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Models of dynamical phenomena for certain moons in the solar system

Abstract

Models for unusual motions of moons and planets are presented. Results are compared with other dynamical systems; Motorcycle-wobbling, for which facts are certain, possible configurations of Gas giants at formation, and a deterministic analysis for oscillation of non-spherical satellites^[1].

Keywords

Celestial mechanics; Phobos; Hyperion; Wobbling; Nonlinear rotation; Deterministic chaos; Conservation law; Configuration torque.

INTRODUCTION

Planets moving in a gravitational field are subjected to forces and other interactions, mainly from the primary, but also from adjacent planets, moons and other matter. For most of the moons, rotation is bounded such that orbit angular velocity, equals the own axis rotation. This is since there is a coupling tidal force between moon and primary. From these rules, there are but a few exceptions. Phoebe, one of the outermost moons of Saturn, does not have a bounded rotation. The moon Hyperion, with non-symmetric shape, is found to rotate chaotic about own axis, (but the orbital rotation is notified as bounded). Saturn has about 60 moons, and planetary rings, in which so-called shepherds moves with a different velocity than the rings, e.g. the moon Daphnis in Keeler Gap in Saturn.

For the orbit around a primary, in vicinity i.e. sufficiently close, satellites rotate with the same angular velocity, and far rotation is gravitational according to Kepler's law. In between, there are regions, e.g. a so-called Roche-region, where other forces act, that may cause objects to disintegrate, and create gaps in planetary rings. Outside the Roche limit is the limit for a synchronous orbit, and in between a region where objects may have increased angular velocities. It is found that the moon

Phobos, close to Mars, is moving very fast, about 3 times its primary. Another exceptional phenomenon notified is that two moons in Saturn change orbits at passing each other.

In^[1], results for non-spherical satellites, in terms of oscillation in angular velocity, so-called libration, were obtained with a deterministic analysis. In the present context, other mechanical models describing some rare phenomena will be derived and discussed. To calculate an angular velocity for orbital motions, a generalisation of a conservation law, will be exploited. For the sidereal motion of Hyperion, nonlinear wobbling is derived.

ESTIMATIONS FOR ANGULAR VELOCITY, FROM A NOETHER THEOREM

Consider optimization of a functional in a time interval $[S, T]$, with the Lagrangian $\Psi(\omega)$ where $\omega = \omega(t)$ is the angular velocity in the orbit. From the general variation of a functional, a conservation law is obtained when the Lagrangian function is homogeneous of 2nd order^[1]. In one dimension, functions hom. of 2nd order reads $C\omega^2$ where C is constant. Variational calculus with Euler-Lagrange's equations give that also $\omega(t)$ is constant. Here, it is assumed that Euler-Lagrange's equations are not exactly fulfilled at all points in the time interval $[S, T]$. This

may be denoted a weak optimization and admits that the angular velocity ω and Ψ depend on t . Subsequently, Ψ will be denoted a potential and an energy-conjugated variable to ω is defined as $P = \Psi_{,\omega}$

Assumption

With the preliminaries above, we suppose an equation for the dynamical variables, at $t = S$, and $t = T$.

When Ψ is proportional to ω^2 , the general variation of a functional gives a conservation law

$$[\Psi t - (\omega t - I / 2) P]_S^T = 0 \tag{1}$$

where I denotes the angle $I = \int \omega dt$, and subscript S and superscript T denotes the value at time points T and S , and $P = \Psi_{,\omega}$, is the conjugated force.

This formula is derived from the conservation law given in^[2] for linear elasticity. The present format is obtained by replacing spatial dependence with time t , and displacements with angular velocity ω . Initial time S and terminal time T correspond to boundary of the elastic body.

To apply the formula (1), it is assumed that the conjugated force P , (generally also I and ω) may couple, with corresponding values connected to an adjacent rotating system.

INCREASED ORBITAL VELOCITY OF THE MOON PHOBOS

It appears that interactions and oscillation phenomena may increase the velocity. Mars has 2 moons. The closest, Phobos, is orbiting, (so-called super-synchronous) with increased angular velocity, 0.31d. This is somewhat faster, but the same magnitude as the Gas giants, Jupiter and Saturn. In a kinematic description, the moon may achieve an increased absolute radial acceleration, since Mars have an eccentric orbit. However the magnitude will not be so large as to describe the 3 times higher angular velocity. Instead, a framework with coupling will be assumed. The moon is non-symmetric, such that chaotic motion or wobbling could be possible, but tidal interaction with Mars dominates and the orbit is locked. It is possible that before tidal locking, a coupling to a chaotic sidereal motion created the increased orbital angular velocity.

Application of the conservation law

Let $S=0$ and $I(0)=0$. Then, the potential is given by $\Psi = (\omega - I / (2T))P$. The variables ω , I and P may either be assumed as the values of the own system, or as values that couples with outer adjacent system at time point T . When chaotic rotation, values for angular velocity components are large, at times. If coupling to these are assumed, such that $P_{orbit} = P_{sidereal}$, the potential for orbital

tation given by (1), will be increased to read $\Psi = \omega P_{sidereal} / 2 = \omega \omega_{sidereal} / 2$. After coupling, the new Ψ determines the new angular velocity ω_{new} , as $\Psi = \omega_{new}^2 / 2$.

ASSUMPTIONS ON COUPLING FOR TWO MOONS THAT CHANGE ORBITS

One of the moons has an elliptical orbit, which occasionally is very close to the circular orbit of the other moon. Briefly, two models will be outlined:

- In a simplified approach, the above conservation law, together with additional coupling conditions, e.g. that the two moons, when sufficiently close, move as a dumbbell and details of the interaction may be described by the calculations in^[1].
- To describe the eccentric orbit, additional d.o.f:s are required. For this purpose, a more complex version of (1), is to be derived.

NONLINEAR EQUATION DESCRIBING A POSSIBLE WOBBLING OF HYPERION

Linear wobbling for Mc-vehicle and Earth

Wobbling usually refers to the oscillations of a vehicle. For a Motorcycle, when no actions from driver, and friction at ground contact is neglected, the model will be linear with 2 d.o.f. The eigenfrequency is the same for both oscillations and depends on the angular velocity of the front wheel, the inertia moment of the front wheel, the inertia moment of fork plus steering device, and the inertia moment of driver plus vehicle minus front wheel. Planets with non-symmetric shape display wobbling. The frequency and amplitude for earth wobble have been calculated by Milankovic.

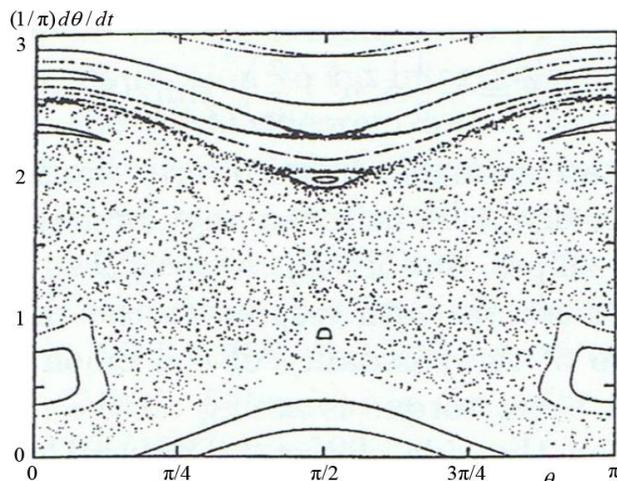
Wobbling model

The moon Hyperion is non-symmetric and the wobbling has a large amplitude such that it faces different sides to its primary. If the equations for planetary wobbling, are modified to also consider the net torque since gravity is larger on the inside part, a nonlinear model is achieved. With 1-direction tangential to the orbit the equations of motions read

$$J_1 d_t \omega_1 + \omega_2 \omega_3 (J_3 - J_2) = M_1 \tag{2}$$

$$J_2 d_t \omega_2 + \omega_1 \omega_3 (J_1 - J_3) = 0 \tag{3}$$

where ω_3 is a constant angular velocity, J_i denotes inertia moments, and the torque M_1 depends the masses, the length of the moon and nonlinear on the rotation-angle in 1-direction. A detailed analysis of a related model is given in^[1], as well as a comparison with possible evolution the moon to the earth. A phase portrait of such



Surface of section of Hyperion (with $\alpha = \sqrt{3(B-A)/C} = 0.89$ and $e = 0.1$). The rate of change of the orientation is plotted against the orientation at every periaapse passage. The spin axis is fixed perpendicular to the orbit plane.

Visualization of model for Hyperion, G.Ohlen, S.Åberg, P.Östberg, Chaos, LTH, 2007.

models is to be found in the literature, cf. figure below.

Energy approach

When the kinetic energy is constant, i.e. independent of t , the amplitude of the oscillation around the part with smaller inertia moment J , will be the larger. In Mc-wobbling, this is a phenomenon known as tank slapper, when the fork and steering device move with very large amplitude^[3].

For a planetary wobble omitting torque, steady state is obtained when two inertia moments are equal. Including torque and other interactions provides solutions also at other shapes. Then tank slapper corresponds to a large rotation oscillation around the axis with smallest J . In the case when this axis points towards the center of the orbit, the orientation is such that the moon has the same side to the center, also at large wobbling amplitude. Hereby, if the planet is very elongated, a tidally locked orbit is admitted also at wobbling.

Uranus has a large inclination angle such that it rolls in its orbit, with a part almost always facing the sun. Presently, it is not rigid and non-symmetric, but the inclination could be a reminiscence of the motion at formation. Also the fact that outer moons of Gas giants move the opposite direction may be a spatial peck of past state, planetary shape and history of oscillation.

Intrinsic properties/interactions related to material forces and configuration forces

In a weak formulation (energy approach), presence of other torques may be tacitly assumed. In a strong formulation of, a priori, nonlinear wobbling, other torques may be included in the equations of motion. In Mc-wobbling these emanate from vehicle contact with ground and actions from driver. Integration and differentiation of equa-

tions of motion, give that

- at ceased contact, torque is zero but integrated torque is nonzero
- at beginning of contact, torque is zero but time differential is nonzero

Hereby initially, the torque is determined by the (free) motion. Since dependent on such intrinsic properties, the interaction may be considered a configuration torque. Such notations are introduced in other modelling, e.g. Fracture mechanics^[4-6]. For planets, there is evidence of significant material torques: In^[7], a core mantle friction is assumed to interact with rotation.

CONCLUSION

The non-stable rotation of Hyperion was described with deterministic chaos, including a torque due to gravity. Then large 'wobbling' amplitude can occur also at tidally locked orbit.

On the planetary scale, it is observed that Mercury^[7] has 3/2 coupling and Venus (-1)/1. Comparing with current dynamical states at the Gas giants, it is possible that Mercury and Venus rotated locked, as moons to the Sun, with Venus in opposite direction, in the beginning, at formation.

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