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# Mathematical analysis of the table tennis hitting process based on dynamics and hydrodynamics

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## ABSTRACT

In this paper, it conducts mechanics analysis on the force condition and motion state during the movement of table tennis ball, and uses the kinetic theory and fluid mechanics theory to describe the mechanical features of the ball. Through the mechanical parameters proves the influencing factors of the ball's movement states, obtains effective mathematical equations, and confirms the scientific principles that the movement process follows. In the attack, in order to receive better attack effect the ball must have high-speed movement and strong rotation; in the defending, you must first determine the speed rotation and drop point of the ball, prepare preparedness to fight response time and improve defense capabilities.



## KEYWORDS

Bernoulli's principle;
Table tennis;
Kinetics;
Hydrodynamics.

## INTRODUCTION

The achievements of Chinese table tennis players in international competitions are not only inseparable with the leaps and bounds improvement of technique in peacetime training, but also closely related with the improvements of table tennis hitting technique. Predecessors through scientific means and analysis correct method of made the hitting table tennis technology continue to improve, but from another point of view we can also draw the appropriate conclusions, and can develop it further.

This article abandon previous research methods in these areas, takes the physical movement mechanics this natural science as a research tool, conducts a detailed introduction on the collision momentum of racket and table tennis, force analysis, trajectory formation during the movement of the ball, and positioning analysis on the falling point of the ball, from the perspective of tennis biomechanics conducts simulation analysis of table tennis trajectory to effectively understand the relationship between force and trajectory in table tennis movement. It promotes the world popularity of "national sport" sports, improves the training efficiency of table tennis, and makes the physical mechanics better serve the table tennis.

## THE MECHANICAL ANALYSIS OF THE COLLISION PROCESS BETWEEN TABLE TENNIS AND RACKET

This paper describes in detail the entire process of the collision between tennis ball and racket, the ball movement in the air, the collision between the tennis ball and the table and fly out the table surface, and conducts the mechanical analysis.

## The role of racket on table tennis

Racket hitting table tennis ball is a complex mechanical process, this article takes the effect of the racket to the ball as the function process of two force, the impact force F perpendicular to the racket surface and the friction $f$ tangential to the racket surface, and the initial velocity and the initial angular velocity is 0. Suppose the mass of the table tennis ball is $m$ , the diameter is D, shown in Figure 1, the racket is bending forward and has a ā angle with the horizontal surface, shoot action time of the ball, tennis racket and speed of impact.

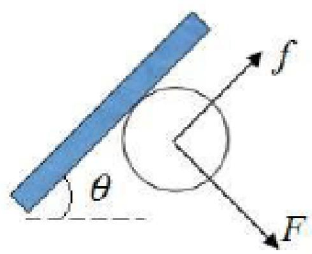


**Figure 1 : Analysis chart of racket initial parameter**

The impulse of $f$ in time $t_0$ is $\int_0^{t_0} fdt$ denoted by $I_f$ , the impulse of $F$ in timeis denoted by. $F$ Go through the centroid of table tennis ball, $f$ is with tangent tennis ball, so friction $f$ produces a torque of $\frac{D}{2}f$ to table tennis, the impulse moment of this moment in time $t_0$ is $\frac{D}{2}\int_0^{t_0} fdt$ , that is $\frac{D}{2}If$ .

Tennis rotation moment of inertia of its own center is $I_e = \frac{2}{3}mr^2 (r = \frac{D}{2})$ , the final velocity in the direction perpendicular to the racket is $v_F$ , the final velocity along a direction parallel to the racket is $v_f$ , the final angular velocity is $\omega$ , according to the momentum conservation law and angular speed law in physics, we can obtain:

$$I_F = mv_F \quad (1)$$

$$I_f = mv_f \quad (2)$$

$$\frac{D}{2}I_f = I_e\omega \quad (3)$$

Through these three equations the following can be successfully solved:

$$v_F = \frac{I_F}{m} \quad (4)$$

$$v_f = \frac{I_f}{m} \quad (5)$$

$$\omega = \frac{DI_f}{2I_e} = \frac{3I_f}{mD} \quad (6)$$

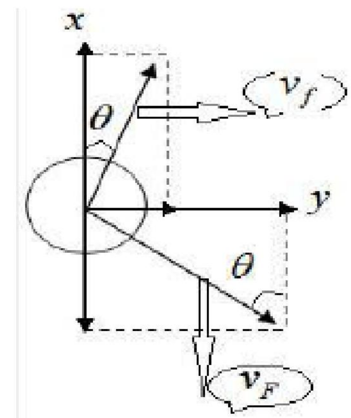


**Figure 2 : Force exploded view of the table tennis in $x$ and $y$ direction**

The horizontal and vertical component $v_F$ and $v_f$ need to be decomposed into the horizontal and vertical directions. As shown in Figure 2, $x$ and $y$ respectively means the horizontal and vertical directions. Thus the following relationship can be obtained:

$$v_x = v_F \sin\theta + v_f \cos\theta = \frac{I_F}{m}\sin\theta + \frac{I_f}{m}\cos\theta \quad (7)$$

$$v_y = v_f \sin\theta - v_F \cos\theta = \frac{I_f}{m}\sin\theta - \frac{I_F}{m}\cos\theta \quad (8)$$

By the formula (7) (8) obtain the table tennis' velocity after the moment of collision with racket.

## Table Tennis movement trajectory

Table tennis trajectory refers to the trajectory from the table tennis leaving racket to just touching the table board, is the important factor to determine the touching point of the ball with the table board. Therefore, the analysis on the moving process is very important and is an important indicator of players to hit good shots. During the flight table tennis mainly suffers the vertically downward gravity, vertical upward buoyancy, also the resistance brought by air in the opposite direction of movement. In the motion process of table tennis, due to the role of the racket, table tennis also rotates around its own axis, so we have to conduct detailed analysis on the rotation of the table tennis.

For the table tennis trajectory analysis, we should assume that table tennis exercises in the plane $xoy$, shown in Figure 3. And the flight speed of the ball is $v$, and the angle between the horizontal line is $\theta$, the angular velocity of the ball rotates around itself is $\omega$, so the linear speed of rotation is $\gamma = \frac{\omega}{2\pi}$, the axis of its rotation is perpendicular to plane $xoy$. Table tennis ball's gravity is $G = mg$, wherein, $g$ is the commonly used $9.8m/s^2$. We assume that the density of air is $\rho$, so buoyancy is $F_{\text{buoyancy}} = \frac{1}{6}\rho g \pi D^3$. Meanwhile, the resistance that table tennis suffers in the motion process is $F_{\text{resistance}} = \frac{1}{2} C \rho A v^2$, where is $C$ the drag coefficient of table tennis, this resistance coefficient is related with the geometry of table tennis, $A = \frac{\pi D^2}{4}$ is the cross sectional area of table tennis. The rotation that table tennis has in motion process is $F_{\text{turn}} = C_l \rho D^3 \gamma v$, and where $C_l$ is the lift coefficient.

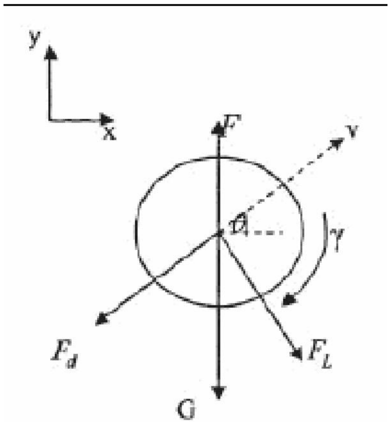


**Figure 3 : The force analysis of table tennis flying in the air**

Figure 3 is the force analysis of table tennis flying in the air, the force analysis directly determines the flight path of table tennis; according to the stress analysis, the force can be decomposed along $x$ and $y$ direction, thus we can list equation expressions of motion as follows.

$$m\frac{d^2x}{dt^2} = C_L \rho D^3 f v \sin\theta + \frac{1}{2} C_d \rho A v^2 \cos\theta \tag{9}$$

$$m\frac{d^2y}{dt^2} = mg - \frac{1}{6}\rho g \pi D^3 + C_L \rho D^3 \gamma v \cos\theta + \frac{1}{2} C_d \rho A v^2 \sin\theta \tag{10}$$

These two equations are the force analysis during the movement process of tennis in the air.

## Analyze the rotation of the ball in the movement process

As shown in Figure 4: apply a force through centroid point O on the sphere, the sphere can only move along the direction parallel to the force.

As shown in Figure 5: apply a force deviate centroid point O on the sphere, the sphere can generate a parallel movement and produce rotation around itself under the action of the F. Its rotation effect is determined by the moment size of the F on point O.

From the above analysis, giving the ball a force not through the center of the sphere can make table tennis rotate up.

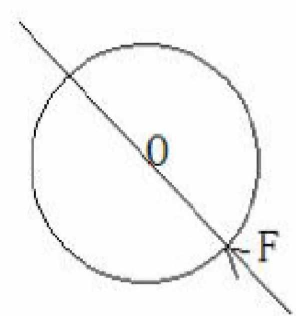


**Figure 4 : Apply a force through centroid point O on the sphere**

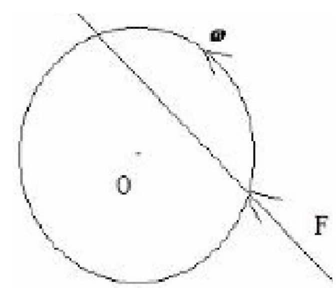


**Figure 5 : Apply a force deviate centroid point O on the sphere**

## Ball's rotation and friction force

From the previous analysis, the key to make table tennis rotate up is that the forces acting on the ball is not passing through the center, and this force is from the friction of the ball on the racket. Shown in Figure 6, when the racket strikes the ball, the racket have relative movement to the ball, this relative movement can produce friction.

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Shown in Figure 6, the moment of hitting the ball, the athletes pull the racket upward with force; the ball suffers the two forces; by this time $F$ is through the center of the sphere, and therefore does not produce torque. The time the ball flight forward under the effect of $F$, $f$ is tangent to the ball, generating the effect that the ball counterclockwise rotates, which is topspin ball in the table tennis movement process. Similarly, if pulling the racket in different directions at the racket hitting instant, it will produce different direction of the ball and friction tangent to the ball.

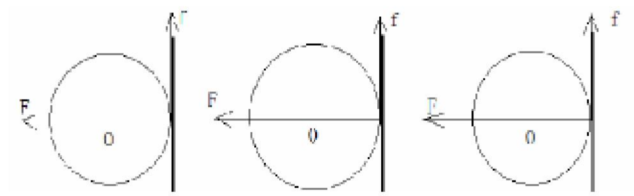


**Figure 6 : Ball's rotation and friction force**

In fact, in table tennis movement process: cut, chop, twist, pull, belt, lift and other technique actions refer to make the racket and the ball produce side relative movement when the racket and the ball contact instant, so that the ball suffers the effect of the lateral friction and produces rotation.

## Bernoulli's Principle and curveball and rotation ball

In a variety of table tennis flight trajectories, often there will be many curve balls whose trajectories are not in the same plane, that is, table tennis' flight is not in a plane, similar to banana kick in the football. So, why the ball will appear different and various arcs, air makes trouble mainly. To solve this problem we must understand the Bernoulli principle. In Figure 7, blow between the freedom of the two pendulous white piece of paper, so that the air occurs flow, we will find two papers will be attracted to each other; according to the Bernoulli principle, the intensity of pressure is small when the fluid flow rate is big, and the intensity of pressure is big when the fluid flow rate is small, so that two pieces of paper will receive lateral pressure $F_1$ and $F_2$ and be attracted to each other.

In the forward process of table tennis, due to the rotation of the ball there will have a similar situation as shown in Figure 8. Rotation ball study: the flow rate of air above the ball is small relative to the ball and the flow rate of air below the ball is big relative to the ball, thus creating downward lateral pressure on the ball. The flight path of the ball is low, and topspin ball is just the opposite. Sidespin ball will appear lateral pressure; the role of this lateral pressure is to make the flight direction of the ball side rotation, similar to the banana kick in football. (Figure 9) trajectory□ is the parabolic trajectory of the no spiral ball, while trajectorya□ is the S-type trajectory of the strong side spiral ball.

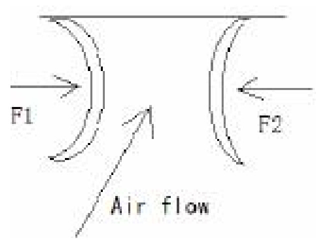


**Figure7 : The Bernoulli principle phenomenon of two white papers**

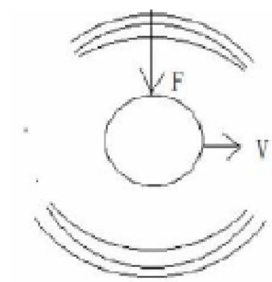


**Figure 8 : The Bernoulli principle phenomenon of rotation**

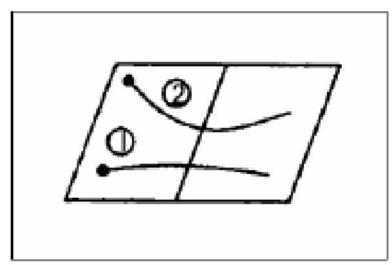


**Figure 9 : The Bernoulli principle movement trajectory of the rotation ball**

# THE MECHANICAL ANALYSIS ON THE HITTING OUT BALL

## Analysis on topspin ball

Topspin ball flies in the air and rotates forward along the horizontal axis (Figure 10); Due to viscosity, the ball will rotate along with the surrounding air. When the ball flights forward, the air circulation above the ball is in contrast with air resistance in the head-on face, the air flow rate of this region is small; the air circulation below the ball is in accordance with air resistance in the head-on face, the air flow rate of this region is larger. According to Bernoulli's theorem, the intensity of pressure is small when the fluid flow rate is big, and the intensity of pressure is big when the fluid flow rate is small, so the upper edge of the ball has a strong pressure, the lower edge of the ball has a small pressure, the differential pressure is generated along the upper edge and the lower edge and the direction is downward; As the sphere circulation of the upper and lower edges is perpendicular to the direction of the air circulation, it does not cause the pressure difference. According to the parallelogram law, the joint force (P) acting on the ball points to the front below of the ball. From the above analysis, topspin ball has a downward trend during the flight, the flight arc is relatively steep and the flight distance is shorter than the non-rotating ball.

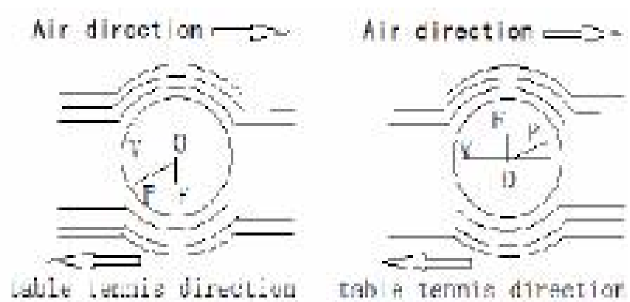


**Figure 10 : Bernoulli mechanical analysis of spin ball**

### Analysis on the under-spin ball

Under-spin ball is on the contrary, it flies in the air and rotates backward along the horizontal axis (Figure 10); the air circulation above the ball is in accordance with the air flow direction, the air circulation below the ball is in contrast with the air flow direction. Therefore, the intensity of pressure of the ball's upper edge is small, the intensity of pressure of the ball's lower edge is big, pressure difference is generated between the upper and lower edges and the direction is upward; The front edge and back edge of the ball have no pressure difference, the joint force (P) acting on the ball points to the top rear f of the ball. Therefore, under-spin ball has an upward trend during the flight, the flight arc is relatively big and the flying speed is more and more slow.

### Analysis on the left and right spin ball

Left spin ball in flight rotates left along the approximate vertical line axis (Figure 11, top view). During flight the air flow of the upper edge and lower edge of the ball does not change, and no pressure difference is generated; the air circulation of the front edge and back edge of the ball is perpendicular to the air flow direction, the pressure difference is also not generated; The air circulation in the ball's left is in the opposite direction of air flow, and the air circulation in the ball's right is in the same direction of air flow. The left and right side of the ball generates a pressure difference, and the direction is right; the joint force (P) acting on the ball points to the rear right of the ball, therefore, left sidespin ball has the right rotation trend in the air. While the flight curvature of right side spin ball is in contrast with the left spin ball and has the left rotation trend.

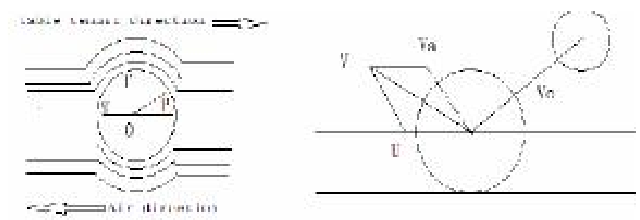

**Figure 11 : Bernoulli mechanical analysis of the left-spin ball**

### The dropping on table analysis of the spin ball

When the spin ball drops on the table, in addition to the interaction force in the vertical direction between the ball and the table (reaction force), the rotation force of the ball in the horizontal direction also acts on the table, the rotational force can acts on the table as frictional force. Thus, the table gives the ball an equal and opposite reaction force, i.e. the reaction of the rotational force.

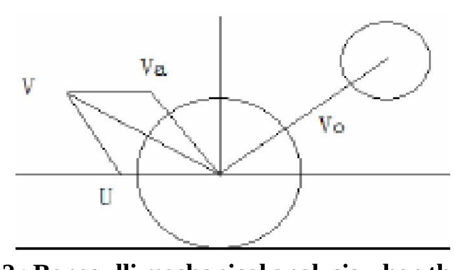


**Figure 12 : Bernoulli mechanical analysis when the rotation ball drops on the table board**

1) The dropping off table board analysis of the upper-spin ball: When the topspin ball drops on the table, the rotation force of the ball acts on the table rearward in the horizontal direction, and the reaction of the rotation force is forward (Figure 12). This force

gives the ball a forward speed U, so the reflected velocity of the ball is no longer V, but V. And it is resultant velocity V with U. The reflection angle of topspin ball significantly is greater than the incidence angle (β> α), but the speed of the ball after bouncing up from the table is fast, the impulsion is big; it remains rotation state in advancing process.

2) The dropping off table board analysis of the under-spin ball: When the under-spin ball drops on the table, opposite to the topspin ball, the rotational force of the ball is forward along the horizontal direction acting on table board, the rotation reaction force of table-board applied to the ball is backward (Figure 12). This force gives the ball to increase a backward speed υ, so the reflection speed of the ball is V. And it is resultant velocity V with υ. After the ball drops on the table, the reflection angle is smaller than the angle of incidence (β <α), the forward speed of the ball after a rebound is weakened and the impulsion is reduced, the ball becomes higher after dropping on the table. If the ball's under-spin is very strong, the forward force of the ball itself is very small, and the backward reaction force that tables gives the ball is larger than the forward force, then the ball will appear rebound after dropping on the table.

3) Dropping table-board analysis of side spin ball: When side spin ball drops on the table, because the ball is rotating approximate to the vertical axis, the ball does not change its force on the table-board after being dropped because of left and right side spin. Therefore, after the side-spin ball drops off the table its flight arc continues turning according to the original direction.

### Flat block and touching the racket analysis of the spin ball

When the topspin ball touches the racket, the continuing rotation ball will apply a friction from up to down on a racket, and the racket will give the ball a reaction force from the bottom up (Figure 13), this force increases the angle that the ball bounce upwards. So after the flat block and touching racket the topspin ball will rebound upward and easy to be out of bounds.

The under-spin ball is opposite to the upper-spin ball; when the ball hits the racket face, the reaction force direction of rotation is downward; the effect of this force is to decrease the angle of the ball rebound. So the under-spin ball after flat block and touching racket can easily off the net.

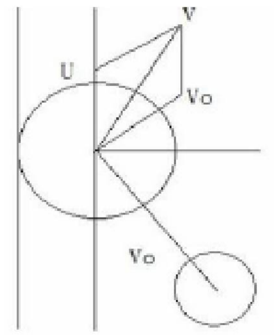


**Figure 13 : The mechanical analysis of the spin ball after the flat block and touching racket**

Based on the above analysis, when flat blocking (catching the ball) the left-spin ball, the ball's bounce direction is right, and the ball is easily out of bounds to the right; when flat blocking the right-spin ball, the ball's bounce direction is left, and the ball is easily out of bounds to the left.

## THE FORCE WHEN THE BALL CONTACTS WITH THE TABLE-BOARD

Ball receives friction of the table when the ball touching the table board, the size of the friction force is related with primary inertial force of the ball and the friction coefficient between the ball and the table board, the relevance with the value of X is small (unless X is very large). The direction of friction force is perpendicular to the front and back direction of the ball and opposite to the speed direction of the ball relatively to the table.

After touching the table the ball bounces, the reflection angle is greater than the incidence angle, shown in Figure 14. According to collision theory, the relationship between the incident angle A and the reflection angle B before and after the ball touches the table board:

$$\tan B = \frac{\tan A}{k}$$

Where, k is the coefficient of restitution, less than 1 (which may be determined experimentally). It is thus clear that, there is always B> A, i.e. the moving route of ball bouncing after touching the table is below the line that the ball comes; For loop drive ball, because the ball rotates intensely even after touching the platform, the movement route of ball bouncing after touching the table needs to plus the action on the ball, which continues to downward bend and side bend.

Suppose the speed of the coming ball off the table is v, v and the angle between v and the vertical line is α; the instantaneous the coming ball drops on the table, the angular velocity of the ball around the horizontal axis O is, $\omega_0$ after the collision with the table, due to the instantaneous friction effect, the horizontal velocity of the ball at the contact point is 0, recovery coefficient of the ball is K, the radius of the ball is r, the mass of the ball is m.

By the collision centroid motion theorem:

$$mu - mv = s \qquad (11)$$

Wherein s is the impulse acting on the ball, u is the sphere center speed when the ball bounces. The projection of the formula (11) into x axis is:

$$mu_x - mv_x = s_s \qquad (12)$$

Take sphere center O as the centroid, by the impulsive moment theorem:

$$J_c(\omega - \omega_0 = s_x r \qquad (13)$$

$J_c$ is the Moment of inertia of the ball, and the ball is hollow, with $J_c = (2/3)mr^2$; as the horizontal velocity of the touching point after the collision is equal to 0, so:

$$r\omega = -u_x \qquad (14)$$

Substitute the formula (13) into formula (14) and eliminate $s_x$ by the formula (12) to obtain:

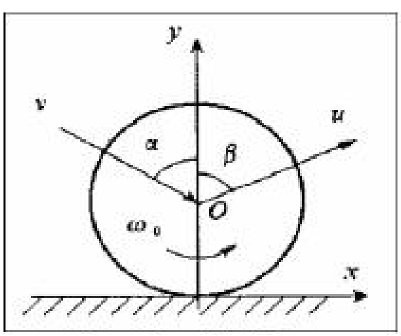


**Figure 14 : Motion state diagram when the ball drops on the table board**

$$u_x = \frac{mr^2 v_x - J_c r\omega_0}{J_c + mr^2} = \frac{1}{5}(3v\sin\alpha - 2r\omega_0) \qquad (15)$$

After the collision the ball's movement in the vertical direction is related to the recovery factor, therefore:

$$u_y = -Kv_y = Kv\cos\alpha \qquad (16)$$

So the bounce angle $\beta$ when the ball rebounded from the table-board is:

$$\tan\beta = \frac{u_x}{u_y} = \frac{1}{5K}\left(3\tan\alpha - \frac{2r\omega_0}{v\cos\alpha}\right) \qquad (17)$$

## CONCLUSIONS

In Table tennis, when the athlete is on the offensive, to receive a better offensive effect the ball must have high-speed movement and strong rotation. The main way is increasing the hitting power of the pat on the ball to increase a greater acceleration of the ball, making the ball produce a greater speed in an instant. In defense, you must first determine the speed rotation and placement of the ball, prepare preparedness to fight for the reaction time and improve defense capabilities.

This paper conducted detailed mechanics analysis and specific physics calculation on the various stages of the table tennis movement process, obtained the detailed calculation formula of table tennis motion law, made the tennis fans have a deeper understanding and understanding of physics knowledge; at the same time, also provided some theoretical foundation for table tennis players and tennis enthusiasts to better grasp the technology, made the mechanics theoretical knowledge in physics be more closely applied to real life.

## REFERENCES

**[1]** Bing Zhang, Yan Feng; The Special Quality Evaluation of the Triple Jump and the Differential Equation Model of Long Jump Mechanics Based on Gray Correlation Analysis. International Journal of Applied Mathematics and Statistics, **40(10)**, 136-143 **(2013)**.

**[2]** Cai Cui; Application of Mathematical Model for Simulation of 100-Meter Race. International Journal of Applied Mathematics and Statistics, **42(12)**, 309-316 **(2013)**.

**[3]** Fang Jie; The studies on Table Tennis Collision using computer simulation.Journal of Tianjin Institute of Physical Education, **18(3)**, 47-52 **(2003)**.

**[4]** Haibin Wang, Shuye Yang; An Analysis of Hurdle

Performance Prediction Based On Mechanical Analysis and Gray Prediction Model. International Journal of Applied Mathematics and Statistics, **39(9)**, 243-250 **(2013)**.

**[5]** Hongwei Yang; Evaluation Model of Physical Fitness of Young Tennis Athletes Based On AHP-TOPSIS Comprehensive Evaluation. International Journal of Applied Mathematics and Statistics, **39(9)**, 188-195 **(2013)**.

**[6]** Sun Zai, Yu Guang-xin, Guo Mei; The aerodynamics principle and flight trajectory simulation of loop table tennis ball. China Sport Science, **(4)**, 69-75 **(2008)**.

**[7]** Zhou Yu-qing, Ye Zhao-ning, Hao Zong-han; The calculation and analysis of air resistance in ball movement. Physics and Engineering, **(1)**, 55-60 **(2002)**.