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Matlab numerical simulation-based tennis drop point judgment model research

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ABSTRACT

In tennis competition "Hawk-Eye" actually is capturing three-dimensional coordinate, speed size, direction when tennis is flying, calculating its flight trajectory and defining its drop points. According to "Hawk-Eye" principle, the paper firstly assumes it's breezeless and has even air density, tennis generated deformation by hitting is ignored, it accurately makes tennis flight suffered gravity, air resistance, decomposes tennis movements into vertical direction and horizontal direction, utilizes calculus thought to solve, and then gets tennis shooting range, dropping time and dropping position. Finally it increases tennis rotation, according to Bernoulli conservation formula, pressure generated Magnus force that is vertical to speed direction and angular speed direction, applies integral principle to solve Magnus force coefficient, by calculating vertical direction's tennis rising height required time, it establishes mathematical model and detailed describes flight trajectory and drop point in case tennis is rotating, and utilizes Matlab software to draw intuitional flight trajectory graph.

KEYWORDS

Tennis drop point; Matlab simulation; Bernoulli equation; Magnus force; Mathematical model.

INTRODUCTION

Tennis event has swept the world, is a kind of traditional elegant event that integrates competitiveness and appreciation. In competitive process, except for referees, it needs multiple linesmen to judge drop point through getting close to line-out. Even so, due to tennis running speed in the air is so fast, after landing, it often has disputes on players' drop point is in line or out of line.

"Hawk-eye" is called instant review system, it utilizes high speed camera to simultaneous capture tennis flight trajectory basic data from different perspectives; then generates the data into three-dimensional images ;finally it utilizes instant imaging technique to clearly presents tennis movement route and drop point. "Hawk-eye" indicated drop point is a shadow, is not achieved by shooting but by precise calculating flight data. Zhu Zheng-Yu, Song Zi-Xia (2009) by statistics and analyzing, they thought tennis doubles service was more important than singles, service drop point selection ratio and service ace ratio had inseparable relations[1]. Yang Yu-Jian, Wu Xia-Wei (2012) researched on world excellent men tennis players receiving drop points and efficiency[2]. Zhao Yang (2013) analyzed tennis players drop point anticipation accuracy influence factors, explored high level athletes and low level athletes' tennis drop points prediction and judgment differences generation factors[3]. All above literatures are analyzing from drop point importance, and little literatures are starting from tennis drop point prediction and judgment. According to "Hawk-eye" principle, the paper establishes tennis flight route models in case with rotation, non-rotation, and considering air resistance, judges tennis drop point, utilizes Matlab software to draw intuitional flight trajectory graph.

TENNIS FORCE ANALYSIS

Tennis belongs to one kind of balls events; it has common features of ball kind event. If it ignores air resistance, then in case there is no rotation, only suffers gravity effects, its trajectory is standard parabola. But tennis soaks in the air, air has viscosity, so it has resistance on moving objects, in ball kind event, air resistance has great impacts on ball horizontal displacement, calculate tennis shooting range should consider air resistance effects.

According to fluid mechanics principle, air resistance can divide into friction resistance and pressure resistance. For tennis, its surface is not smooth, so it has friction resistance. And due to its movement speed is fast, Reynolds number is larger, then it surely has pressure resistance[4],the two items resistance compounds into air resistance that is always in the opposite direction of tennis movement speed. $f = \frac{1}{2}c\rho Av^2$, as Figure 1 shows.

Figure 1 : Tennis force graph

Air resistance is closely linked to speed, so its size and direction are constantly changing. It causes inconvenience in computation. Therefore, it should decompose tennis movement, use calculus thought to solve its speed size, direction, horizontal displacement, vertical displacement at one moment, and then get trajectory mathematical expression. For the third topic, tennis adds rotation, tennis surface is not smooth, when rotating, ball surface drives surrounding gas to move, in this way in previous two questions when tennis moves forward, it led to gas separation to be greatly reduces, and it causes pressure resistance reducing. At the same time ball surrounding air speed generates changes, according to Bernoulli conservation formula, pressure differences generate Magnus force that is vertical to speed direction and angular speed direction.

$F_M = 8\pi\rho a^3 v / 3$

Due to coefficient ω is unknown, it should calculate its size by inputting trajectory passed coordinate point (0.2215,1.5517,1.0485),then can define trajectory equation, its trajectory equation is equal to a section of vertical direction's parabola that opens with certain arc. By establishing calculus solved mathematical model, and utilize Matlab software, respectively draw its vertical direction parabola trajectory and horizontal direction deflection trajectory, then it can work out tennis drop point in case it has rotation.

TENNIS FORCE FLIGHT MODEL ESTABLISHMENT

At first, set one point three-dimensional coordinate as (x_0, y_0, z_0), speed size as v_0 ,direction is ($p, q, 1$),for the purpose of computation convenience, it can firstly regulate the point as new established coordinate system origin, take speed direction horizontal projection as *x* axis direction,*z* axis changes into *y* axis direction, then tennis speed direction is ($\sqrt{p^2+q^2}$,0,1). Take new coordinate system as calculation criterion, then tennis movement trajectory changes into two-dimensional figure that is figure in 0-*xz* plane, decompose speed, from which f size is in direct proportion to speed V quadratic, it can be simplified into $f = kv^2$. It has following equation :

x direction:

$$\mathbf{m}\frac{\partial^2 \mathbf{x}}{\partial \mathbf{t}^2} = -\mathbf{k}\left(\frac{\partial \mathbf{x}}{\partial \mathbf{t}}\right)^2 \tag{1}$$

y direction:

$$m\frac{\partial^2 y}{\partial t^2} = k\left(\frac{\partial y}{\partial t}\right)^2 - mg \qquad \frac{\partial y}{\partial t} \geq 0$$

$$\mathbf{m}\frac{\partial^2 \mathbf{y}}{\partial \mathbf{t}^2} = -\mathbf{k}\left(\frac{\partial \mathbf{y}}{\partial \mathbf{t}}\right)^2 - \mathbf{mg} \qquad \frac{\partial \mathbf{y}}{\partial \mathbf{t}} \leq \mathbf{0} \tag{2}$$

In formula, y equation due to projectile upward movement and downward movement instant air resistant direction and y positive direction change from reverse to forward direction, so it has air resistance front symbol changes[5].

Input substituend $v_x = \dfrac{\partial x}{\partial t}$ into formula (1),it has:

$$\mathbf{m}\frac{\partial \mathbf{v_x}}{\partial \mathbf{t}} = -\mathbf{k}\mathbf{v_x}^2 \tag{3}$$

Separate formula (3)variables and simultaneously make integral in two sides, and apply initial boundary condition $v_x |_{t=0} = v_{0x}$, it can get partial differential equation analytic solution:

$$\mathbf{v_x} = \frac{\mathbf{m}\mathbf{v_{0x}}}{\mathbf{m} + \mathbf{k}\mathbf{v_{0x}}\mathbf{t}} \tag{4}$$

Continue to adopt method of separation of variables, and adopt initial condition $x\big|_{t=0}=0$,it can get:

$$x=\frac{m}{k}\ln\left(\frac{kv_{0x}t}{m}+1\right) \tag{5}$$

Input substituend $v_y=\frac{\partial y}{\partial t}$ into formula (2.1)it has:

$$m\frac{\partial v_y}{\partial t}=-kv_y{}^2-mg \tag{6}$$

Similarly adopt method of separation of variables, and add initial condition $v_y\big|_{t=0}=v_{0y}$,it can get:

$$\mathbf{v_y}=\sqrt{\frac{\mathbf{mg}}{\mathbf{k}}}\,\mathbf{tan}\left[\mathbf{arctan}\left(\sqrt{\frac{\mathbf{k}}{\mathbf{mg}}}\mathbf{v_{0y}}\right)-\mathbf{t}\sqrt{\frac{\mathbf{gk}}{\mathbf{m}}}\right] \tag{7}$$

By substituend $v_y=\frac{\partial y}{\partial t}$, continue to separate variables integrals,and add initial condition $y\big|_{t=0}=h$,it can get:

$$\mathbf{y}=\mathbf{h}-\frac{\mathbf{m}}{\mathbf{k}}\ln\left|\frac{\cos\left[\arctan\left(\sqrt{\frac{\mathbf{k}}{\mathbf{mg}}}\mathbf{v_{0y}}\right)\right]}{\cos\left[\arctan\left(\sqrt{\frac{\mathbf{k}}{\mathbf{mg}}}\mathbf{v_{0y}}\right)-\mathbf{t}\sqrt{\frac{\mathbf{gk}}{\mathbf{m}}}\right]}\right| \tag{8}$$

Formula (8) in formula (7),when $v_y=0$,it gets maximum value $y_{\max}$, which also has:

$$\mathbf{t}=\mathbf{t_1}=\sqrt{\frac{\mathbf{m}}{\mathbf{gk}}}\,\mathbf{arctan}\left(\sqrt{\frac{\mathbf{k}}{\mathbf{mg}}}\mathbf{v_{0y}}\right) \tag{9}$$

$$\mathbf{y_{max}}=\mathbf{h}-\frac{\mathbf{m}}{\mathbf{k}}\ln\left\{\cos\left[\arctan\left(\sqrt{\frac{\mathbf{k}}{\mathbf{mg}}}\mathbf{v_{0y}}\right)\right]\right\} \tag{10}$$

Due to t_1 moment equation (8) gets maximum value that projectile arrives at top point, in equation (8), time variable t range is $t\in[0,t_1]$.

Input $v_y=\frac{\partial y}{\partial t}$ into formula (2.2),and adopt method of separation of variables, and add initial condition $v_y\big|_{t=t_1}=0$,it can get:

$$v_y=\frac{\sqrt{\frac{mg}{k}}\left(1-e^{\sqrt{\frac{kg}{m}}\left(t-t_1\right)}\right)}{1+e^{\sqrt{\frac{kg}{m}}\left(t-t_1\right)}} \tag{11}$$

Make following variable substitution:

$$a = e^{\sqrt{\frac{kg}{m}}\left(t-t_1\right)} \quad a > 0 \tag{12}$$

Utilize $v_y = \frac{\partial y}{\partial t}$, and formula (12), equation (11)can make variable substitution as :

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial a}\frac{\partial a}{\partial t} = \frac{\partial y}{\partial a} \bullet a\sqrt{\frac{kg}{m}} = \frac{\sqrt{\frac{kg}{m}}\left(1-a\right)}{1+a} \tag{13}$$

Continue to use method of separation of variables, and apply initial condition: $y\big|_{t=t1} = y_{\max}$, $a\big|_{t=t1} = 1$,formula (13)can make integral, and substitute variables back and get:

$$y = y_{\max} + \frac{m}{k}\ln\left\{\frac{4e^{\sqrt{\frac{kg}{m}}\left(t-t_1\right)}}{\left[1+e^{\sqrt{\frac{kg}{m}}\left(t-t_1\right)}\right]^2}\right\} \tag{14}$$

Both equation (11) and (14) variable t values range are $t \in \left[t_1, \infty\right]$. When projectile lands, it has $y = 0$, input it into formula (14) then can calculate projectile air movement total time T (take proper root from them):

$$T = t_1 + \sqrt{\frac{m}{kg}}\ln\left(\frac{2 - e^{-\frac{k}{m}y_{\max}} + 2\sqrt{1 - e^{-\frac{k}{m}y_{\max}}}}{e^{-\frac{k}{m}y_{\max}}}\right) \tag{15}$$

Input solved T value into formula (5),then it can get shooting range.

Finally it can get drop point coordinate (*x*,*y*,*z*),because in the beginning, it has already made coordinate transformation, when calculate drop point coordinate, it should also transform coordinates back, now drop point coordinate is (x_t, y_t, z).

$$x_t = x_0 + x \cdot \frac{p_0}{\sqrt{{p_0}^2 + {q_0}^2}},\ y_t = y_0 + x \cdot \frac{q_0}{\sqrt{{p_0}^2 + {q_0}^2}},\ z = 0.$$

MATLAB NUMERICAL SIMULATIONS

Make specific quantization on initial values data, it also can make specific quantization on drop point coordinate, similarly, utilize Matlab software, it can accurate describe tennis flight trajectory in the air. In the following, list known quantities one by one:

$h = 1m, m = 200g, k = 1/2c\rho A = 3.982 \times 10^{-3}, g = 9.8m/s^2,$
$v_{0x} = 0.9749m/s, v_{0y} = 15.7198m/s$

Input formula (5), (8),it can get:

$$x = 50.226\ln(0.3130t + 1),\ y = 1 - 50.226\ln\left|\frac{\cos(\arctan 0.0439)}{\cos(\arctan 0.0439 - 0.4417t)}\right|$$

It is tennis flight trajectory in the interval $t \in [0, t_1]$.
Input formula (9),it can get: $t_1 = 0.0993s$
Input formula (10),it can get: $y_{\max} = 1.0503m$
Input formula (5)(14),it can get:

$$x = 50.226\ln(0.3130t + 1),\ \ y = 1.0503 + 50.226\ln\{\frac{4e^{0.4417(t-t_1)}}{\left[1 + e^{0.4417(t-t_1)}\right]^2}\}$$

It is tennis flight trajectory in the interval $t \in [t_1, T]$.
Combine with two intervals, then as following Figure 2 shows:

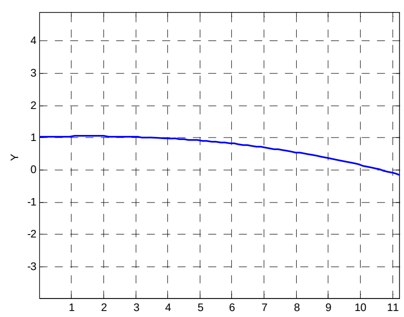

Figure 2 : Tennis flight two-dimensional trajectory figure

Now it needs to transform coordinates back to original coordinates:

$$x_t = x_0 + x \cdot \frac{p_0}{\sqrt{{p_0}^2 + {q_0}^2}},\ y_t = y_0 + x \cdot \frac{q_0}{\sqrt{{p_0}^2 + {q_0}^2}},\ z_t = y$$

Then make (x_t, y_t, z_t) three-dimensional figure as following Figure 3:

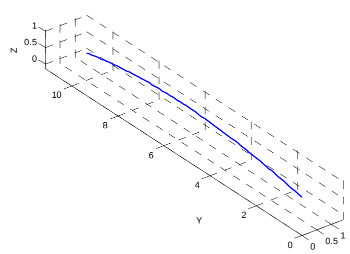

Figure 3 : Tennis flight three-dimensional trajectory figure

Finally, tennis drop point is (1.5249,12.1991,0)

MODEL EXPANSIONS

In tennis playing process, due to it should generate horizontal deflection, it often exerts rotation so that ball will increase Magnus Force that is vertical to speed direction and angular speed direction in movement, force changes, and trajectory naturally will change, and its flight route is not just in one plane but is a space curve. Magnus force size is in direct proportional to speed size, for convenience, it can be written into $F_M = Gv$,G is related to air density, tennis radius, rotational speed, its size is unknown, so it needs to solve by another coordinate point that trajectory goes through.

Due to two coordinates points height difference is almost zero, it can be thought to do horizontal move, first assume tennis rotational axis is parallel to z axis, so in research process, air resistance, speed and Magnus force are in the same horizontal plane, tennis force is as Figure 4.

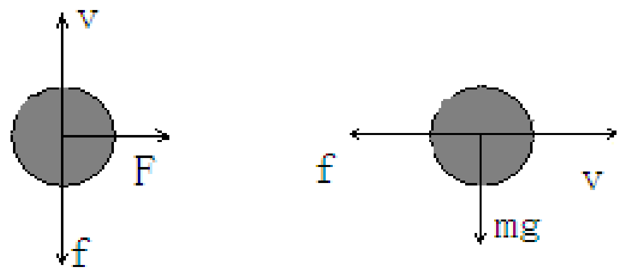

Figure 4 : Tennis force graph

For the purpose of calculation convenience, it still needs to transform coordinate system, take speed horizontal projection as *x* axis direction, offset direction as *y* axis direction, vertical direction z axis remains unchanged, then after axis transformation, it has following relationship :

$$x' = x\cos\alpha + y\sin\alpha \text{ , } y' = x\sin\alpha - y\cos\alpha$$

$$\cos\alpha = \frac{2}{\sqrt{260}}, \sin\alpha = \frac{16}{\sqrt{2^2+16^2}} = \frac{16}{\sqrt{260}}$$

After coordinates succeed in transformation, then it can list differential equation and then describe tennis.

Vertical direction: $\mathbf{m\frac{d^2z}{dt^2} = -mg}$ **(16)**

Horizontal tangent direction: $\mathbf{m\frac{dv}{dt} = -kv^2}$ **(17)**

Horizontal normal direction: $\mathbf{m\frac{v^2}{\rho} = Gv}$ **(18)**

Make integral of formula (17), add initial conditions $t_0 = 0$, $v|_{t=0} = v_0$, it can get:

$$\mathbf{v = \frac{mv_0}{m + kv_0 t}} \quad \textbf{(19)}$$

Because $ds = vdt$,then:

$$\mathbf{s = \int_0^r \frac{mv_0}{m + kv_0 t} dt = \frac{m}{k} \ln \frac{m + kv_0 t}{m}} \qquad \textbf{(20)}$$

Input $\rho = \frac{ds}{d\theta} = \frac{ds}{dt}\frac{dt}{d\theta} = v\frac{dt}{d\theta}$ into formula (18),it gets:

$$\mathbf{\frac{d\theta}{dt} = \frac{G}{m}} \qquad \textbf{(21)}$$

So it has:

$$\mathbf{\theta = \int_0^\theta d\theta = \int_0^t \frac{G}{m} dt = \frac{G}{m} t} \qquad \textbf{(22)}$$

Therefore it has:

$$\mathbf{v_{x'} = v\cos\theta = \frac{mv_o}{m + kv_0 t} \cos \frac{G}{m} t} \qquad \textbf{(23)}$$

$$\mathbf{v_{y'} = v\sin\theta = \frac{mv_0}{m + kv_0 t} \sin \frac{G}{m} t} \qquad \textbf{(24)}$$

Considering $kv_0 t$ is smaller by comparing to m [6],it can be ignored, approximately has :

$$\mathbf{v_{x'} = v_0 \cos \frac{G}{m} t} \qquad \textbf{(25)}$$

$$\mathbf{v_{y'} = v_0 \sin \frac{G}{m} t} \qquad \textbf{(26)}$$

Then:

$$\mathbf{x' = \frac{mv_0}{G} \sin \frac{G}{m} t} \qquad \textbf{(27)}$$

$$\mathbf{y' = \frac{mv_0}{G} \left(1 - \cos \frac{Gt}{m} \right)} \qquad \textbf{(28)}$$

By (26)(27) two formulas eliminate *t* and can get:

$$\mathbf{y'^2 - \frac{2mv_0}{G} y' + x'^2 = 0} \qquad \textbf{(29)}$$

According to trajectory one point (0.2215,1.5517,1.0485), input into formula (28)(after coordinate transformation), and can get:

$G = 0.0699\omega kg$

Input result into formula (26), solve the distance flight time $t = 0.0996s$,now verify tennis rotational axis is parallel to z axis or not:

Due to tennis almost has no displacement in vertical direction, air resistance components in vertical direction can be ignored, so, it only suffers gravity impacts in vertical direction, decompose initial time speed, and can get: $v_z = 0.975m/s$,in vertical direction, rising $1.0485 - 1 = 0.0485m$,it needs time $t' = 0.0990s$,by calculating, two time phases relative error is only 0.6%,it can be ignored, so assume that tennis rotational axis being parallel to z axis is at work. After that, transform formula of coordinates:

$$x' = x\cos\alpha + y\sin\alpha \text{ , } y' = x\sin\alpha - y\cos\alpha$$

Input formula (27)(28), then solve trajectory equation under original coordinate:

$$x = 5.5896(8 - 8\cos 0.3495t + \sin 0.3495t)$$

$$y = 5.5896(8\sin 0.3495t - 1 + \cos 0.3495)$$

$$z = -4.9t^2 + 0.975t + 1$$

In the following, calculate tennis dropping time T: $v_{oz}T - \frac{1}{2}gT^2 = -1$

It can get : $T = 0.5621s$. Utilize Matlab to draw flight trajectory in the interval $t \in [0,0.5621]$ as following Figure 5 shows:

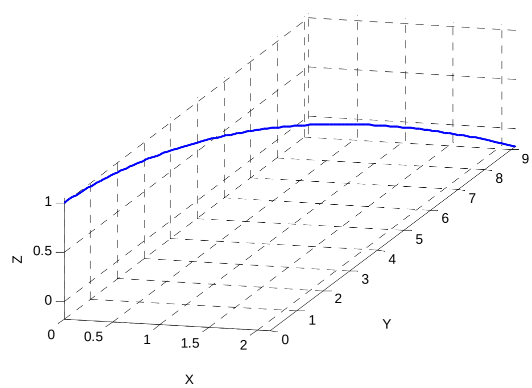

Figure 5 : Tennis flight trajectory simulation graph

Finally, work out tennis drop point as (1.9474,8.6042,0)

CONCLUSION

The paper applies integral principle to correctly describe tennis flight speed changes and trajectory curve; it avoids errors that are caused by unchanged speed and other assumptions. In addition, when establish model, we apply coordinate rotation transformation method, it greatly reduces calculation difficulty, and by coordinate transformation, transform three-dimensional figure into two-dimensional figure, it strengthens tennis flight route intuition to a certain angle, but increases steps of transforming coordinates into original coordinates, it needs more rigorous logic. The paper result is more specific and accurate comparing to other literatures. Apply mathematical software Matlab to solve calculation and drawing problems, it let result to be more intuitional.

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