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Mechanics model of highest basketball goal percentage based on Matlab values simulation

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ABSTRACT

Goal percentage is key to winning a basketball game, which is closely related to the position of the player, shooting angle and shooting velocity of the ball. By analyzing the free throw scenario, the paper discusses how changes of distance from the centre of ball to the centre of the hoop, the height of centre of ball and shooting velocity influence the goal percentage, without the difficulty brought to the player by motion. This paper simplifies the situation as a upward projection movement. It discusses the law of movement of horizontal direction as a uniform motion, while the one of vertical direction as a uniformly retarded motion with the shooting velocity as the initial velocity v and g as the acceleration. Following conclusions are found: The variations of shooting angle poses more influence to the goal percentage than the shooting velocity. When the shooting height is between 1.8~2.1m, the shooting velocity should be higher than 8m/s. The angle of incidence should be larger than 33.1^{o}. The higher the shooting velocity, the less the allowable deviation there is for shooting angle. While the allowable deviation of shooting velocity increases, there is a stricter requirement on shooting angle than shooting velocity. When the shooting velocity is fixed, more shooting height means less allowable deviation of shooting angle and more allowable deviation of shooting velocity.

KEYWORDS

Values simulation;
Basketball;
Mechanics modal;
Goal percentage.

INTRODUCTION

Shooting is the main attacking technique in basketball and the only scoring method. All techniques and strategies exist to serve for a better chance to shoot and a higher percentage. A game is often decided by who can manage to make more shoots. Therefore, it is crucial to learn shooting technique and put it into practical use for a higher percentage of goals.

Two crucial factors determining whether the shoot would convert into a goal are shooting angle and shooting velocity. This paper starts from the free throw scenario, which is free from interferences brought to player by motion, and builds a relation model among shooting velocity, shooting angle and shooting height. Thus it provides some rational advices on basketball training in terms on improving goal percentage.

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ANALYSIS OF PROBLEM AND ESTABLISHMENT OF MODEL

Model hypothesis

Suppose that ball has no spin after leaving the hands; Situation in which the ball contacts the board not considered; air resistance not considered;

Symbols

d Diameter of basketball, d=24.6cm

D Diameter of hoop, D=45cm

L Horizontal distance from free throw position to centre of hoop, L=4.6m

H Height of centre of hoop, H=3.05m

h Height of the ball when player shoots

v Velocity of the ball when player shoots

Analysis of problem and establishment of model

Size of ball and basket not considered

In a simple situation that the size of ball and basket are not considered, the ball is regarded as a particle in oblique projectile. Set the origin of coordinates at centre of ball *P*, and the trajectory of the ball can be solved by equations of motion of *x* (horizontal) direction and *y* (vertical) direction. Therefore, the condition that the ball goes into the hoop can be explained by relations between the shooting angle and shooting velocity, shooting height, as well as between angle of incidence and shooting angle. Therefore, for various shooting velocity and shooting height, shooting angle and angle of incidence can be found as shown in Figure 1.

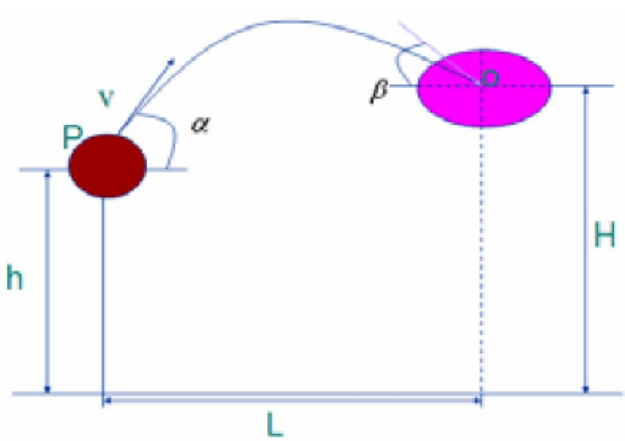

Figure 1 : Model of shooting

The sizes of ball and hoop and air resistance are not considered. It is an oblique projectile with *P* as its origin of coordinates, *x* as horizontal direction, *y* as the vertical direction. The ball is shot when $t=0$ with shooting velocity of v and shooting angle of α. The equation of motion is familiar to us:

$$x(t) = vt\cos\alpha\ ;\ y(t) = vt\sin\alpha - \frac{gt^2}{2} \tag{1}$$

g is the gravity. And the trajectory of the centre of ball is a parabola:

$$y = x\tan\alpha - x^2\frac{g}{2v^2\cos^2\alpha} \tag{2}$$

With substitutions of $x=L, y=H-h$, the condition that centre of ball goes onto the centre of hoop is here:

$$\tan\alpha = \frac{v^2}{gL}\left[1\pm\sqrt{1-\frac{2g}{v^2}\left(H-h+\frac{gL^2}{2v^2}\right)}\right] \tag{3}$$

It can be seen that two shooting angle will satisfy the condition with shooting velocity *v* and shooting height *h* fixed. The premise that previous equation does have solution(s) is:

$$1-\frac{2g}{v^2}\left(H-h+\frac{gL^2}{2v^2}\right)\geq 0 \tag{4}$$

Find *v*:

$$v^2 \geq g\left[H-h+\sqrt{L^2+(H-h)^2}\right] \tag{5}$$

Therefore, for certain *h*, a minimum *v* that balances the equation is a decreasing function of *h*.

From equation (3) two shooting angles, α_1 and α_2 can be found and let $\alpha_1 > \alpha_2$. It can be seen that α_1 is a decreasing function of h and v. β, which is the angle of incidence when the ball goes into the hoop, can be found in:

$$\tan\beta = \left.\frac{dy}{dx}\right|_{x=L} \tag{6}$$

The derivative here can be found by substitutions in equation (2)

$$\tan\beta = \tan\alpha - \frac{2(H-h)}{L} \tag{7}$$

So for α_1 and α_2, there are β_1 and β_1. Let $\beta_1 > \beta_2$

When the sizes of ball and hoop are considered:

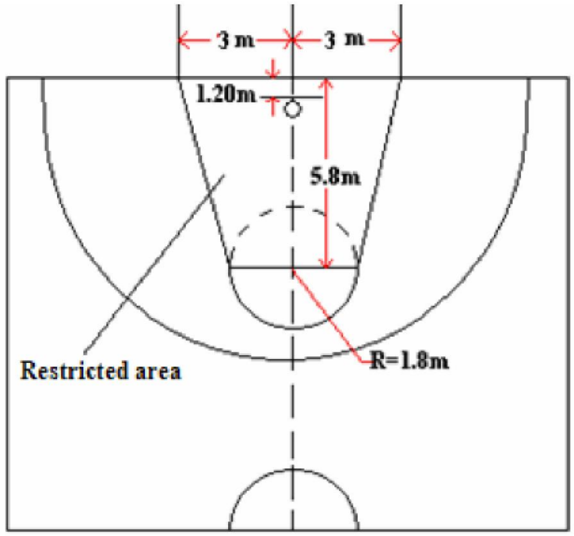

Figure 2 : Diagram of a half basketball court

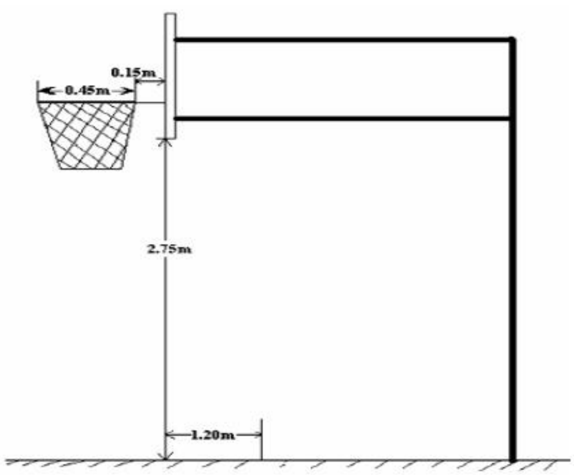

Figure 3 : Diagram of side view of board and hoop

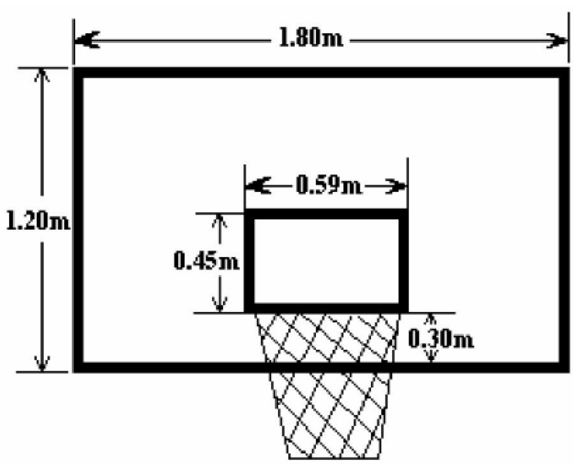

Figure 4 : Diagram of front view of board and hoop

When the sizes of ball and hoop are considered, the diameter of ball is d and the diameter of hoop is D. It is obvious that the ball would still not go in even if the centre of ball goes onto the centre of hoop when the angle of incidence β is too small and the ball will contact A, as shown in Figure 5:

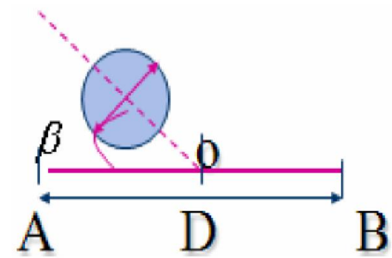

Figure 5 : Model when ball goes into hoop

It is easy to find the condition of β that would make centre of ball goes onto centre of hoop and the ball go in.

$$\sin\beta > \frac{d}{D} \tag{8}$$

Let $d = 24.6cm$, $D = 45cm$ and thus $\beta > 33.1°$. All previous calculation results that don't satisfy this should be neglected.

When the ball goes into the hoop, the centre of ball can deviate from the centre of hoop. The maximum deviation distance is Δx which can be found by angle of incidence β. From β and the relation between x and α in the trajectory of ball, the maximum allowable deviation Δx for shooting angle α can be found. The maximum allowable deviation Δv for shooting velocity v can be found in the same way.

The maximum front deviation (the back deviation is the same) distance when the ball goes into the hoop is:

$$\Delta x = \frac{D}{2} - \frac{d}{2\sin\beta} \tag{9}$$

The maximum allowable deviation of shooting angle can be calculated by substituting L with L+Δx in (1.3). However, as Δx includes β, so α does too. Therefore, this method doesn't work.

If we start from (2) and let y=H-h, there is:

$$x^2\frac{g}{2v^2cos^2\alpha} - x\tan\alpha + H - h = 0 \tag{10}$$

Find derivative of α and let x=L, there is:

$$\frac{dx}{d\alpha} = \frac{L\left(v^2 - gLtan\alpha\right)}{gL - v^2 sin\alpha cos\alpha} \tag{11}$$

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Substitute the derivative on the left with $\frac{\Delta x}{\Delta \alpha}$, the relationship between the deviation of shooting angle $\Delta\alpha$ and Δx is :

$$\Delta\alpha = \frac{gL - v^2 sin\alpha cos\alpha}{L(v^2 - gLtan\alpha)}\Delta x \quad (12)$$

Relative deviation can be calculated by $\Delta\alpha$ and already known α.

Similarly, find derivative of v in (10) and let x=L, the maximum allowable deviation of shooting velocity, $\frac{\Delta x}{\Delta \alpha}$ can be found.

$$\Delta v = \frac{gL - v^2 \sin\alpha\cos\alpha}{gL^2} v\Delta x \quad (13)$$

Relative deviation of (12), (13):

$$\left|\frac{\Delta v}{v}\right| = \left|\Delta\alpha\left(\frac{v^2}{gL} - \tan\alpha\right)\right| \quad (14)$$

SOLUTIONS OF MODEL AND ANALYSIS OF RESULTS

Minimum shooting velocity and corresponding shooting angle for different shooting heights

When v is the minimum shooting velocity, v_{min}, (5) will be an equation. Then, the shooting angle α_0 can be found by (3):

$$\tan\alpha_0 = \frac{v^2}{gL} \quad (15)$$

Let h=1.8~2.1(m). Use the formula $v^2 = g\left[H - h + \sqrt{L^2 + (H-h)^2}\right]$ to find v_{min}. Then α can be found by $\tan\alpha_0 = \frac{v^2}{gL}$. MATLAB is used to find solutions. Results are in TABLE 1.

Therefore, the minimum shooting angle matches the minimum shooting velocity. They are all decreasing when the shooting height is increasing. The shooting velocity should be less than 8m/s.

TABLE 1 : Minimum shooting velocities and corresponding shooting angles for different shooting heights

h(m)	v_{min} (m/s)	(α degrees)
1.8	7.6789	52.6012
1.9	7.5985	52.0181
2.0	7.5186	51.4290
2.1	7.4392	50.8344

Shooting angles and angles of incidence for different shooting velocities and heights

For shooting velocity v=8.0~9.0(m/s) and heights 1.8~2.1(m), the formula $\tan\alpha = \frac{v^2}{gL}\left[1 \pm \sqrt{1 - \frac{2g}{v^2}\left(H - h + \frac{gL^2}{2v^2}\right)}\right]$ can be used. MATLAB is used to find α_1 and α_2.

Find all α_2. And use the found α and formula $\tan\beta = \tan\alpha - \frac{2(H-h)}{L}$ to calculate different angle of incidence β_1 and β_2 for different shooting angles α_1 and α_2. Results as following:

TABLE 2 : Shooting angles and angles of incidence for different shooting velocities and heights

v(m/s)	h(m)	α_1	α_2	β_1	β_2
8.0	1.8	62.4099	42.7925	53.8763	20.9213
	1.9	63.1174	40.9188	55.8206	20.1431
	2.0	63.7281	39.1300	57.4941	19.6478
	2.1	64.2670	37.4019	58.9615	19.3698
8.5	1.8	67.6975	37.5049	62.1726	12.6250
	1.9	68.0288	36.0075	63.1884	12.7753
	2.0	68.3367	34.5214	64.1179	13.0240
	2.1	68.6244	33.0444	64.9729	13.3583
9.0	1.8	71.0697	34.1327	67.1426	7.6550
	1.9	71.2749	32.7614	67.7974	8.1663
	2.0	71.4700	31.3881	68.4098	8.7321
	2.1	71.6561	30.0127	68.9840	9.3472

As found in previous calculations, ball would go into the hoop when β is larger than 33.1°. Here, β_2 is always smaller than 33.1° and thus doesn't satisfy the condition. Therefore, when the sizes of ball and hoop are considered, the shooting angle can only be β_1, which matches α_1. It's obvious that at certain velocity, higher shooting height means the shooting angle should be larger. But when the velocity increases, there is less influence from the height to the angle. The influence is around 1°. At a certain shooting height, the shooting angle should increase along with the increase of velocity. The influence of shooting velocity is between 7~9°.

Analysis of the maximum deviation of shooting angle and shooting velocity

Calculate the maximum deviation of shooting angle $\Delta\alpha$ and $\frac{\Delta\alpha}{\alpha}$ by using $\Delta\alpha = \frac{gL - v^2 \sin\alpha\cos\alpha}{L(v^2 - gL\tan\alpha)}\Delta x$ and the already found α_1. Then calculate the maximum deviation of shooting velocity Δv and $\frac{\Delta v}{v}$ with equations (13) and (14). The following Table includes only results when h=1.8(m) and h=2.0(m).

TABLE 3 : Deviation relations between shooting angles and velocities

h(m)	α (degrees)	v(m/s)	$\Delta\alpha$	Δv	$\left\lvert\frac{\Delta\alpha}{\alpha}\right\rvert$	$\left\lvert\frac{\Delta v}{v}\right\rvert$
1.8	62.4099	8.0	-0.7562	0.0528	1.2261	0.6597
	67.6975	8.5	-0.5603	0.0694	0.8276	0.8167
	71.0697	9.0	-0.4570	0.0803	0.6431	0.8925
2.0	63.7281	8.0	-0.7100	0.0601	1.1140	0.7511
	68.3367	8.5	-0.5411	0.0734	0.7918	0.8640
	71.4700	9.0	-0.4463	0.0832	0.6244	0.9243

In all, ads are rather small. Further analysis reveals that, the higher the velocity, the less the allowable deviation of angle. While higher the allowable deviation of velocity is, there is stricter requirement on angle than on velocity. At certain shooting velocity, higher shooting height means less allowable deviation for shooting angle and more allowable deviation for shooting velocity. However at this time, there is a more lenient requirement on shooting angle and shooting velocity.

IMPROVEMENTS ON MODEL

When the air resistance on horizontal direction is considered, the trajectory of centre of ball should be calculated by differential equations. As the resistance is very weak, it can be simplified and calculated as previous ones.

If only resistance on horizontal direction is considered and the resistance is direct proportional to velocity, the differential equation of horizontal motion when the proportionality coefficient is k is:

$$\ddot{x} + k\dot{x} = 0,\ x(0) = 0,\ \dot{x}(0) = v\cos\alpha \quad \textbf{(16)}$$

The solution is:

$$x(t) = v\cos\alpha\,\frac{1 - e^{-kt}}{k} \quad \textbf{(17)}$$

As the resistance is not strong and t is small too (around 1 sec), so a Taylor expansion on (17) and omission of terms above second order would bring following (as vertical resistance is not considered, $y(t)$ is still the same with equation (18):

$$x(t) = vt\cos\alpha - \frac{kvt^2\cos\alpha}{2};\ y(t) = v\sin\alpha t - \frac{gt^2}{2} \quad \textbf{(18)}$$

When sizes of ball and hoop are not considered, the condition that centre of ball goes onto the centre of hoop is expressed in following function set:

$$vt\cos\alpha - \frac{kvt^2\cos\alpha}{2} - L = 0;\ vt\sin\alpha - \frac{gt^2}{2} - (H - h) = 0 \quad \textbf{(19)}$$

Solutions can be found in same way in model 1) and 2). Different shooting angels and angle of incidence can be found for different shooting velocities and shooting heights.

CONCLUSION

Goal percentage is crucial in any basketball game. This paper presents physical motion analysis with mathematical theories, and finds optimal shooting height and shooting velocity by differential methods. It also points out the shooting angle and angle of incidence at certain heights, as well as the maximum deviations for angle and velocity by extremism principle. MATLAB tool is used to solve complicated equations. The paper finds that: The variations of shooting angle poses more influence to the goal percentage than those of shooting velocity. When the shooting height is between 1.8~2.1, the ideal shooting velocity is higher than 8m/s and angle of incidence should be larger than 33.1o. The higher the shooting velocity, the less the allowable deviation there is for shooting angle. While the allowable deviation of shooting velocity increases, there is a stricter requirement on shooting angle than shooting velocity. When the shooting velocity is fixed, more shooting height means less allowable deviation of shooting angle and more allowable deviation of shooting velocity. This model provides rational advices to the training of basketball players.

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