_2} - \delta_{k_1}} - (\delta_{k_1} - \delta_{k_2})\dfrac{f(\delta_{k_2}, \delta_{k_3}) - f(\delta_{k_1}, \delta_{k_2})}{\delta_{k_3} - \delta_{k_1}}}{(\delta_{k_1} - \delta_{k_2})(\delta_{k_1} - \delta_{k_3})},$$

$$B = \frac{f(\delta_{k_2}, \delta_{k_3}) - f(\delta_{k_1}, \delta_{k_2})}{\delta_{k_3} - \delta_{k_1}} - (\delta_{k_1} + \delta_{k_2} + \delta_{k_3}) \frac{f'(\delta_{k_1}) - \dfrac{f(\delta_{k_2}) - f(\delta_{k_1})}{\delta_{k_2} - \delta_{k_1}} - (\delta_{k_1} - \delta_{k_2})\dfrac{f(\delta_{k_2}, \delta_{k_3}) - f(\delta_{k_1}, \delta_{k_2})}{\delta_{k_3} - \delta_{k_1}}}{(\delta_{k_1} - \delta_{k_2})(\delta_{k_1} - \delta_{k_3})},$$

$$C = \frac{f(\delta_{k_2}) - f(\delta_{k_1})}{\delta_{k_2} - \delta_{k_1}} - \frac{f(\delta_{k_2}, \delta_{k_3}) - f(\delta_{k_1}, \delta_{k_2})}{\delta_{k_3} - \delta_{k_1}}(\delta_{k_1} + \delta_{k_2}) + \frac{f'(\delta_{k_1}) - \dfrac{f(\delta_{k_2}) - f(\delta_{k_1})}{\delta_{k_2} - \delta_{k_1}} - (\delta_{k_1} - \delta_{k_2})\dfrac{f(\delta_{k_2}, \delta_{k_3}) - f(\delta_{k_1}, \delta_{k_2})}{\delta_{k_3} - \delta_{k_1}}}{(\delta_{k_1} - \delta_{k_2})(\delta_{k_1} - \delta_{k_3})}(\delta_{k_1}\delta_{k_2} + \delta_{k_2}\delta_{k_3} + \delta_{k_1}\delta_{k_3}),$$

$$D = f(\delta_{k_1}) - \frac{f(\delta_{k_2}) - f(\delta_{k_1})}{\delta_{k_2} - \delta_{k_1}}\delta_{k_2} + \frac{f(\delta_{k_2}, \delta_{k_3}) - f(\delta_{k_1}, \delta_{k_2})}{\delta_{k_3} - \delta_{k_1}}\delta_{k_1}\delta_{k_2} - \delta_{k_1}\delta_{k_2}\delta_{k_3}\frac{f'(\delta_{k_1}) - \dfrac{f(\delta_{k_2}) - f(\delta_{k_1})}{\delta_{k_2} - \delta_{k_1}} - (\delta_{k_1} - \delta_{k_2})\dfrac{f(\delta_{k_2}, \delta_{k_3}) - f(\delta_{k_1}, \delta_{k_2})}{\delta_{k_3} - \delta_{k_1}}}{(\delta_{k_1} - \delta_{k_2})(\delta_{k_1} - \delta_{k_3})},$$

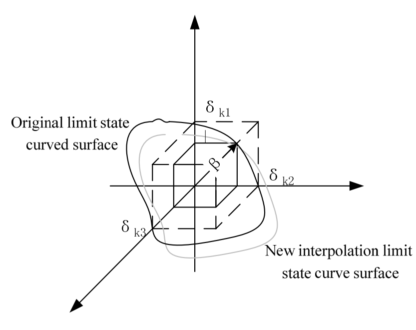

Figure 1 : Structures non-probabilistic reliability β

From the above computing equation, we can know that the specific value of the structure reliability index β can be identified by computing the value of the function $f(\delta_{k_1})$, $f(\delta_{k_2})$, $f(\delta_{k_3})$ and $f'(\delta_{k_1})$. Existing references propose multiple improved methods, but several single-variant equations for δ should be solved. In standardization of the parameter variants,

$$\delta_i = \frac{X_i - X_i^c(\alpha)}{X_i^r(\alpha)} \tag{12}$$

It is easy to find that the computed δ_i reaches the minimum when $|X_i^c(\alpha)|$ reaches the maximum. Based on the functional analysis, the shortest distance from the origin to the limit state curved surface of the standardization functionality function must be met in the reference[4]. The super ray from the origin to the limit state curved surface vertex of the functionality function interacts with each other at the limit state curved surface of the standardization space functionality function. If the partial derivative of the standardization functionality function to δ_i is positive or negative simultaneously, it indicates that the vector $\overrightarrow{\delta_i}$ is in one gradient direction and its value is positive. On the contrary, the values have reverse symbols.

Computing steps of improved methods

Based on the theory proof and analysis, the computing steps of the improved one-dimensional optimization algorithm are described as follows:

Analyze the structure computing parameter variant $X_i = X_i^c(\alpha) + X_i^r(\alpha)\delta_i$

Get $\max|X_i^c(\alpha)|$ and compute $g(0,\cdots,\delta_i,\cdots,0) = 0$.

3) The interaction of limit state curved surface and coordinate axis should meet $g(0,\cdots,\delta_j,\cdots,0) = 0$ in the limitless-dimension space.

4) $\delta_{k_2} = \min(|\delta_j|)$, if $\frac{\partial Z}{\partial \delta_i} > 0$ and $\frac{\partial Z}{\partial \delta_j} > 0$, we can know that the structure reliability index $\delta_{k_1} = |\delta_i|$. If $\frac{\partial Z}{\partial \delta_i} < 0$, $\frac{\partial Z}{\partial \delta_j} < 0$ indicates that the structure reliability index $\delta_{k_1} = |\delta_i|$. If $\frac{\partial Z}{\partial \delta_i} > 0$ and $\frac{\partial Z}{\partial \delta_j} < 0$, the structure reliability index $\delta_{k_2} = -|\delta_i|$. If $\frac{\partial Z}{\partial \delta_i} < 0$ and $\frac{\partial Z}{\partial \delta_j} > 0$, the structure reliability index $\delta_{k_2} = -|\delta_i|$ and the value of $g(\delta_{k_1})$, $g(\delta_{k_2})$ and $g'(\delta_{k_1})$ are computed.

Based on the value scope $[-\delta_{k_1}, \delta_{k_1}]$ and $[-\delta_{k_2}, \delta_{k_2}]$ of the structure reliability index, take any $\delta_{k_3} \in [-\delta_{k_1}, \delta_{k_1}] \cap [-\delta_{k_2}, \delta_{k_2}]$, meet the condition $g(0,\cdots,\delta_{k_3},\cdots,0) = 0$ and compute the value of $g(\delta_i)$.

Substitute the values of δ_{k_1}, δ_{k_2}, δ_{k_3} and the corresponding $g(\delta_i)$ and $g'(\delta_i)$ into the equation (10) and compute the interpolation curved surface $f(\delta)$.

7) Substitute $f(\delta_i) i = 1,2,3$, compute the equation (11) and get the structure reliability index β.

EXAMPLE AND ANALTSIS

E.g. shown as the Figure 2, for the five-bar plane truss under the external force, the limit state function under the invalidity mode is $M = 3R_1 + 2R_2 + 2R_3 + \frac{2}{\sqrt{2}}R_4 + \frac{1}{\sqrt{2}}R_5 - 5S$ and the value interval of the resistance and load parameter R_1, R_2, R_3, R_4, R_5 and S are $[10,30]$, $[15,35]$, $[20,50]$, $[30,50]$, $[20,40]$ and $[5,45]$ respectively.

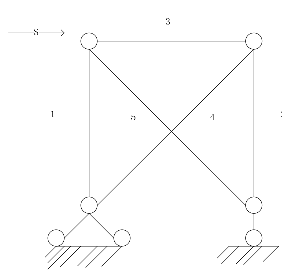

Figure 2 : Five-bar plane truss

Based on the computing method of the existing reference[5], the interval variants are standardized as $R_1 = 20 + 10\delta_{R_1}$, $R_2 = 25 + 10\delta_{R_2}$, $R_3 = 35 + 15\delta_{R_3}$, $R_4 = 40 + 10\delta_{R_4}$, $R_5 = 30 + 10\delta_{R_5}$ and $S = 25 + 20\delta_S$. The following limit state functions can be established:

$$M = 55 + \frac{110}{\sqrt{2}} + 30\delta_{R_1} + 20\delta_{R_2} + 30\delta_{R_3} + \frac{20}{\sqrt{2}}\delta_{R_4} + \frac{10}{\sqrt{2}}\delta_{R_5} - 100\delta_S = 0$$

Next determine the partial derivative of the limit state function to variants:

$$\frac{\partial M}{\partial \delta_{R_1}} > 0, \frac{\partial M}{\partial \delta_{R_2}} > 0, \frac{\partial M}{\partial \delta_{R_3}} > 0, \frac{\partial M}{\partial \delta_{R_4}} > 0, \frac{\partial M}{\partial \delta_{R_5}} > 0, \frac{\partial M}{\partial \delta_S} < 0$$

Based on $|X_i^c(\alpha)|$ size, the value range of the structure reliability index is determined as $[-9.3885, 9.3885]$:

Finally compute multiple single-variant equations:

$$55 + \frac{110}{\sqrt{2}} + 30\delta + 20\delta + 30\delta = 0$$

$$55 + \frac{110}{\sqrt{2}} - 30\delta - 20\delta - 30\delta = 0$$

Get $\beta = 1.66$

Based on the improved computing method in this paper,

Make $\delta_{k_1} = \delta_{R_4} = 83.28$, $\delta_{k_2} = \delta_S = 1.328$, $\delta_{k_3} = 1.327$, $M_{\delta_{k1}} = 1310.73$, $M_{\delta_{k2}} = 0$, $M_{\delta_{k1}} = 0$,

$$\begin{aligned} f(\delta) &= 1310.73 + 15.994(\delta - 1.328) + 17.614(\delta - 83.28)(\delta - 1.328) \\ &+ 0.21(\delta - 83.28)(\delta - 1.328)(\delta - 1.327) \\ &= 0.21\delta^3 - 0.43\delta^2 - 1521.09\delta + 3268.34 \end{aligned}, \quad \beta = 2.15.$$

To compare two methods, this paper solves one single-variant three-order fitting equation instead to several multiple single-variant equation to identify the reliability index value. This method reduces the computing time from $O(n)$ to $O(1)$, improves the computing efficiency of non-probability reliability index, increases the value of the reliability index, and approaches to the standard value in structure design much.

CONCLUSIONS

Generally the limit state function indicates to design the highly non-linear inexplicit function of the variants in the structure reliability analysis. For a large complicated structure, the structure analysis is a complicated process. The structure is analyzed several times in the structure reliability analysis process. The computing burden is heavy for a general structure reliability analysis method. It is very meaningful for actual engineer application of the structure reliability index computing to find a structure reliability analysis method meeting the engineering precision requirement and easy to compute. This paper gradually reduces the value scope of the index value, avoids solution of many single-variant equations based on one-dimensional optimization algorithm of non-probability reliability index. The interpolation one-dimensional optimization method is more precise than old method and can reduce the complexity compared to the intelligent optimization method.

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