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Logistic bounce model-based table tennis rebound process energy change influence factors research

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ABSTRACT

Table tennis makes friction work with air during falling process, one part energy is converted into internal energy, ball kinetic energy reduces, and converted gravity potential energy also reduces, it cannot arrive at original height. In case that want to let cast table tennis to bounce into position that is higher than that when cast, it should exert an external force, let cast table tennis to have larger kinetic energy, and kinetic energy under the external force should be above bounce (rebound) potential energy after table tennis landing, that is to say, now kinetic energy should be above potential energy so that only exert larger external force on table tennis and cast it out then can let it to bounce to position higher than that when cast. The paper established table tennis bounced logistic model, and u tilize Matlab to make simulation, which provides theoretical evidence for athlete better grasping racket swinging force.

KEYWORDS

Logistic model; Matlab simulation; Bounce; Energy conservation.

INTRODUCTION

Table tennis free falls from height, bounced maximum height after rebounding through ground always is lower than original height when fall, is because table tennis and air generate friction during falling process, gravity potential energy is converted into kinetic energy and its partial energy works at air, these internal energy consumes original gravity potential energy, losses partial energy, according to energy conservation law, table tennis mechanical energy reduces, it cannot arrive at original energy when it free falls, so table tennis free falls from height, bounced maximum height after rebounding through ground always is lower than original height when fall[1,2]. To let cast table tennis to bounce to position that is higher than that when cast, it can only exert external force on it, explain according to energy conservation law and energy transformation principle as well as energy transformation view :That is it should follow energy conservation law and energy transformation basic principle, to let cast table tennis to bounce to position that is higher than that when cast, it should exert an external force, let cast table tennis to have larger kinetic energy, and kinetic energy under the external force effecting should be above bounce (rebound) potential energy after table tennis landing, that is to say, now the kinetic energy should be larger than potential energy, so that only exert larger external force on table tennis and cast is out then it can bounce to position that is higher than that when cast. Table tennis player should well control racket swinging force so as to get good results in competitions, the paper aims to establish table tennis logistic bounce model, and use Matlab to make simulation, so that better guide to athletes.

PROBLEM ANALYSES

Height that a table tennis from racket is h_0, it falls in racket and rebounds, set restitution coefficient is e, without calculating air resistance.
(1) If e is constant, discuss ball height change rules. If e^2 and height h_n are in linear relations:

$$e^2 = \mu(1 - h_n/H_0) \tag{1}$$

Among them, H_0 is largets height, μ is parameter. To different parameters, discuss small ball height change rules.
(2) When parameter is constantly changing, analyze final distributed height.
(3) Calculat previous branch points.
(4) Use Lyapunov exponent to judge occurrence of chaos.
When ball falls from height h_n to racket, previous speed is:

$$v_n = \sqrt{2gh_n} \tag{2}$$

After ball collides with racket, rebound speed is:

$$v'_n = ev_n \tag{3}$$

Ball rebound height is:

$$h_{n+1} = e^2 h_n \tag{4}$$

If $e < 1$, then ball rebound height is constantly diminishing with times;if $e = 1$, then after ball rebounding, it always keep initial height;if $e > 1$, for example racket adds a upward impulsive force every time, then ball height is constantly increasing with times.

e^2 and height linear relations show:If ball height is larger, then restitution coefficient is smaller, on the contrary, it is larger. Set relative height is $x_n = h_n/H_0$, then next time rising relative height is

$$x_{n+1} = \mu(1 - x_n)x_n, (n = 0, 1, 2, \ldots) \quad (5)$$

That is famous logistic model. Due to relative height $0 \le x_n \le 1$, and $(1 - x_n)x_n$ maximum value is 1/4, parameter values is between 0and4. Ball height intensily relies on parameters.
First take a parameter, then take a relative height, accorign to iterative algorithm, calculate height after colliding next time, draw out height point, and so on. Then take another height parameter, recalculate height according to iterative algorithm, draw out height point, and so on.

PROGRAM SIMULATIONS

Program is as following:

```
clear
u=input(' Please input parameter (Reference value:0.5, 2, 3.25, 3.5, 3.56, 3.8):');
xn=0.9;
figure
plot(0, xn, '.')
text(0, xn, num2str(xn), 'FontSize', 10)
grid minor
title([' Collision height of table tennis ball with racket (\it\mu\rm=', num2str(u), ')'], 'FontSize', 10)
n=50;
axis([0, n, 0, 1])
hold on
for j=1:n
xn=u*(1-xn)*xn;
plot(j, xn, '.')
end
xn=0.1;
plot(0, 'ro')
text(0, xn, num2str(xn), 'FontSize', 10)
for j=1:nxn=u*(1-xn)*xn;
plot(j, xn, 'ro')
end
```

[Illustration]
(1) When exert program, user uses keyboard to input parameters, provide six parameters for selecting.
(2) Take the first larger initial height.
(3) Iterative calculate next height.
(4) Take the second smaller initial height. When illustrate chaos, revise the sentence as following, let the second height to be slightly larger than the first height.

```
e=1e-8;
xn=0.9+e;
text(0, 0, num2str(e), 'FontSize', 10)
```

(5) Similarly iterative calculate next height.
(1) As Figure 1shows, when parameter μis0.5, if take initial relative height as 0.9, after ball and racket colliding, height is constantly diminishing, final height is zero. Even take initial relative height as 0.2, after ball and racket colliding, height is also constantly diminishing, final height is zero.
(2) As Figure 2 shows, when $\mu$is 2, if take initial relative height as0.9, after ball first colliding, height is diminishing, height rises in future colliding, it keeps certain height in final colliding. If take initial

relative height as 0.2, then collding height is constantly increasing, finally it remains in certain heights. Process before height remaining stable is called transient process or temporary process, transient process is related to initial height, but stabilization of final height is unrelated to initial height. Height value is called fixed point that is point of repeating self trajectory.

(3) As Figure 3shows, when $\mu$is 3.25, no matter how initial height is, after going through process period, ball is finally alternately changing in two heights. Fixed points numbers will increase with parameters increasing.

(4) As Figure 4 shows, when μis3.5, no matter how initial height is, after going through process period, ball is finally alternately changing in four heights, only transient process to be slightly longer. Fixed points' numbers are further increasing with parameters increasing.

(5) As Figure 5 shows, when μis3.57, no matter how initial height is, after going through transitional period, ball is finally alternately changing in eight heights.

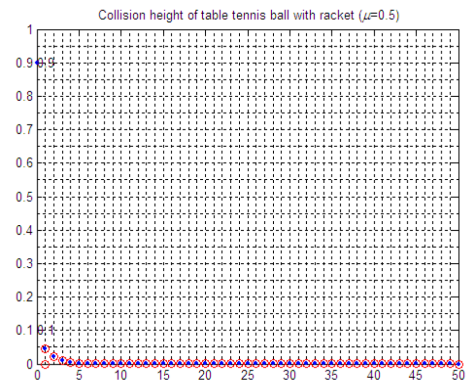


Figure 1 : When parameter $\mu$is 0.5

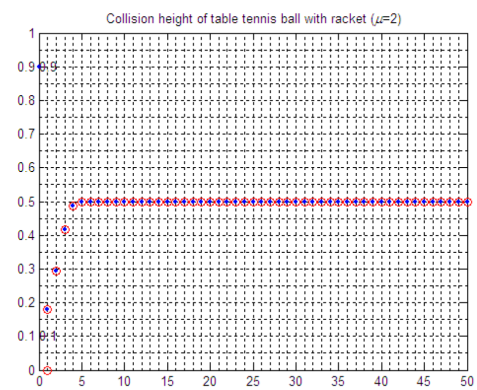


Figure 2 : When $\mu$is 2

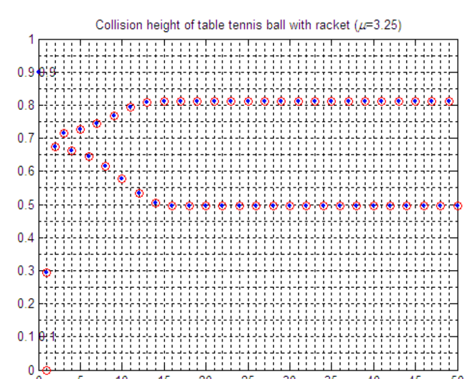


Figure 3 : When μis3.25

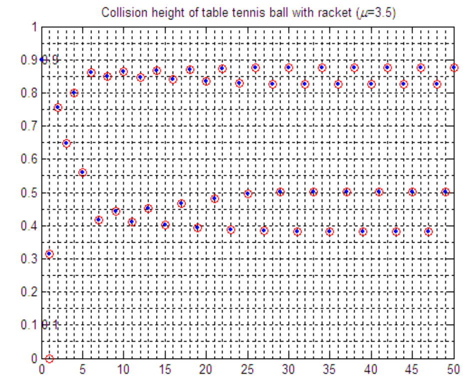


Figure 4 : When μis3.5

(6) As Figure 6 shows, when μis3.8, take the first initial relative height as 0.9, take the second initial relative height as 0.2, ball heights are desultorily chaning.

(7) As Figure 7 shows, μ is unchanged, the first initial relative height is unchanged, the second initial relative height only increases 10^{-5}, future heights changes are also widely different.

(8) As Figure 8 shows, $\mu$is unchanged, the first initial relative height is unchanged, the second initial relative height only slightly to be larger (10^{-8}), future heights changes are also widely different. Such motion that is very sensitive to initial conditions is called chaotic motion.

[Analysis](2) When parameter is constantly changing, similarly utilize (2.5) formula to calculate height. When iterations n is more enough, for periodic fixed point, x_n represents stable value x_∞. Take μ as independent variable, take $x = x_\infty$ as function, it can draw μ-$x$curve.

μ constantly takes value from 0to 4, first screen out transitional value by iterative algorithm, and further get iteration result by using iterative algorithm, draw out iteration chart.

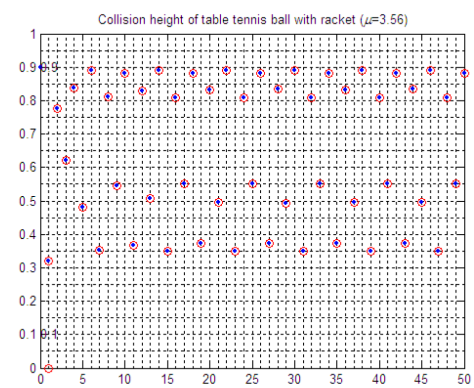


Figure 5 : When $\mu$is 3.57

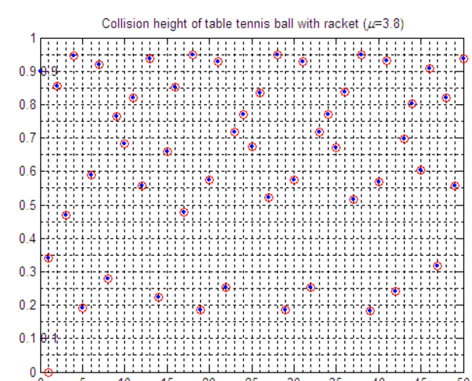


Figure 6 : When μis3.8

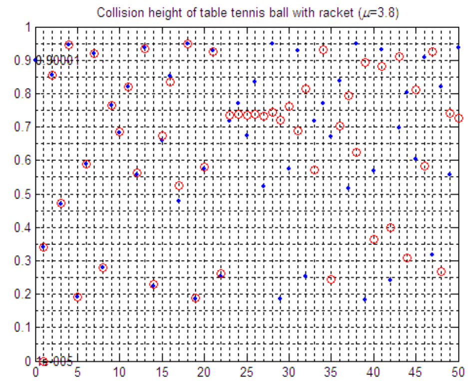


Figure 7 : The second initial relative height only increases 10^{-5}

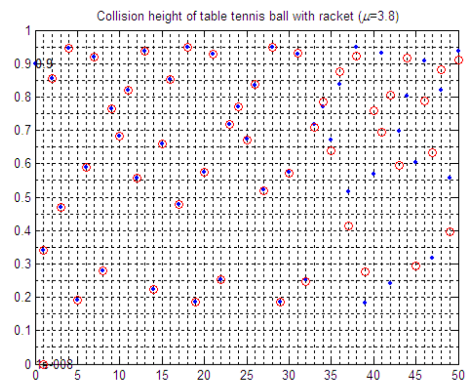


Figure 8 : The second initial relative height to be slightly larger (10^{-8})

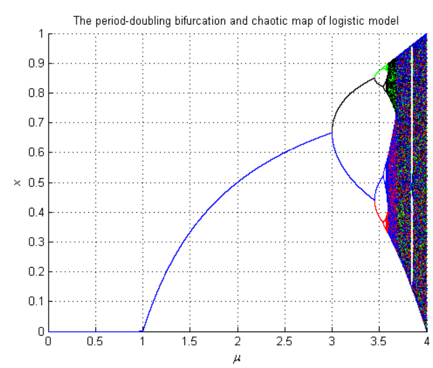


Figure 9 : When parameter *μ is from* 0to 1

(5) When draw points in circulation, for periodic motion, draw same height; for chaotic motion, then draw different heights. Every cycle four rounds use same color to draw points.

[Graphical representation] As Figure 9shows, when parameter μ is from 0to 1, height is zero;when $\mu$is from 0to3, height has a nonzero value ;when $\mu > 3$, height firstly has two values, then it bifurcates as four values, then it bifurcates as eight values, …, such case is called period double bifurcation; When μ arrives at one value, system enters into chaotic state. Chaotic chart has complex structures.

[Analysis](3) In period doubling bifurcation, bifurcation points classify period range. Set binary function:

$$f(\mu, x) = \mu(1 - x)x \tag{6}$$

For period 1 fixed point, when $n\to\infty$, it has $x_{n+1}\to x_\infty$, $x_n\to x_\infty$, x_∞ is fixed point, use x to express x_∞, it can get:

$$\boldsymbol{x = f(\mu, x) = \mu(1 - x)x} \quad \textbf{(7)}$$

Thereupon, it solves:

$$\boldsymbol{x^{(1)} = 0, x^{(2)} = 1 - 1/\mu} \quad \textbf{(8)}$$

Fixed point $x^{(1)}$ is unrelated to parameters, it is called ordinary fixed point. Fixe point $x^{(2)}$ is related to paramters, it is called intrinsic fixed point. Due to $0 \le x_n \le 1$, $x^{(2)} \ge 1$.
Derivative of function to independent variable is:

$$f_x(\mu,x) = \frac{\partial f}{\partial x} = \mu(1-2x) \quad \textbf{(9)}$$

Fixed point steady condition is:

$$\boldsymbol{|f_x| < 1} \quad \textbf{(10)}$$

For ordinary fixed point, due to:

$$\boldsymbol{f_x(\mu, x^{(1)}) = f_x(\mu, 0) = \mu} \quad \textbf{(11)}$$

It is clear: When $\mu < 1$, $x^{(1)}$ is stable fixed point;when $\mu > 1$, $x^{(1)}$ is unstable fixed point, or it is said that $x^{(1)}$ lacks of stability. $\mu_1 = 1$ is a branch point.
For intrinsic fixed point, due to:

$$f_x(\mu,x^{(2)}) = f_x(\mu,1-\frac{1}{\mu}) = 2-\mu \quad \textbf{(12)}$$

Only meet $-1 < f_x < 1$ condition, then th point is stable, so when $1 < \mu < 3$, $x^{(2)}$ is stable fixed point. When $\mu > 3$, $f_x < -1$, $x^{(2)}$ lacks of stability, therefore $\mu_2 = 3$ is a branch point. Bifurcation value is:

$$\boldsymbol{x^{(2)} = 2/3} \quad \textbf{(13)}$$

For period 2 fixed point, then it has $x_{n+2}\to x_n\to x_\infty$, use function to express as $f[\mu, f(\mu, x)]= x$, denote as :

$$\boldsymbol{f^2(\mu, x) = x} \quad \textbf{(14)}$$

Exponent represents function nest. According to (2.6) formula, it can get:

$$\mu[1 -\mu(1 - x)x]\mu(1 - x)x = x$$

Factor and get:

$$\boldsymbol{x[\mu x - (\mu - 1)][\mu^2x^2 - (\mu + 1)\mu x + \mu + 1] = 0} \quad \textbf{(15)}$$

Equation has another two solutions except for $x^{(1)}$ and $x^{(2)}$ two solutions:

$$x^{(3)(4)}=\frac{1}{2\mu}[\mu+1\pm\sqrt{(\mu+1)(\mu-3)}] \quad \textbf{(16)}$$

It is clear:When $\mu > 3$, x will have unequal real number solution, which is generating period 2 fixed point. Fixed point stability condition is:

$$|f_x(\mu, x^{(3)})f_x(\mu, x^{(4)})| < 1 \quad \textbf{(17)}$$

Set $g(\mu) = f_x(\mu, x^{(3)})f_x(\mu, x^{(4)})$, utilize (2.9) formula and can get:

$g(\mu) = \mu^2(1 - 2x^{(1)})(1 - 2x^{(2)}) = \mu^2[1 - 2(x^{(1)} + x^{(2)}) + 4x^{(1)}x^{(2)}]$

Then utilize formula (2.16) again and can get:

$g(\mu) = -\mu^2 + 2\mu + 4$

When $g(\mu) = -1$, it solves $\mu=1\pm\sqrt{6}$. By $g(\mu) > -1$, it can get:

$[\mu-(1+\sqrt{6})][\mu-(1-\sqrt{6})]<0$

Due to $\mu+\sqrt{6}-1>0$, it surely has:

$$\mu<1+\sqrt{6}=\mu_3 \quad \textbf{(18)}$$

When $g(\mu) = 1$, it solves $\mu = 3$and -1. By $g(\mu) < 1$, it can get $\mu > 3$. Therefore, in the range $\mu_2 < \mu < \mu_3$, it has period 2 stable fixed points. When $\mu > \mu_3$, $x^{(3)}$and $x^{(4)}$ lacks of stability, therefore μ_3 is branch point, bifurcation value is :

$$x^{(3)} = 0.440, x^{(4)} = 0.8499 \quad \textbf{(19)}$$

[Algorithm](3) When iterations n is more enough, for periodic fixed point, x_n represents stable value x_∞.μ *takes continuous values from* 0toμ_3, finally it gets stable point x *by iterative algorithm*, which can compare to solution of analysis. μ *takes continuous values from* $\mu_3$to4, first screen out transitional values by iterative algorithm, continue to get iterative result by using iterative algorithm.

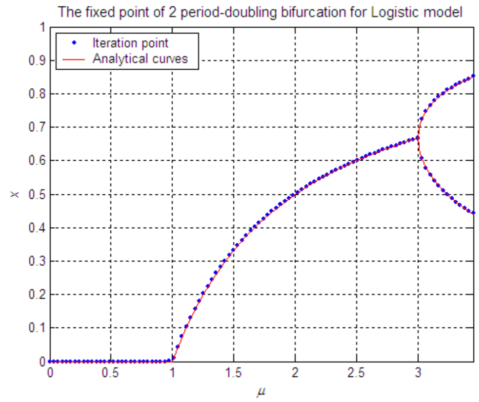
(1) As Figure 10shows, calulation results by using iterative method and analysis method are totally the same. In $\mu_1 = 1$, it occurs to once bifurcation. In μ2 = 3, it occurs 2 period doubling bifurcation, it is branch bifurcation.

(2) As Figure 11 shows, with parameters increasing, after 2 period doubling bifurcation, it occurs to four period doubling bifurcation, eight period doubling bifurcation, …, such continuous bifurcation is also called Feigenbaum bifurcation. When period infinitely increases, it will lose of periodicity, period doubling bifurcation will move towards chaos. In chaos, there is also bifurcation, most obvious one is period 5 and period 3 bifurcations, every branch moves toward chaos again by period doubling bifurcation. In $\mu \approx 3.678$, two main branches start to come across. Before coming across, upper branch is similar to lower branch; two branches are similar to entity. Not only that, smaller partial branch will similar to entity. The similarity is called self-similarity.

[Analysis](4) To one-dimensional mapping

$$x_{n+1} = f(x_n) \quad \textbf{(20)}$$

It can use initial value x_0 and neighbouring value $x_0 + \delta x_0$ to calculate separation. After once iteration, distance is:



Figur 10 : Iterative method and analysis method computational results

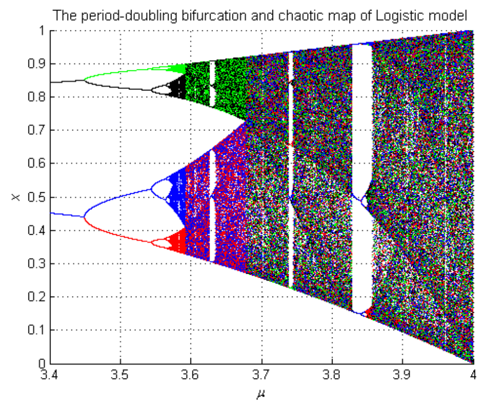


Figure 11 : Feigenbum bifurcation

$$\delta x_1 = f(x_0 + \delta x_0) - f(x_0) = \frac{\mathrm{d}f(x_0)}{\mathrm{d}x}\delta x_0 \quad \textbf{(21)}$$

Then make once iteration again, distance is:

$$\delta x_2 = f(x_1 + \delta x_1) - f(x_1) = \frac{\mathrm{d}f(x_1)}{\mathrm{d}x}\delta x_1 = f'(x_1)f'(x_0)\delta x_0 \quad \textbf{(22)}$$

After *m* times of iteration, distance is:

$$\delta x_m = \delta x_0 \prod_{n=0}^{m-1} f'(x_n) \quad \textbf{(23)}$$

Let:

$$\delta x_m = \delta x_0 \exp(\lambda_m m) \quad \textbf{(24)}$$

Then it gets:

$$\lambda_m = \frac{1}{m}\ln(\frac{\delta x_m}{\delta x_0}) = \frac{1}{m}\sum_{n=1}^{m-1}\ln|f'(x_n)| \quad \textbf{(25)}$$

Lyapunov exponent is:

$$\lambda = \lim_{m\to\infty}\frac{1}{m}\sum_{n=1}^{m-1}\ln|f'(x_n)| \tag{26}$$

When $\lambda < 0$, it indicates orbit is stable in local that represents period motion ; when $\lambda > 0$, it indicates orbit separation that it has sensitivity to value that represents chaos. When $\lambda$is changing from negative value to positive value, it indicatesperiodic motion turns to chaos.
To logistic model, due to:

$x_{n+1} = \mu(1 - x_n)x_n$, $(n = 0, 1, 2, \ldots)$

That

$$f(\mu, x_n) = \mu(1 - x_n)x_n \tag{27}$$

Therefore:

$$\frac{\partial f(\mu, x_n)}{\partial x_n} = \mu(1 - 2x_n) \tag{28}$$

CONCLUSION

Force is cause of object generating accelerated speed, racket to ball acting force size is in direct proportion to racket accelerated speed. In case ball mass is unchanged, athlete used racket weight is unchaged; the hitting force size is mainly up to hitting instantaneous racket swinging speed. Ball bouncing height gets higher, ball forward flight speed is fast; only rely on experiences to estimate its drop point and flight time. If opponent lacks of experiences, then it is prone to cause judgment mistakes. It forces opponent to fast move, it shortens oppponet time of preparing for hitting, which causes his returning mass diminishing, and even causes directly fault. It increases ball to opponent racket acting force, and also increases opponent adjust racket shape difficulties. But if hitting strength is too big, then it is prone to cause self fault, the paper provides theoretical support for athlete effective controlling racket swinging force and bouncing height, it can improve competition winning rate.

REFERENCES

[1] Alireza Fadaei Tehrani, Ali Mohammad Doosthosseini, Hamid Reza Moballegh, Peiman Amini, Mohammad Mehdi DaneshPanah; A New Odometry System to Reduce Asymmetric Errors for Omnidirectional Mobile Robots[J]. RoboCup, 600- 610 **(2003)**.
[2] R.E.Kalman; A New Approach to Linear Filtering and Prediction Problems [J]. Transaction of the ASME - Journal of Basic Engineering, **82**, 35- 45 **(1960)**.
[3] Carlos F.Marques, Pedro U.Lima; A Localization Method for a Soccer Robot Using a Vision- Based Omni-Directional Sensor [J]. RoboCup, 96- 107 **(2000)**.
[4] S.Thrun, D.Fox, W.Burgard, F.Dellaert; Robust Monte Carlo localization for mobile robots [J]. Artificial Intelligence Journal, **128**, 99- 41 **(2001)**.
[5] Kan Li-ping; Evaluation on Technical data of Free Kick in Impose Fine Region in Football Game[J]. Bulletin of Sport Science & Technology, **19(3)**, 19-20 **(2011)**.
[6] Zheng Wei; On the Training of Football Shooting[J]. Sport Science and Technology, **3**, 23-26, 33 **(2000)**.
[7] Yang Jilin et al.; Research on shooting in the 17th World Cup football semi-finals[J].Journal of Shandong Physical Education Institute, **18(3)**, 51-53 **(2002)**.
[8] Wang Xin; Analysis on the best region of shoot [J]. Journal of Nanjing Institute of Physical Education, **16(5)**, 96-97 **(2002)**.
[9] Zhang Ji, xiang; A Study on Effect of Application of the Skills of Side-to-middle Court Passing of Chinese Football Team[J].Journal of Hubei Sports Science, **21(1)**, 74-75, 79 **(2002)**.
[10] Li Ning, Zhou Jiandong; Statistical Analysis of Goals at 19th FIFA World Cup [J]. Journal of Jilin Institute of Physical Education, **27(3)**, 45-47 **(2011)**.