



Nano Science and Nano Technology

An Indian Journal

Full Paper

NSNTAJ, 10(1), 2016 [013-029]

On approach of optimization of manufacturing of a heterotransistors with two gates to decrease their dimensions

E.L.Pankratov^{1*}, E.A.Bulaeva^{1,2}

¹Nizhny Novgorod State University, 23 Gagarin avenue, Nizhny Novgorod, 603950, (RUSSIA)

²Nizhny Novgorod State University of Architecture and Civil Engineering, 65 Il'insky street, Nizhny Novgorod, 603950, (RUSSIA)

E-mail: elp2004@mail.ru

ABSTRACT

In this paper we introduce an approach to manufacture a field-effect heterotransistor with two gates. Framework the approach we consider a heterostructure with required configuration, doping of required parts of the heterostructure by diffusion and/or ion implantation and optimization of annealing of dopant or radiation defects. The introduced approach of manufacturing of transistor gives us possibility to decrease area of surface and thickness of the transistor. In this paper we also introduce an approach to make prognosis of mass and heat transport with account variation of parameters of these processes in space in time and nonlinearity of these processes.

© 2016 Trade Science Inc. - INDIA

INTRODUCTION

Development of solid state electronic (increasing of integration rate of elements of integrated circuits, their reliability and frequency characteristics) leads to elaboration new and optimization existing technological processes^[1-8]. During elaboration of new and optimization of existing technological processes it is attracted an interest modeling of appropriate physical processes. The modeling gives us possibility to make prognosis of the processes and characteristics of elaborating devices. Using mathematical modeling leads to necessity to development approaches of solving of adequate models.

In this paper we consider a heterostructure, which consist of a substrate and an epitaxial layer. The epitaxial layer includes into itself several sections manufactured by using another materials (see Figure 1). These sections have been doped by diffu-

sion or ion implantation to produce required type of conductivity (p or n) during manufacture the required transistors. Farther we consider annealing of dopant and/or radiation defects. Main aim of the present paper is analysis of dynamics of redistribution of dopant with account redistribution of radiation defects to determine conditions, which corresponds to decreasing of dimensions of transistors. The accompanying aim of the present paper is development of approaches of modeling of technological processes accounting required quantity of influencing factors.

Method of solution

To solve our aim we determine spatio-temporal distributions of concentrations of dopants. We calculate the required distributions by solving the second Fick's law in the following form^[9,10]

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D_c \frac{\partial C(x, y, z, t)}{\partial x} \right] +$$

Full Paper

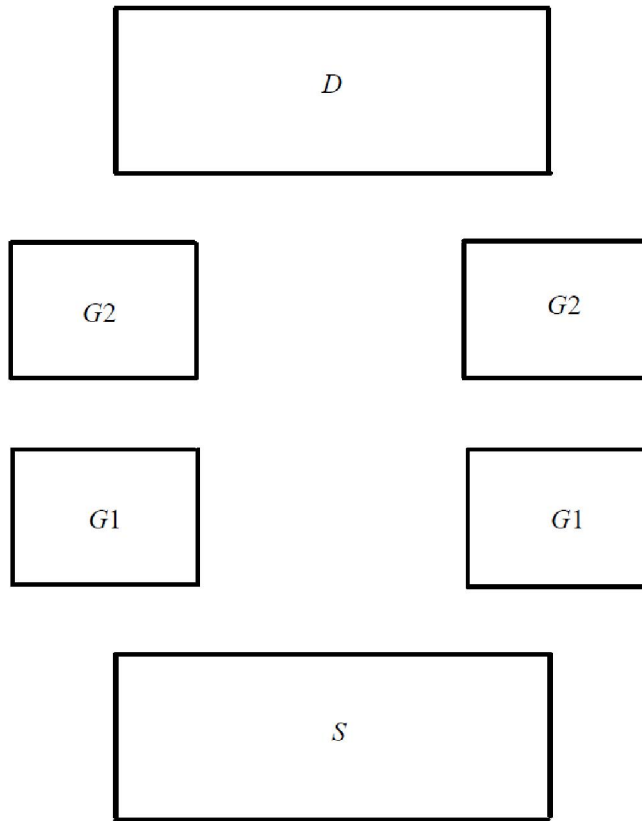


Figure 1 : Structure of epitaxial layer.View from top

$$+ \frac{\partial}{\partial y} \left[D_c \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_c \frac{\partial C(x, y, z, t)}{\partial z} \right] \tag{1}$$

with boundary and initial conditions

$$\begin{aligned} \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad C(x, y, z, 0) = f(x, y, z). \end{aligned} \tag{2}$$

Here $C(x,y,z,t)$ is the spatio-temporal distribution of concentration of dopant; T is the temperature of annealing; D_c is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on properties of materials of heterostructure, speed of heating and cooling of heterostructure (with account Arrhenius law). Dependences of dopant diffusion coefficient on parameters could be approximated by the following relation^[11-13]

$$D_c = D_L(x, y, z, T) \left[1 + \xi \frac{C^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \right] \left[1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right], \tag{3}$$

where $D_L(x,y,z,T)$ is the spatial (due to presents several layers in heterostructure) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficient; $P(x, y, z, T)$ is the limit of solubility of dopant; parameter γ depends on properties of materials and could be integer in the following interval γ

$\in [1,3]^{[11]}$; $V(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation vacancies; V^* is the equilibrium distribution of concentration of vacancies. Concentrational dependence of dopant diffusion coefficient has been described in details in^[11]. It should be noted, that using diffusion type of doping did not generation radiation defects. In this situation $\zeta_1 = \zeta_2 = 0$. We determine spatio-temporal distributions of concentrations of radiation defects by solving the following system of equations^[12,13]

$$\begin{aligned} \frac{\partial I(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial y} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial z} \right] - k_{I,V}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) - \\ &- k_{I,I}(x,y,z,T) I^2(x,y,z,t) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial V(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial y} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial z} \right] - k_{I,V}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) - \\ &- k_{V,V}(x,y,z,T) V^2(x,y,z,t) \end{aligned}$$

with boundary and initial conditions

$$\begin{aligned} \left. \frac{\partial \rho(x,y,z,t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \rho(x,y,z,t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \rho(x,y,z,t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \rho(x,y,z,t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \rho(x,y,z,t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \rho(x,y,z,t)}{\partial z} \right|_{z=L_z} = 0, \quad \rho(x,y,z,0) = f_p(x,y,z). \end{aligned} \quad (5)$$

Here $\rho = I, V$; $I(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation interstitials; $D_p(x,y,z,T)$ are the diffusion coefficients of point radiation defects; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation divacancies and diinterstitials; $k_{I,V}(x,y,z,T)$ is the parameter of recombination of point radiation defects; $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ are the parameters of generation of simplest complexes of point radiation defects.

We determine spatio-temporal distributions of concentrations of divacancies $\Phi_V(x,y,z,t)$ and dinterstitials $\Phi_I(x,y,z,t)$ by solving the following system of equations^[12,13]

$$\begin{aligned} \frac{\partial \Phi_I(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial y} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial z} \right] + k_{I,I}(x,y,z,T) I^2(x,y,z,t) - k_I(x,y,z,T) I(x,y,z,t) \end{aligned} \quad (6)$$

Full Paper

$$\frac{\partial \Phi_v(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D_{\Phi_v}(x, y, z, T) \frac{\partial \Phi_v(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_v}(x, y, z, T) \frac{\partial \Phi_v(x, y, z, t)}{\partial y} \right] +$$

$$+ \frac{\partial}{\partial z} \left[D_{\Phi_v}(x, y, z, T) \frac{\partial \Phi_v(x, y, z, t)}{\partial z} \right] + k_{v,v}(x, y, z, T) V^2(x, y, z, t) - k_v(x, y, z, T) V(x, y, z, t)$$

with boundary and initial conditions

$$\left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0,$$

$$\left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0,$$

$$\Phi_I(x, y, z, 0) = f_{\Phi_I}(x, y, z), \quad \Phi_v(x, y, z, 0) = f_{\Phi_v}(x, y, z). \quad (7)$$

Here $D_{\Phi_p}(x, y, z, T)$ are the diffusion coefficients of the above complexes of radiation defects; $k_I(x, y, z, T)$ and $k_v(x, y, z, T)$ are the parameters of decay of these complexes.

It should be noted, that nonlinear equations with space and time varying coefficients are usually used to describe physical processes. Although the equations are usually solved in different limiting cases^[14-17]. One way to solve the problem is solving the Eqs. (1), (4), (6) by the Bubnov-Galerkin approach^[18] after appropriate transformation of these transformation. To determine the spatio-temporal distribution of concentration of dopant we transform the Eq.(1) to the following integro- differential form

$$\frac{x y z}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z C(u, v, w, t) d w d v d u = \frac{y z}{L_y L_z} \int_0^t \int_{L_y}^y \int_{L_z}^z \left[1 + \varsigma_1 \frac{V(x, v, w, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, v, w, \tau)}{(V^*)^2} \right] \times$$

$$\times D_L(x, v, w, T) \left[1 + \xi \frac{C^\gamma(x, v, w, \tau)}{P^\gamma(x, v, w, T)} \right] \frac{\partial C(x, v, w, \tau)}{\partial x} d \tau + \frac{x z}{L_x L_z} \int_0^t \int_{L_x}^x \int_{L_z}^z D_L(u, y, w, T) \times$$

$$\times \left[1 + \varsigma_1 \frac{V(u, y, w, \tau)}{V^*} + \varsigma_2 \frac{V^2(u, y, w, \tau)}{(V^*)^2} \right] \left[1 + \xi \frac{C^\gamma(u, y, w, \tau)}{P^\gamma(x, y, z, T)} \right] \frac{\partial C(u, y, w, \tau)}{\partial y} d \tau +$$

$$+ \frac{x y}{L_x L_y} \int_0^t \int_{L_x}^x \int_{L_y}^y D_L(u, v, z, T) \left[1 + \varsigma_1 \frac{V(u, v, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(u, v, z, \tau)}{(V^*)^2} \right] \left[1 + \xi \frac{C^\gamma(u, v, z, \tau)}{P^\gamma(x, y, z, T)} \right] \times$$

$$\times \frac{\partial C(u, v, z, \tau)}{\partial z} d \tau + \frac{x y z}{L_x L_y L_z} \int_{L_x}^x \int_{L_y}^y \int_{L_z}^z f(u, v, w) d w d v d u. \quad (1a)$$

We determine solution of the above equation by using Bubnov-Galerkin approach^[18]. Framework the approach we determine solution of the Eq.(1a) as the following series

$$C_0(x, y, z, t) = \sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t),$$

where $e_{nC}(t) = \exp\left[-\pi^2 n^2 D_{0C} t \left(L_x^{-2} + L_y^{-2} + L_z^{-2}\right)\right]$, $c_n(\chi) = \cos(\pi n \chi/L_\chi)$. The above series includes into itself finite number of terms N. The considered series is similar with solution of linear Eq.(1) (i.e. with $\xi = 0$) and averaged dopant diffusion coefficient D_0 . Substitution of the series into Eq.(1a) leads to the following result

$$\begin{aligned} & \frac{x y z}{\pi^2} \sum_{n=1}^N \frac{a_{nC}}{n^3} s_n(x) s_n(y) s_n(z) e_{nC}(t) = -\frac{y z}{L_y L_z} \int_0^t \int_0^y \int_0^z \left\{ 1 + \left[\sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(w) e_{nC}(\tau) \right]^\gamma \times \right. \\ & \left. \times \frac{\xi}{P^\gamma(x, y, w, T)} \right\} \left[1 + \zeta_1 \frac{V(x, y, w, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, w, \tau)}{(V^*)^2} \right] D_L(x, y, w, T) \sum_{n=1}^N a_{nC} s_n(x) c_n(y) \times \\ & \times n c_n(w) e_{nC}(\tau) d\tau - \frac{x z}{L_x L_z} \int_0^t \int_0^x \int_0^z \left\{ 1 + \left[\sum_{m=1}^N a_{mC} c_m(u) c_m(y) c_m(w) e_{mC}(\tau) \right]^\gamma \frac{\xi}{P^\gamma(u, y, w, T)} \right\} \times \\ & \times D_L(u, y, w, T) \left[1 + \zeta_1 \frac{V(u, y, w, \tau)}{V^*} + \zeta_2 \frac{V^2(u, y, w, \tau)}{(V^*)^2} \right] \sum_{n=1}^N n c_n(u) s_n(y) c_n(w) e_{nC}(\tau) d\tau \times \\ & \times a_{nC} - \frac{x y}{L_x L_y} \int_0^t \int_0^x \int_0^y D_L(u, v, z, T) \left\{ 1 + \frac{\xi}{P^\gamma(u, v, z, T)} \left[\sum_{n=1}^N a_{nC} c_n(u) c_n(v) c_n(z) e_{nC}(\tau) \right]^\gamma \right\} \times \\ & \times \left[1 + \zeta_1 \frac{V(u, v, z, \tau)}{V^*} + \zeta_2 \frac{V^2(u, v, z, \tau)}{(V^*)^2} \right] \sum_{n=1}^N n a_{nC} c_n(u) c_n(v) s_n(z) e_{nC}(\tau) d\tau + \frac{x y z}{L_x L_y L_z} \times \\ & \times \int_{L_x L_y L_z}^{x y z} f(u, v, w) d w d v d u, \end{aligned}$$

where $s_n(\chi) = \sin(\pi n \chi/L_\chi)$. We determine coefficients a_n by using orthogonality condition of terms of the considered series framework scale of heterostructure. The condition gives us possibility to obtain relations for calculation of parameters a_n for any quantity of terms N. In the common case the relations could be written as

$$\begin{aligned} & -\frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{nC}}{n^6} e_{nC}(t) = -\frac{L_y L_z}{2 \pi^2} \int_0^t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} D_L(x, y, z, T) \left\{ 1 + \left[\sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(\tau) \right]^\gamma \times \right. \\ & \left. \times \frac{\xi}{P^\gamma(x, y, z, T)} \right\} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \sum_{n=1}^N \frac{a_{nC}}{n} s_n(2x) c_n(y) c_n(z) e_{nC}(\tau) \times \\ & \times \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} d z d y d x d \tau - \int_0^t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} D_L(x, y, z, T) \times \end{aligned}$$

Full Paper

$$\begin{aligned}
 & \times D_L(x, y, z, T) \left\{ 1 + \left[\sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(\tau) \right]^\gamma \frac{\xi}{P^\gamma(x, y, z, T)} \right\} \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \right. \\
 & \left. + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \sum_{n=1}^N \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \frac{a_{nC}}{n} \times \\
 & \times \frac{L_x L_z}{2\pi^2} c_n(x) s_n(2y) c_n(z) e_{nC}(\tau) \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} d z d y d x d \tau - \frac{L_x L_y}{2\pi^2} \times \\
 & \times \int_0^t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left\{ 1 + \left[\sum_{n=1}^N a_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(\tau) \right]^\gamma \frac{\xi}{P^\gamma(x, y, z, T)} \right\} \left[1 + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} + \right. \\
 & \left. + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} \right] D_L(x, y, z, T) \sum_{n=1}^N \frac{a_{nC}}{n} c_n(x) c_n(y) s_n(z) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \times \\
 & \times \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} e_{nC}(\tau) d z d y d x d \tau + \sum_{n=1}^N \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \times \\
 & \times \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} f(x, y, z) d z d y d x .
 \end{aligned}$$

As an example for $\gamma = 0$ we obtain

$$\begin{aligned}
 a_{nC} &= \int_0^{L_x} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} f(x, y, z) d z d y \left\{ x s_n(x) + \right. \\
 & \times \left. [c_n(x) - 1] \frac{L_x}{\pi n} \right\} d x \left(\frac{n}{2} \int_0^t \int_0^{L_x} s_n(2x) \int_0^{L_y} c_n(y) \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} D_L(x, y, z, T) \times \right. \\
 & \times \left. \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left[1 + \frac{\xi}{P^\gamma(x, y, z, T)} \right] \times \right. \\
 & \times \left. c_n(z) d z d y d x e_{nC}(\tau) d \tau + \int_0^t e_{nC}(\tau) \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} s_n(2y) \int_0^{L_z} c_n(z) \times \right. \\
 & \times \left. \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} \left[1 + \frac{\xi}{P^\gamma(x, y, z, T)} \right] \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times
 \end{aligned}$$

$$\begin{aligned} & \times D_L(x, y, z, T) dz dy dx d\tau + \int_0^t e_{nC}(\tau) \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} c_n(y) \{ s_n(y) \times \\ & \times y + \frac{L_y}{\pi n} [c_n(y) - 1] \} \int_0^{L_z} s_n(2z) D_L(x, y, z, T) \left[1 + \frac{\xi}{P^\gamma(x, y, z, T)} \right] \left[1 + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} + \right. \\ & \left. + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} \right] dz dy dx d\tau \left\{ - \frac{L_x^2 L_y^2 L_z^2}{\pi^5 n^6} e_{nC}(t) \right\}^{-1}. \end{aligned}$$

For $\gamma = 1$ one can obtain the following relation to determine required parameters

$$a_{nC} = - \frac{\beta_n}{2\alpha_n} \pm \sqrt{\beta_n^2 + 4\alpha_n \int_0^{L_x} c_n(x) \int_0^{L_y} c_n(y) \int_0^{L_z} c_n(z) f(x, y, z) dz dy dx},$$

$$\begin{aligned} \text{where } \alpha_n &= \frac{\xi L_x L_z}{2\pi^2 n} \int_0^t e_{nC}(\tau) \int_0^{L_x} s_n(2x) \int_0^{L_y} c_n(y) \int_0^{L_z} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\ & \times c_n(z) \frac{D_L(x, y, z, T)}{P(x, y, z, T)} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} dz dy dx d\tau + \\ & + \frac{\xi L_x L_z}{2\pi^2 n} \int_0^t e_{nC}(\tau) \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} s_n(2y) \int_0^{L_z} \left\{ z s_n(z) - \frac{L_z}{\pi n} [c_n(z) - 1] \right\} \times \\ & \times c_n(z) \frac{D_L(x, y, z, T)}{P(x, y, z, T)} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] dz dy dx d\tau + \frac{\xi L_x L_y}{2\pi^2 n} \times \\ & \times \int_0^t e_{nC}(\tau) \int_0^{L_x} c_n(x) \int_0^{L_y} c_n(y) \int_0^{L_z} s_n(2z) \frac{D_L(x, y, z, T)}{P(x, y, z, T)} \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\ & \times \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} dz dy dx d\tau, \beta_n = \frac{L_y L_z}{2n\pi^2} \times \\ & \times \int_0^t e_{nC}(\tau) \int_0^{L_x} s_n(2x) \int_0^{L_y} c_n(y) \int_0^{L_z} c_n(z) D_L(x, y, z, T) \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\ & \times \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} dz \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} dy dx d\tau + \frac{L_x L_z}{2n\pi^2} \int_0^t e_{nC}(\tau) \times \end{aligned}$$

Full Paper

$$\begin{aligned} & \times \int_0^{L_x} c_n(x) \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} s_n(2y) \int_0^{L_z} \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\ & \times c_n(z) D_L(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} d z d y d x d \tau + \frac{L_x L_y}{2 n \pi^2} \int_0^t e_{nC}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \right. \\ & \left. + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\ & \times s_n(2z) D_L(x, y, z, T) d z c_n(y) d y c_n(x) d x d \tau - L_x^2 L_y^2 L_z^2 e_{nC}(t) / \pi^5 n^6. \end{aligned}$$

Analogous way could be used to calculate values of parameters a_n for larger values of parameter γ . However the relations will not be present in the paper because the relations are bulky. Advantage of the approach is absent of necessity to join dopant concentration on interfaces of heterostructure.

Equations of the system (4) have been also solved by using Bubnov-Galerkin approach. Previously we transform the differential equations to the following integro- differential form

$$\begin{aligned} & \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z I(u, v, w, t) d w d v d u = \int_0^t \int_0^y \int_0^z D_I(x, v, w, T) \frac{\partial I(x, v, w, \tau)}{\partial x} d w d v d \tau \times \\ & \times \frac{y z}{L_y L_z} + \frac{x z}{L_x L_z} \int_0^t \int_0^x \int_0^z D_I(u, y, w, T) \frac{\partial I(u, y, w, \tau)}{\partial x} d w d u d \tau - \int_0^x \int_0^y \int_0^z k_{I,V}(u, v, w, T) \times \\ & \times I(u, v, w, t) V(u, v, w, t) d w d v d u \frac{x y z}{L_x L_y L_z} + \frac{x y}{L_x L_y} \int_0^t \int_0^x \int_0^y \frac{\partial I(u, v, z, \tau)}{\partial z} \times \\ & \times D_I(u, v, z, T) d v d u d \tau - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{I,I}(u, v, w, T) I^2(u, v, w, t) d w d v d u + \\ & + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z f_I(u, v, w) d w d v d u \end{aligned} \quad (4a)$$

$$\begin{aligned} & \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z V(u, v, w, t) d w d v d u = \int_0^t \int_0^y \int_0^z D_V(x, v, w, T) \frac{\partial V(x, v, w, \tau)}{\partial x} d w d v d \tau \times \\ & \times \frac{y z}{L_y L_z} + \frac{x z}{L_x L_z} \int_0^t \int_0^x \int_0^z D_V(u, y, w, T) \frac{\partial V(u, y, w, \tau)}{\partial x} d w d u d \tau + \int_0^t \int_0^x \int_0^y D_V(u, v, z, T) \times \\ & \times \frac{\partial V(u, v, z, \tau)}{\partial z} d v d u d \tau \frac{x y}{L_x L_y} - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{I,V}(u, v, w, T) I(u, v, w, t) \times \end{aligned}$$

$$\begin{aligned} & \times V(u, v, w, t) d w d v d u - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{v,v}(u, v, w, T) V^2(u, v, w, t) d w d v d u + \\ & + \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z f_v(u, v, w) d w d v d u. \end{aligned}$$

Farther we determine solutions of the above equations as the following series

$$\rho_0(x, y, z, t) = \sum_{n=1}^N a_{np} c_n(x) c_n(y) c_n(z) e_{np}(t),$$

where a_{np} are not yet known coefficients. Substitution of the series into Eqs.(4a) leads to the following results

$$\begin{aligned} & \frac{x y z}{\pi^3} \sum_{n=1}^N \frac{a_{nl}}{n^3} s_n(x) s_n(y) s_n(z) e_{nl}(t) = - \frac{y z \pi}{L_x L_y L_z} \sum_{n=1}^N a_{nl} \int_0^t \int_{L_y}^y c_n(y) \int_{L_z}^z c_n(z) D_I(x, v, w, T) d w d v \times \\ & \times e_{nl}(\tau) d \tau s_n(x) - \frac{x z \pi}{L_x L_y L_z} \sum_{n=1}^N a_{nl} s_n(y) \int_0^t e_{nl}(\tau) \int_{L_x}^x c_n(x) \int_{L_z}^z c_n(z) D_I(u, y, w, T) d w d u d \tau - \\ & - \frac{x y \pi}{L_x L_y L_z} \sum_{n=1}^N a_{nl} s_n(z) \int_0^t e_{nl}(\tau) \int_{L_x}^x c_n(x) \int_{L_y}^y c_n(y) D_I(u, v, z, T) d v d u d \tau - \int_0^x \int_0^y \int_0^z k_{I,I}(u, v, v, T) \times \\ & \times \left[\sum_{n=1}^N a_{nl} c_n(u) c_n(v) c_n(w) e_{nl}(t) \right] d w d v d u - \frac{x y z}{L_x L_y L_z} - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z \sum_{n=1}^N a_{nl} c_n(u) c_n(v) c_n(w) \times \\ & \times e_{nl}(t) \sum_{n=1}^N a_{nv} c_n(u) c_n(v) c_n(w) e_{nv}(t) k_{I,V}(u, v, v, T) d w d v d u + \int_0^x \int_0^y \int_0^z f_I(u, v, w) d w d v d u \times \\ & \times x y z / L_x L_y L_z \\ & \frac{x y z}{\pi^3} \sum_{n=1}^N \frac{a_{nv}}{n^3} s_n(x) s_n(y) s_n(z) e_{nv}(t) = - \frac{y z \pi}{L_x L_y L_z} \sum_{n=1}^N a_{nv} \int_0^t \int_{L_y}^y c_n(y) \int_{L_z}^z c_n(z) D_V(x, v, w, T) d w d v \times \\ & \times e_{nv}(\tau) d \tau s_n(x) - \frac{x z \pi}{L_x L_y L_z} \sum_{n=1}^N a_{nv} s_n(y) \int_0^t e_{nv}(\tau) \int_{L_x}^x c_n(x) \int_{L_z}^z c_n(z) D_V(u, y, w, T) d w d u d \tau - \\ & - \frac{x y \pi}{L_x L_y L_z} \sum_{n=1}^N a_{nv} s_n(z) \int_0^t e_{nv}(\tau) \int_{L_x}^x c_n(x) \int_{L_y}^y c_n(y) D_V(u, v, z, T) d v d u d \tau - \int_0^x \int_0^y \int_0^z k_{V,V}(u, v, v, T) \times \\ & \times \left[\sum_{n=1}^N a_{nv} c_n(u) c_n(v) c_n(w) e_{nl}(t) \right] d w d v d u - \frac{x y z}{L_x L_y L_z} - \frac{x y z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z \sum_{n=1}^N a_{nl} c_n(u) c_n(v) c_n(w) \times \\ & \times e_{nl}(t) \sum_{n=1}^N a_{nv} c_n(u) c_n(v) c_n(w) e_{nv}(t) k_{I,V}(u, v, v, T) d w d v d u + \int_0^x \int_0^y \int_0^z f_V(u, v, w) d w d v d u \times \\ & \times x y z / L_x L_y L_z \end{aligned}$$

Full Paper

$\times x y z / L_x L_y L_z$.

We determine coefficients a_{np} by using orthogonality condition on the scale of heterostructure. The condition gives us possibility to obtain relations to calculate a_{np} for any quantity N of terms of considered series. In the common case equations for the required coefficients could be written as

$$\begin{aligned}
 & -\frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{nl}}{n^6} e_{nl}(t) = -\frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{nl}}{n^2} \int_0^t \int_0^{L_x} [1 - c_n(2x)] \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
 & \times \int_0^{L_z} D_l(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} dz dy dx e_{nl}(\tau) d\tau - \frac{1}{2\pi L_y} \sum_{n=1}^N \frac{a_{nl}}{n^2} \int_0^t \int_0^{L_x} \left\{ x s_n(2x) + \right. \\
 & \left. + L_x + \frac{L_x}{\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} \int_0^{L_z} D_l(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz [1 - c_n(2y)] \times \\
 & \times dy dx e_{nl}(\tau) d\tau \int_0^{L_z} D_l(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx e_{nl}(\tau) d\tau - \\
 & -\frac{1}{2\pi L_z} \sum_{n=1}^N \frac{a_{nl}}{n^2} \int_0^t \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
 & \times \int_0^{L_z} [1 - c_n(2z)] D_l(x, y, z, T) dz dy dx e_{nl}(\tau) d\tau - \sum_{n=1}^N a_{nl}^2 e_{nl}(2t) \int_0^{L_x} \left\{ L_x + \frac{L_x}{2\pi n} [c_n(2x) - 1] + \right. \\
 & \left. + x s_n(2x) \right\} \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \int_0^{L_z} k_{l,l}(x, y, z, T) \left\{ L_z + \frac{L_z}{2\pi n} [c_n(2z) - 1] + \right. \\
 & \left. + z s_n(2z) \right\} dz dy dx - \sum_{n=1}^N a_{nl} a_{nV} e_{nl}(t) e_{nV}(t) \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} \left\{ L_y + \right. \\
 & \left. + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \int_0^{L_z} k_{l,v}(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz \times \\
 & \times dy dx + \sum_{n=1}^N \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} f_l(x, y, z, T) \times \\
 & \times \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx \\
 & -\frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{nV}}{n^6} e_{nV}(t) = -\frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{nV}}{n^2} \int_0^t \int_0^{L_x} [1 - c_n(2x)] \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times
 \end{aligned}$$

$$\begin{aligned}
& \times \int_0^{L_z} D_V(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} dz dy dx e_{nV}(\tau) d\tau - \frac{1}{2\pi L_y} \sum_{n=1}^N \frac{a_{nV}}{n^2} \int_0^{L_x} \int_0^{L_z} \{ x s_n(2x) + \\
& + L_x + \frac{L_x}{\pi n} [c_n(2x) - 1] \} \int_0^{L_y} \int_0^{L_z} D_V(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz [1 - c_n(2y)] \times \\
& \times dy dx e_{nV}(\tau) d\tau \int_0^{L_z} D_V(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx e_{nV}(\tau) d\tau - \\
& - \frac{1}{2\pi L_z} \sum_{n=1}^N \frac{a_{nV}}{n^2} \int_0^{L_x} \int_0^{L_y} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_z} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\
& \times \int_0^{L_z} [1 - c_n(2z)] D_V(x, y, z, T) dz dy dx e_{nV}(\tau) d\tau - \sum_{n=1}^N a_{nV}^2 e_{nV}(2t) \int_0^{L_x} \left\{ L_x + \frac{L_x}{2\pi n} [c_n(2x) - 1] + \right. \\
& + x s_n(2x) \} \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \int_0^{L_z} k_{V,V}(x, y, z, T) \left\{ L_z + \frac{L_z}{2\pi n} [c_n(2z) - 1] + \right. \\
& + z s_n(2z) \} dz dy dx - \sum_{n=1}^N a_{nI} a_{nV} e_{nI}(t) e_{nV}(t) \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} \left\{ L_y + \right. \\
& + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \} \int_0^{L_z} k_{I,V}(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz \times \\
& \times dy dx + \sum_{n=1}^N \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} f_V(x, y, z, T) \times \\
& \times \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx.
\end{aligned}$$

In the final form relations for required parameters could be written as

$$a_{nI} = -\frac{b_3 + A}{4b_4} \pm \sqrt{\frac{(b_3 + A)^2}{4} - 4b_4 \left(y + \frac{b_3 y - \gamma_{nV} \lambda_{nI}^2}{A} \right)}, \quad a_{nV} = -\frac{\gamma_{nI} a_{nI}^2 + \delta_{nI} a_{nI} + \lambda_{nI}}{\chi_{nI} a_{nI}},$$

$$\begin{aligned}
\text{where } \gamma_{n\rho} &= e_{n\rho}(2t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_{\rho,\rho}(x, y, z, T) \left\{ L_x + x s_n(2x) + \frac{L_x}{2\pi n} [c_n(2x) - 1] \right\} \left\{ y s_n(2y) + L_y + \right. \\
& + \frac{L_y}{2\pi n} [c_n(2y) - 1] \} \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx, \quad \delta_{n\rho} = \frac{1}{2\pi L_x n^2} \int_0^t e_{n\rho}(\tau) \times
\end{aligned}$$

Full Paper

$$\begin{aligned}
& \times \int_0^{L_x} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} D_\rho(x, y, z, T) dz dy [1 - \\
& - c_n(2x)] dx d\tau + \frac{1}{2\pi L_y n^2} \int_0^t e_{n\rho}(\tau) \int_0^{L_x} \left\{ L_x + x s_n(2x) + \frac{L_x}{\pi n} [c_n(2x) - 1] \right\} \int_0^{L_y} [1 - c_n(2y)] \int_0^{L_z} \{L_z + \\
& + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1]\} D_\rho(x, y, z, T) dz dy dx d\tau + \frac{1}{2\pi L_z n^2} \int_0^t e_{n\rho}(\tau) \int_0^{L_x} \{x s_n(2x) + \\
& + L_x + \frac{L_x}{\pi n} [c_n(2x) - 1]\} \int_0^{L_y} \left\{ L_y + y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} [1 - c_n(2z)] D_\rho(x, y, z, T) dz \times \\
& \times dy dx d\tau - \frac{L_x^2 L_y^2 L_z^2}{\pi^5 n^6} e_{n\rho}(t), \chi_{nV} = \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ L_y + \frac{L_y}{2\pi n} [c_n(2y) - 1] + \right. \\
& \left. + y s_n(2y) \right\} \int_0^{L_z} k_{I,V}(x, y, z, T) \left\{ L_z + z s_n(2z) + \frac{L_z}{2\pi n} [c_n(2z) - 1] \right\} dz dy dx e_{nI}(t) e_{nV}(t), \\
& \lambda_{n\rho} = \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{\pi n} [c_n(z) - 1] \right\} \times \\
& \times f_\rho(x, y, z, T) dz dy dx, b_4 = \gamma_{nV} \gamma_{nI}^2 - \gamma_{nI} \chi_{nI}^2, b_3 = 2\gamma_{nV} \gamma_{nI} \delta_{nI} - \delta_{nI} \chi_{nI}^2 - \delta_{nV} \chi_{nI} \gamma_{nI}, \\
& A = \sqrt{8y + b_3^2 - 4b_2}, b_2 = \gamma_{nV} \delta_{nI}^2 + 2\lambda_{nI} \gamma_{nV} \gamma_{nI} - \delta_{nV} \chi_{nI} \delta_{nI} + (\lambda_{nV} - \lambda_{nI}) \chi_{nI}^2, b_1 = 2\lambda_{nI} \times \\
& \times \gamma_{nV} \delta_{nI} - \delta_{nV} \chi_{nI} \lambda_{nI}, y = \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q} - \frac{b_3}{3b_4}, p = \frac{3b_2 b_4 - b_3^2}{9b_4^2}, \\
& q = (2b_3^3 - 9b_2 b_3 + 27b_1 b_4^2) / 54b_4^3.
\end{aligned}$$

We determine spatio-temporal distributions of concentrations of complexes of radiation defects in the following form

$$\Phi_{\rho 0}(x, y, z, t) = \sum_{n=1}^N a_{n\Phi\rho} c_n(x) c_n(y) c_n(z) e_{n\rho}(t),$$

where $a_{n\Phi\rho}$ are not yet known coefficients. Let us previously transform the Eqs.(6) to the following integro-differential form

$$\begin{aligned}
& \frac{xy z}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z \Phi_I(u, v, w, t) dw dv du = \int_0^t \int_0^y \int_0^z D_{\Phi I}(x, v, w, T) \frac{\partial \Phi_I(x, v, w, \tau)}{\partial x} dw dv d\tau \times \\
& \times \frac{yz}{L_y L_z} + \frac{xz}{L_x L_z} \int_0^t \int_0^x \int_0^z D_{\Phi I}(u, y, w, T) \frac{\partial \Phi_I(u, y, w, \tau)}{\partial y} dw du d\tau + \frac{xy}{L_x L_y} \int_0^t \int_0^x \int_0^y D_{\Phi I}(u, v, z, T) \times
\end{aligned}$$

$$\begin{aligned}
& \times \frac{\partial \Phi_I(u, v, z, \tau)}{\partial z} d v d u d \tau + \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} k_{I,I}(u, v, w, T) I^2(u, v, w, \tau) d w d v d u - \quad (6a) \\
& - \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} k_I(u, v, w, T) I(u, v, w, \tau) d w d v d u + \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} f_{\Phi_I}(u, v, w) d w d v d u \\
& \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} \Phi_V(u, v, w, t) d w d v d u = \int \int \int_{0 L_y L_z}^t D_{\Phi_V}(x, v, w, T) \frac{\partial \Phi_V(x, v, w, \tau)}{\partial x} d w d v d \tau \times \\
& \times \frac{y z}{L_y L_z} + \frac{x z}{L_x L_z} \int \int \int_{0 L_x L_z}^t D_{\Phi_V}(u, y, w, T) \frac{\partial \Phi_V(u, y, w, \tau)}{\partial y} d w d u d \tau + \frac{x y}{L_x L_y} \int \int \int_{0 L_x L_y}^t D_{\Phi_V}(u, v, z, T) \times \\
& \times \frac{\partial \Phi_V(u, v, z, \tau)}{\partial z} d v d u d \tau + \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} k_{V,V}(u, v, w, T) V^2(u, v, w, \tau) d w d v d u - \\
& - \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} k_V(u, v, w, T) V(u, v, w, \tau) d w d v d u + \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} f_{\Phi_V}(u, v, w) d w d v d u .
\end{aligned}$$

Substitution of the previously considered series in the Eqs.(6a) leads to the following form

$$\begin{aligned}
& - x y z \sum_{n=1}^N \frac{a_{n\Phi_I}}{\pi^3 n^3} s_n(x) s_n(y) s_n(z) e_{nI}(t) = - \frac{y z \pi}{L_x L_y L_z} \sum_{n=1}^N n a_{n\Phi_I} s_n(x) e_{nI}(t) \int \int \int_{0 L_y L_z}^t c_n(v) c_n(w) \times \\
& \times D_{\Phi_I}(x, v, w, T) d w d v d \tau - \frac{x z \pi}{L_x L_y L_z} \sum_{n=1}^N a_{n\Phi_I} \int \int \int_{0 L_x L_z}^t c_n(u) c_n(w) D_{\Phi_I}(u, v, w, T) d w d u d \tau \times \\
& \times n s_n(y) e_{n\Phi_I}(t) - \frac{x y \pi}{L_x L_y L_z} \sum_{n=1}^N n a_{n\Phi_I} s_n(z) e_{n\Phi_I}(t) \int \int \int_{0 L_x L_y}^t c_n(u) c_n(v) D_{\Phi_I}(u, v, z, T) d v d u d \tau + \\
& + \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} k_{I,I}(u, v, w, T) I^2(u, v, w, \tau) d w d v d u + \int \int \int_{L_x L_y L_z} f_{\Phi_I}(u, v, w) d w d v d u \times \\
& \times \frac{x y z}{L_x L_y L_z} - \frac{x y z}{L_x L_y L_z} \int \int \int_{L_x L_y L_z} k_I(u, v, w, T) I(u, v, w, \tau) d w d v d u \\
& - x y z \sum_{n=1}^N \frac{a_{n\Phi_V}}{\pi^3 n^3} s_n(x) s_n(y) s_n(z) e_{nV}(t) = - \frac{y z \pi}{L_x L_y L_z} \sum_{n=1}^N n a_{n\Phi_V} s_n(x) e_{nV}(t) \int \int \int_{0 L_y L_z}^t c_n(v) c_n(w) \times \\
& \times D_{\Phi_V}(x, v, w, T) d w d v d \tau - \frac{x z \pi}{L_x L_y L_z} \sum_{n=1}^N n \int \int \int_{0 L_x L_z}^t c_n(u) c_n(w) D_{\Phi_V}(u, v, w, T) d w d u d \tau \times \\
& \times a_{n\Phi_V} s_n(y) e_{n\Phi_V}(t) - \frac{x y \pi}{L_x L_y L_z} \sum_{n=1}^N n s_n(z) e_{n\Phi_V}(t) \int \int \int_{0 L_x L_y}^t c_n(u) c_n(v) D_{\Phi_V}(u, v, z, T) d v d u d \tau \times
\end{aligned}$$

Full Paper

$$\begin{aligned} & \times a_{n\Phi V} + \frac{xyz}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_{V,V}(u, v, w, T) V^2(u, v, w, \tau) d w d v d u + \int_0^x \int_0^y \int_0^z f_{\Phi V}(u, v, w) d w d v d u \times \\ & \times \frac{xyz}{L_x L_y L_z} - \frac{xyz}{L_x L_y L_z} \int_0^x \int_0^y \int_0^z k_V(u, v, w, T) V(u, v, w, \tau) d w d v d u . \end{aligned}$$

We determine coefficients $a_{n\Phi p}$ by using orthogonality condition on the scale of heterostructure. The condition gives us possibility to obtain relations to calculate $a_{n\Phi p}$ for any quantity N of terms of considered series. In the common case equations for the required coefficients could be written as

$$\begin{aligned} & - \frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^6} e_{n\Phi I}(t) = - \frac{1}{2\pi L_x} \sum_{n=1}^N \int_0^t \int_0^{L_x} [1 - c_n(2x)] \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\ & \times \frac{a_{n\Phi I}}{n^2} \int_0^{L_z} D_{\Phi I}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x e_{n\Phi I}(\tau) d \tau - \frac{1}{2\pi} \sum_{n=1}^N \int_0^t \int_0^{L_x} \{ x s_n(2x) + \\ & + L_x + \frac{L_x}{2\pi n} [c_n(2x) - 1] \} \int_0^{L_y} [1 - c_n(2y)] \int_0^{L_z} D_{\Phi I}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x \times \\ & \times a_{n\Phi I} \frac{e_{n\Phi I}(\tau)}{n^2 L_y} d \tau - \frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^2} \int_0^t \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] + \right. \\ & + L_y \left. \right\} \int_0^{L_z} [1 - c_n(2y)] D_{\Phi I}(x, y, z, T) d z d y d x e_{n\Phi I}(\tau) d \tau + \frac{1}{\pi^3} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^3} \int_0^t e_{n\Phi I}(\tau) \int_0^{L_x} \left\{ \frac{L_x}{2\pi n} [c_n(x) - 1] + \right. \\ & + x s_n(x) \left. \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} I^2(x, y, z, t) k_{I,I}(x, y, z, T) \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + \right. \\ & + z s_n(z) \left. \right\} d z d y d x - \frac{1}{\pi^3} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^3} \int_0^t e_{n\Phi I}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ \frac{L_y}{2\pi n} [c_n(y) - 1] + \right. \\ & + y s_n(y) \left. \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} k_I(x, y, z, T) I(x, y, z, t) d z d y d x + \frac{1}{\pi^3} \sum_{n=1}^N \frac{a_{n\Phi I}}{n^3} \times \\ & \times \int_0^t e_{n\Phi I}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + \right. \\ & + z s_n(z) \left. \right\} f_{\Phi I}(x, y, z) d z d y d x \\ & - \frac{L_x^2 L_y^2 L_z^2}{\pi^5} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^6} e_{n\Phi V}(t) = - \frac{1}{2\pi L_x} \sum_{n=1}^N \int_0^t \int_0^{L_x} [1 - c_n(2x)] \int_0^{L_y} \left\{ L_y + y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] \right\} \times \\ & \times \frac{a_{n\Phi V}}{n^2} \int_0^{L_z} D_{\Phi V}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} d z d y d x e_{n\Phi V}(\tau) d \tau - \frac{1}{2\pi} \sum_{n=1}^N \int_0^t \int_0^{L_x} \{ x s_n(2x) + \end{aligned}$$

$$\begin{aligned}
 &+ L_x + \frac{L_x}{2\pi n} [c_n(2x) - 1] \int_0^{L_y} [1 - c_n(2y)] \int_0^{L_z} D_{\Phi V}(x, y, z, T) \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} dz dy dx \times \\
 &\times a_{n\Phi V} \frac{e_{n\Phi V}(\tau)}{n^2 L_y} d\tau - \frac{1}{2\pi L_x} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^2} \int_0^t \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(2y) + \frac{L_y}{2\pi n} [c_n(2y) - 1] + \right. \\
 &+ L_y \left. \right\} \int_0^{L_z} [1 - c_n(2y)] D_{\Phi V}(x, y, z, T) dz dy dx e_{n\Phi V}(\tau) d\tau + \frac{1}{\pi^3} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^3} \int_0^t e_{n\Phi V}(\tau) \int_0^{L_x} \left\{ \frac{L_x}{2\pi n} [c_n(x) - 1] + \right. \\
 &+ x s_n(x) \left. \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} V^2(x, y, z, t) k_{V,V}(x, y, z, T) \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + \right. \\
 &+ z s_n(z) \left. \right\} dz dy dx - \frac{1}{\pi^3} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^3} \int_0^t e_{n\Phi V}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ \frac{L_y}{2\pi n} [c_n(y) - 1] + \right. \\
 &+ y s_n(y) \left. \right\} \int_0^{L_z} \left\{ z s_n(z) + \frac{L_z}{2\pi n} [c_n(z) - 1] \right\} k_V(x, y, z, T) V(x, y, z, t) dz dy dx + \frac{1}{\pi^3} \sum_{n=1}^N \frac{a_{n\Phi V}}{n^3} \times \\
 &\times \int_0^t e_{n\Phi V}(\tau) \int_0^{L_x} \left\{ x s_n(x) + \frac{L_x}{2\pi n} [c_n(x) - 1] \right\} \int_0^{L_y} \left\{ y s_n(y) + \frac{L_y}{2\pi n} [c_n(y) - 1] \right\} \int_0^{L_z} \left\{ \frac{L_z}{2\pi n} [c_n(z) - 1] + \right. \\
 &+ z s_n(z) \left. \right\} f_{\Phi V}(x, y, z) dz dy dx.
 \end{aligned}$$

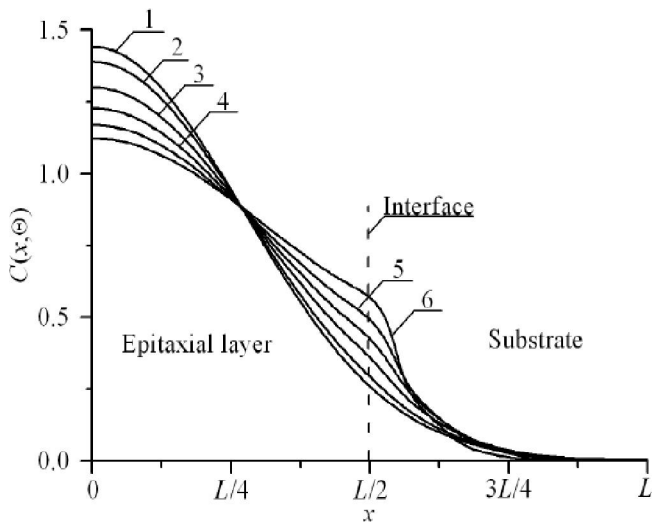


Figure 2a : Distributions of concentration of infused dopant in heterostructure from Figures 1 and 2 in direction, which is perpendicular to interface between epitaxial layer substrate. Increasing of number of curve corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate

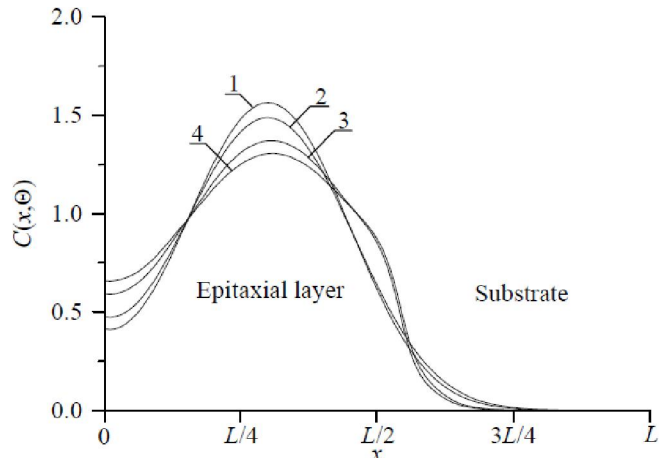


Figure 2b : Distributions of concentration of implanted dopant in heterostructure from Figures 1 and 2 in direction, which is perpendicular to interface between epitaxial layer substrate. Curves 1 and 3 corresponds to annealing time $\Theta = 0.0048(L_x^2 + L_y^2 + L_z^2)/D_0$. Curves 2 and 4 corresponds to annealing time $\Theta = 0.0057(L_x^2 + L_y^2 + L_z^2)/D_0$. Curves 1 and 2 corresponds to homogenous sample. Curves 3 and 4 corresponds to heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate

Full Paper

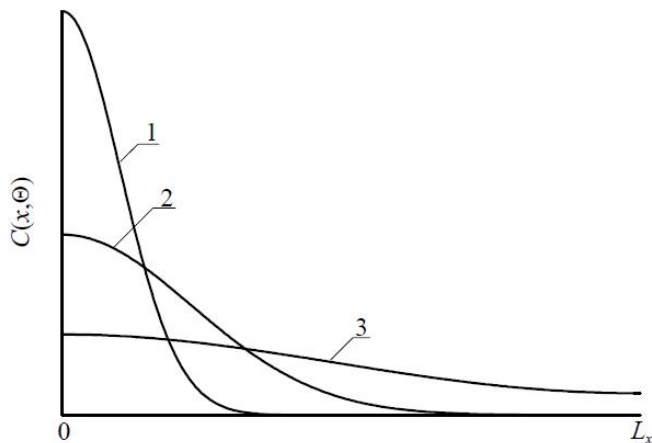


Figure 3a : Spatial distributions of concentration of implanted dopant in heterostructure from Figures 1. Curves 1-3 are distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

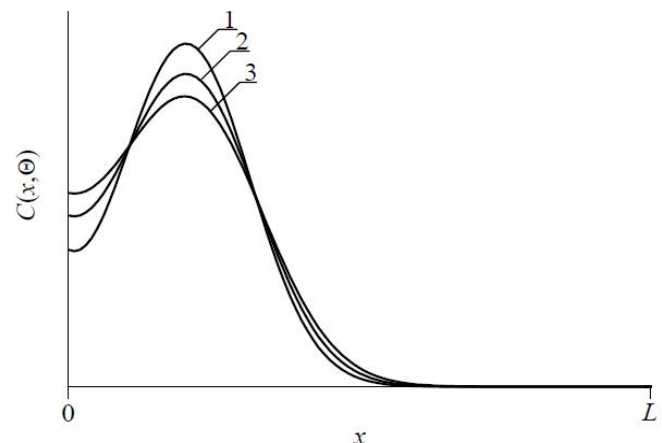


Figure 3b : Spatial distributions of concentration of in-fused dopant in heterostructure from Figures 1. Curves 1-3 are the real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

DISCUSSION

We use the calculated in the previous section relations for the analysis of redistribution of dopant with account redistribution of radiation defects. Figures 2 and 3 shows typical distributions of concentrations of dopants for diffusion and ion types of doping, respectively. The figures correspond to larger value of dopant diffusion coefficient in the epitaxial layer comparison with dopant diffusion coefficient in the substrate. Figures 2 and 3 shows, that presents of interface between layers of heterostructure gives us possibility to increase absolute value of gradient of concentration of dopant out of doped area. In this situation one can obtain increasing of compactness of transistor. At the same time one can obtain increasing of homogeneity of concentration of dopant in doped area. The first effect gives us possibility to increase performance of manufactured device. The second effect gives us possibility to decrease local overheats of the doped materials during functioning of these devices, which were manufactured based on the materials.

It is known, that increasing of annealing time leads to too large diffusion of dopant into nearest materials from doped one (see Figures 4 and 5). Decreasing of annealing time leads to decreasing of homogeneity of concentration of dopant in doped area (see Figures 4 and 5). Let us determine compromise annealing time framework recently introduced approach^[19-26]. Framework the approach we approximate real distribution of concentration of dopant by step-wise function $\psi(x, y, z)$ ^[19-26]. Farther the required compromise annealing time by minimization of the following mean-squared error

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [C(x, y, z, \Theta) - \psi(x, y, z)]^2 dz dy dx \quad (8)$$

Analysis of optimal value of annealing time shows, that optimal value of annealing time for diffusion type of doping is larger, than optimal value of annealing time for ion type of doping. Reason of the difference is necessity to anneal radiation defects. In the ideal case dopant achieves nearest interface between layers of heterostructure during annealing of radiation defects. If the dopant did not achieves the interface during the annealing, it is attracted an interest additional annealing of dopant for the considered achievement.

It should be noted, that using diffusion type of doping did not leads to so serious damage of materials as one can obtain during ion implantation. However ion implantation gives us possibility gives us possibility to decrease mismatch-induced stress in heterostructure^[27].

CONCLUSION

In this paper we introduce an approach to manufacture a multi-gate transistors and at the same time to decrease their dimensions. In this paper we also discussed (i) an approach to optimize annealing time, (ii) an analytical approach to model nonlinear mass and heat transport with account variation parameters of the transports in space and time. The analytical approach gives us possibility to model mass and heat transport without crosslinking solutions at any interface between the layers of the heterostructure.

ACKNOWLEDGMENTS

This work is supported by the contract 11.G34.31.0066 of the Russian Federation Government, grant of Scientific School of Russia, the agreement of August 27, 2013 № 02.B.49.21.0003 between The Ministry of education and science of the Russian Federation and Lobachevsky State University of Nizhni Novgorod and educational fellowship for scientific research of Nizhny Novgorod State University of Architecture and Civil Engineering.

REFERENCES

- [1] M.J.Kumar, T.V.Singh; *Int.J.Nanoscience*, **7(2-3)**, 81-84 (2008).
- [2] D.Fathi, B.Forouzandeh; *Nano*, **4(3)**, 171-176 (2009).
- [3] D.Fathi, B.Forouzandeh, N.Masoumi, *Nano*, **4(4)**, 233-238 (2009).
- [4] A.O.Ageev, A.E.Belyaev, N.S.Boltovets, V.N.Ivanov, R.V.Konakova, Ya.Ya.Kudrik, P.M.Litvin, V.V.Milenin, A.V.Sachenko; *Semiconductors*, **43(7)**, 897-903 (2009).
- [5] Jung-Hui Tsai, Shao-Yen Chiu, Wen-Shiung Lour, Der-Feng Guo. *ФТНТ*, **43(7)**, 971-974 (2009).
- [6] P.Sinsermsuksakul, K.Hartman, S.B.Kim, J.Heo, L.Sun, H.H.Park, R.Chakraborty, T.Buonassisi, R.G.Gordon; *Appl.Phys.Lett.*, **102(5)**, 053901-053905 (2013).
- [7] J.G.Reynolds, C.L.Reynolds, Jr.A.Mohanta, J.F.Muth, J.E.Rowe, H.O.Everitt, D.E.Aspnes; *Appl.Phys.Lett.*, **102(15)**, 152114-152118 (2013).
- [8] S.A.Chachuli, P.N.A.Fasyar, N.Soin, N.M.Karim, N.Yusop; *Mat.Sci.Sem.Proc.* **24**, 9-14 (2014).
- [9] V.I.Lachin, N.S.Savelov. *Electronics (Phoenix, Rostov-na-Donu)*, (2001).
- [10] N.A.Avaev, Yu.E.Naumov, V.T.Frolkin; *Basis of microelectronics (Radio and communication, Moscow)*, (1991).
- [11] Z.Yu.Gotra; *Technology of microelectronic devices (Radio and communication, Moscow)*, (1991).
- [12] V.L.Vinetskiy, G.A.Kholodar'; *Radiative physics of semiconductors*, ("Naukova Dumka", Kiev, 1979, in Russian).
- [13] P.M.Fahey, P.B.Griffin, J.D.Plummer; *Rev.Mod.Phys.*, **61(2)**, 289-388 (1989).
- [14] W.X.Ni, G.V.Hansson, J.E.Sundgren, L.Hultman, L.R.Wallenberg, J.Y.Yao, L.C.Markert, J.E.Greene; *Phys.Rev.B.* **46(12)**, 7551-7558 (1992).
- [15] B.A.Zon, S.B.Ledovsky, A.N.Likholet; *Tech.Phys.*, **45(4)**, 419-424 (2000).
- [16] F.Faupel, W.Frank, M.-P.Macht, H.Mehrer, V.Naundorf, K.Ratzke, H.R.Schober, S.K.Sharma, H.Teichler; *Reviews of modern physics.* **75(1)**, 237-280 (2003).
- [17] S.A.Bahrani, Y.Jannot, A.Degiovanni; *J.Appl.Phys.* **114(14)**, 143509-143516 (2014).
- [18] M.L.Krasnov, A.I.Kiselev, G.I.Makarenko; *Integral equations ("Science", Moscow)*, (1976).
- [19] E.L.Pankratov; *Russian Microelectronics*, **36(1)**, 33-39 (2007).
- [20] E.L.Pankratov; *Int.J.Nanoscience*, **7(4-5)**, 187-197 (2008).
- [21] E.L.Pankratov; *J.Comp.Theor.Nanoscience*, **7(1)**, 289-295 (2010).
- [22] E.L.Pankratov, E.A.Bulaeva; *Int.J.Nanoscience*, **11(5)**, 1250028-1-1250028-8 (2012).
- [23] E.L.Pankratov, E.A.Bulaeva; *J.Comp.Theor.Nanoscience*, **10(4)**, 888-893 (2013).
- [24] E.L.Pankratov, E.A.Bulaeva; *Int.J.Micro-Nano Scale Transp.*, **4(1)**, 17-31 (2014).
- [25] E.L.Pankratov, E.A.Bulaeva; *Applied Nanoscience*, **4(5)**, 537-541 (2014).
- [26] E.L.Pankratov, E.A.Bulaeva; *Int.J.Rec.Adv.Phys.*, **3(2)**, 55-73 (2014).
- [27] E.L.Pankratov, E.A.Bulaeva; *J.Comp.Theor.Nanoscience.*, **11(1)**, 91-101 (2014).