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## Volleyball best spike area research based on lagrange extreme value analysis

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### ABSTRACT

Volleyball players will use a variety of basic motions when fight in the court, from which spike motion is generally recognized as the most ornamental and destruction technique, the technique can interference with opponent team scoring, and also provide opportunities for its own team organize new round attacking. This paper analyzes volleyball spike technique, in the hope of exploring volleyball expected landing point after spiking, spike process action principle and reasonable hitting angle, it provides theoretical basis for athletes' training process and researchers' parsing process. This paper proposes volleyball expected landing point area after spiking, and analyzes expected landing point area difficult defensiveness, then analyzes athletes' action features and action biomechanical principle in five segments from the perspective of spike process, finally it establishes programming equation for volleyball spike expected landing point, and applies Lagrange conditional extremum algorithm in carrying out data simulation on programming equation, it gets reasonable hitting angle range under reasonable parameters, which provides theoretical basis for accurate hitting and scientific hitting.

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### KEYWORDS

Spike technique;  
Expected landing point;  
Kinematics equation;  
Hitting angle;  
Louis lagrange;  
Conditional extremum.

### INTRODUCTION

Volleyball fighting process proceeds with 12 people movement, the two parties try their best to interference with opponents' ace to gain success, then strives for own party gaining more scores as much as possible; For volleyball technical motions, spike technique is the main way in competitive ace, in order to realize athletes' accurate spiking and scientific spiking, this paper implements analysis of forward spike process athletes' action features and hitting angle generated volleyball landing point problems, in the hope of making contri-

bution s to volleyball further development.

For volleyball spike motion and spike hitting angle, lots of people have made efforts, just by their efforts make it possible to accurate hitting and scientific hitting, from which: Lin Sen etc. (2013) applied insole system testing on Liaoning men volleyball spike take-off technique, and explored volleyball spike take-off technique pelma mechanical feature, which provided theoretical basis and guidance for improving volleyball athletes' spike take-off height and lower limbs special strength training<sup>[1]</sup>; Zhang Hai-Bin etc.(2013)took No.4 was stop-jump before spiking from 12 volleyball players as

research objects, collected athletes' stop process kinematical parameters by force platform, video and myoelectricity multi-machine synchronous testing to do analysis, which provided theoretical basis for stop technique<sup>[2]</sup>; Liu Ju-Ke etc.(1987)took amateurs and volleyball athletes as objects, he defined spike take-off point and spike landing point, according to similar triangles geometric relationship, he established the mathematical model, and got take-off corresponding men and women different spike routes, landing point areas' hitting point height and best hitting angle by electronic computer technology handling<sup>[3]</sup>.

This paper on the basis of previous research, it summarizes spike technique expected landing point area and spike process biomechanical principle, by mathematical programming method, it establishes programming equation that required to arrive at difficult defensive area, and explores Lagrange conditional extremum application in the programming equation solution, which provides theoretical basis for coaches and athletes.

**VOLLEYBALL SPIKE MOTION PURPOSE AND MECHANICAL ANALYSIS**

Volleyball match results is classified with scores, from which one game is stipulated as 25 scores, fighting teams who first gets 25 scores is thought to win, therefore both two parties in fighting process aims at getting more scores and interference with opponents' scores as much as possible. Volleyball players' basic motions can be classified according to their action features as preliminary posture and shift, pass, dig, service, block and spike, for destruction, it is the spike technique be the strongest, the technique is own party team ace important way and also fatal technique to interference with opponent scores. If it want to realistic

realize spike technique power, it needs to make analysis of opponents' players' location distribution and court designing, and play the ball into opponent vulnerable location is the key to effective interfere with opponent ace, therefore the chapter analyzes expected location after volleyball spiking and spike technique action features, in the hope of providing basis for volleyball kinematical analysis and teaching design.

**Expected location analysis after volleyball spiking**

In volleyball fighting, two parties' players' line up in the court shows symmetric type, generally, court middle zone defense ability is the strongest while lies in court back row small partial area and left right two sides' small partial area defensive ability are relative poor, its court space presentation is as Figure 1 show.

In Figure 1, the left side position is regarded as opponent court, right side position is regarded as own party court, white parts in opponent court is middle zone, the zone defensive ability is very stronger, and shadow parts represent back side line and left right side line's difficult defensive area, meanwhile the paper con-

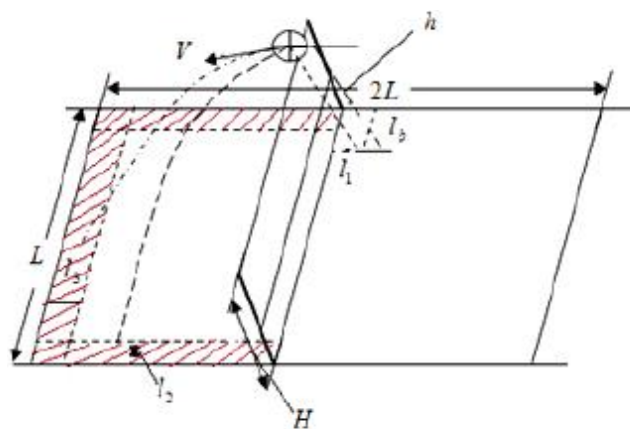


Figure 1 : Volleyball court movement and court location schematic diagram

TABLE 1 : Court and volleyball parameters definition

Symbol	Definition	Symbol	Definition
$l_1$	When volleyball is shot, distance between sphere center to net plane	$H$	Volleyball net height
$l_2$	Left right side line difficult defensive area width	$V$	Volleyball sphere center speed
$l_3$	Right side line difficult defensive area width	$h$	Volleyball sphere center to ground distance
$l_b$	When volleyball is shot, the shortest distance between sphere center to border	$L$	Court effective area total width, that is $\frac{1}{2}$ of total length

## FULL PAPER

trols spike initial conditions with parameters in the figure, and let them drop into shadow parts in the figure, so as to arrive at spike expected Location, parameters definition in figure is as TABLE 1 show.

### Volleyball spike motion technical analysis

Volleyball spike technique effects have remarkable features in the match ace, it can cause difficulties to opponent receive the ball, meanwhile it creates opportunities for own team to attack again, its features are high hitting point, fast speed, big strength and multiple changes, besides it need to closely cooperate with setting.

According to spike action features, it can be divided into forward spike, windmill smash and self-cover spike the three kinds, this paper takes forward spike as an example to carry out volleyball spike motion analysis, as Figure 2 show the forward spike motion process.

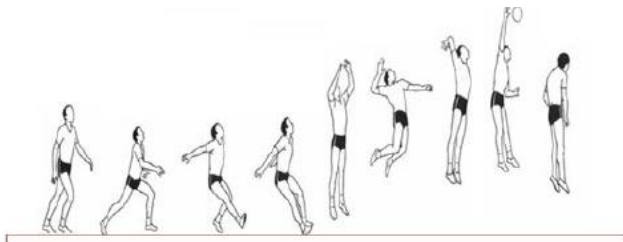


Figure 2 : Forward spike timing motion schematic diagram

Forward spike is the basic type in spike technique, due to athlete can face to net during spike process, and forward spike arm swinging motion is relative more flexible, it can let ball landing point more accurate, and it can carry out proper volleyball movement route controlling according to opponent defense status. As Figure 2 show, forward spike motion completion is composed of following 5 basic segments organic combination.

#### (1) Preliminary posture

The segment is the preparation before running-up, it needs athlete control two legs let left and right open, make right leg moving forward a small step, the small step left and right distance should slightly smaller than front and back distance, athlete two knees slightly bend, let body gravity center line drop in the leg area which is to the running-up direction, human body upper body should slightly lean forward, and two arms should proper

bend elbow and let them fall in body two sides, eyes should fully focus on volleyball movement;

#### (2) Running-up segment

The segment starting to defining is starting running to prepare to jump, when running-up starts, athlete should first control left leg move the first step to ball landing direction, and then let right leg fast cross the second step to ball landing direction, after that left leg follows right leg together with right leg fast draw close; when left leg draws close to right leg, it should land in right leg left side nearly equal to should width and slight half leg forward location, at this time it moves to the third segment take-off process, which is also running-up segment ending node;

#### (3) Take-off segment

Take-off segment is the process that athlete from running-up ending to body soaring, in the beginning of the process, at first it should complete take-off leg braking, brake movement process can be understood as horizontal translation changing into spinning on its axis, while rotation process has moment of resistance effects that lets body complete braking, if take-off leg translational speed is  $v$ , and radius on the axis is  $r$ , then take-off leg rotational angular speed  $\omega$  at this time is as formula (1) shows:

$$\omega = \frac{v}{r} \quad (1)$$

According to formula (2) showed the moment momentum theorem, it is clear that take-off leg braking force process:

$$\Delta M(t_2 - t_1) = I\omega_2 - I\omega_1 \quad (2)$$

In formula (2),  $\Delta M$  represents take-off leg suffered average moment of resistance in braking process, due to  $(t_2 - t_1)$  is quite small, during take-off leg proper angular speed meeting process, it requires greater moment of resistance, so at this time, it is prone to let athletes get injured that needs athletes to carry out scientific training.

After take-off leg completing braking, human body two legs' knee will change into flexion shape and inner buckling, upper body needs forward lean take-off postures, after that athletes' two legs fast and powerful kicking the ground to provide proper speed for take-off; In the moment of take-off, so as to achieve higher vertical

speed, it needs two arms continue to bend arm from back body and swing to body forward upper side, it provides upward accelerated speed for athletes, at the same time athletes should fast stretch abdomen, extend knee and bend ankle raise toe, in this way it achieves reasonable body soaring. To sum up, in take-off process, athlete should pay attention to braking process sports injury and take-off process two arms' cooperation.

**(4) Hitting in the air**

Hitting in the air segment is the fundamental objective of spike motion, which needs athlete just right left hitting arm collides with ball, in the process, athletes after whole body soaring, it needs left arm swing to the front body, make hitting arm bend and control it in one side of head, control elbow location on shoulder height location, carry out stretch abdomen, chest out, open shoulder and body in reversed arc, eyes focus on volleyball, use hitting arm to do whip motion by seizing the opportunities, its big arm forward spins, elbow towards front top side, small arm relaxes and fast vibrates backwards, during the period, it needs to relax wrists so as to provide better transmitter for whip motion speed transferring, then following by small arm upward swinging through back vibration accelerated trends, shakes the whole arm and forward swinging it in arc shape to hitting point and hits; when hitting, it needs the whole palm cover volleyball, and needs to make forward push motion, in the hope of making reasonable controlling of ball direction and route.

**(5) Landing segment;**

Landing segment is forward spike motion ending segment, the segment motion merits don't have big effects on the hitting, but it will have certain effects on next defense and attack, if player lands appear stumbling or injury and other accidents, then it generates unfavorable opportunity for own party next attack, therefore landing segment needs athlete to land stable, in order to pursuit stability and no injuries in landing process, it needs two legs simultaneous landing, use two legs' bearing ability to endure landing ground reverse impulse.

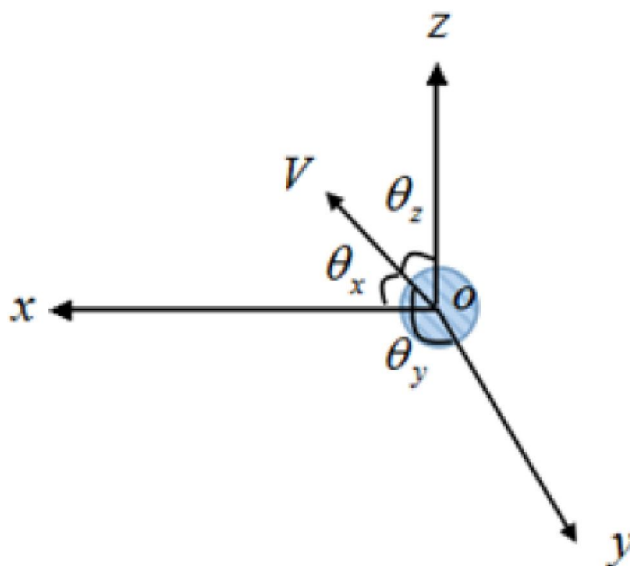
**VOLLEYBALL DROPS INTO EXPECTED LOCATION AFTER SPIKING CONDITION**

**ANALYSIS**

**Volleyball out of hand kinematical equation**

In the moment hit by athlete, volleyball has speed after collision, the speed is the fundamental factor affects volleyball trajectory, and the chapter analyzes initial volleyball mass center speed, and establishes volleyball kinematical equation with kinematical parameters provided by the paper.

Volleyball hit instantaneous speed  $V$  resolution status in triangular rectangular coordinate system is as Figure 3 show, rectangular coordinate system  $x$  axis positive direction is the direction along opponent court left and right side line,  $y$  positive direction is own party team left side along net up and down side line direction,  $z$  axis positive direction is vertical and upwards direction, origin location is volleyball sphere center.



**Figure 3 : Volleyball hit instantaneous sphere center speed three dimensional resolution schematic diagram**

From Figure 3 showed volleyball mass center speed resolution status, it can get three coordinate axis upward component speed relations, as formula (3) show:

$$\begin{cases} v_x = V \cdot \cos \theta_x \\ v_y = V \cdot \cos \theta_y \\ v_z = V \cdot \cos \theta_z \\ V^2 = v_x^2 + v_y^2 + v_z^2 \end{cases} \quad (3)$$

If it makes following four hypothesizes on volleyball after hitting, it can get as formula (4) showed vol-

## FULL PAPER

leyball kinematical equation.

- 1) Volleyball after hitting, no rotation exists.
- 2) Without considering other external force effects besides gravity;
- 3) Opponent player hasn't touched the ball in blocking process.
- 4) Regard volleyball movement as movement of particle with mass

$$\begin{cases} s_x = V \cdot \cos \theta_x \cdot t & , & s_y = V \cdot \cos \theta_y \cdot t \\ s_z = V \cdot \cos \theta_z \cdot t - \frac{1}{2}gt^2 \end{cases} \quad (4)$$

In formula(4),  $(s_x, s_y, s_z)$  represents volleyball mass center at time  $t$  three dimensional space coordinate.

### Arrive at expected location mathematical constraints

From spike motion proceeding features, it is clear that volleyball movement form after hitting is from the top down, which is can also judge volleyball mass center vertical upward movement status, and volleyball in  $xoy$  plane movement only needs one direction drift angle, therefore in order to define volleyball movement status, only measures and gets ball instantaneous speed and  $\theta_x, \theta_z$ , it can write out volleyball kinematical equation, when sphere speed is fixed, volleyball drift angle in  $x$  axis and that of  $z$  axis will decide volleyball movement trajectory. In this way it can get as formula(5) showed solving angular extremum programming expression:

$$\begin{cases} \min \theta_x & \max \theta_x & \min \theta_z & \max \theta_z \\ \text{s.t} & & & \\ (\mathbf{L} - \mathbf{l}_3 + \mathbf{l}_1) \leq s_x \leq (\mathbf{L} + \mathbf{l}_1) & & & \\ -\mathbf{l}_b \leq s_y \leq (\mathbf{L} - \mathbf{l}_b) & & & \\ \mathbf{H} \leq s_z & & & \end{cases} \quad (5)$$

Volleyball landing moment  $z$  axis direction coordinate should be  $-h$ , establish as formula (6) showed equation, it can get two solutions with regard to time, according to parabola features, it can know the solutions are certain one positive and one negative, it should

get rid of negative solution, take positive solution:

$$\begin{aligned} -\frac{1}{2}gt^2 + V \cdot \cos \theta_z \cdot t + h &= 0 \\ \Downarrow \\ \begin{cases} t_1 = \frac{-V \cos \theta_z + \sqrt{(V \cdot \cos \theta_z)^2 + 2gh}}{g} > 0 \\ t_2 = \frac{-V \cos \theta_z - \sqrt{(V \cdot \cos \theta_z)^2 + 2gh}}{g} < 0 \end{cases} \end{aligned} \quad (6)$$

Analyze subject meaning, it is known that can take  $t_1$  solution. Similarly it can get as formula(6) showed the time when ball vertical height and net height are the same:

$$\begin{aligned} -\frac{1}{2}gt^2 + V \cdot \cos \theta_z \cdot t + (h - \mathbf{H}) &= 0 \\ \Downarrow \\ \begin{cases} t_3 = \frac{-V \cos \theta_z + \sqrt{(V \cdot \cos \theta_z)^2 + 2g(h - \mathbf{H})}}{g} > 0 \\ t_4 = \frac{-V \cos \theta_z - \sqrt{(V \cdot \cos \theta_z)^2 + 2g(h - \mathbf{H})}}{g} < 0 \end{cases} \end{aligned} \quad (7)$$

Similarly it takes solution  $t_3$ .

Thereupon, it is clear that when hitter volleyball expected landing point is back side line difficult defensive area, constraint condition is as formula (8) show, expected landing point further from hitting location two lateral side lines' difficult defensive area constraint condition is as formula (9) show, expected landing point hitting location two lateral side lines' difficult defensive area constraint condition is as formula (10) show:

$$\begin{cases} (\mathbf{L} - \mathbf{l}_3 + \mathbf{l}_1) \leq v_x \cdot t_1 \leq (\mathbf{L} + \mathbf{l}_1) \\ -\mathbf{l}_b \leq v_y \cdot t_1 \leq (\mathbf{L} - \mathbf{l}_b) \\ \mathbf{l}_1 \leq v_x \cdot t_2 \end{cases} \quad (8)$$

$$\begin{cases} \mathbf{l}_1 \leq v_x \cdot t_1 \leq (\mathbf{L} + \mathbf{l}_1) \\ (\mathbf{L} - \mathbf{l}_2 - \mathbf{l}_b) \leq v_y \cdot t_1 \leq (\mathbf{L} - \mathbf{l}_b) \\ \mathbf{l}_1 \leq v_x \cdot t_2 \end{cases} \quad (9)$$

$$\begin{cases} \mathbf{l}_1 \leq v_x \cdot t_1 \leq (\mathbf{L} + \mathbf{l}_1) \\ |\mathbf{l}_2 - \mathbf{l}_b| \leq v_y \cdot t_1 \leq \mathbf{l}_b \\ \mathbf{l}_1 \leq v_x \cdot t_2 \end{cases} \quad (10)$$

**Lagrange extremum solution principle**

Programming model provided in the paper is a conditional extremum solving application problem, this paper adopts Lagrange multiplier method to solve the problem. If objective function  $f(x_1, x_2, \dots, x_n)$  has  $m$  pieces as formula(11) showed constraint conditions and formula(11) has continuous partial derivative then it can get as formula(12) showed Jacobi matrix:

$$g_i(x_1, x_2, \dots, x_n) = 0, (i = 1, 2, \dots, m; m < n) \tag{11}$$

$$J = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \dots & \frac{\partial g_1}{\partial x_n} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \dots & \frac{\partial g_2}{\partial x_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial g_m}{\partial x_1} & \frac{\partial g_m}{\partial x_2} & \dots & \frac{\partial g_m}{\partial x_n} \end{bmatrix} \tag{12}$$

If met constraint condition point area has Jacobi matrix  $rank(J) = m$ , and then it can get following two conclusions.

Conclusion 1: If point  $x_0 = (x_1^0, x_2^0, \dots, x_n^0)$  is function  $f(x)$  meeting constraint condition extremum point then it surely exists  $m$  pieces of constants  $\lambda_1, \lambda_2, \dots, \lambda_m$  let  $x_0$  point relationship as formula(13) shows is at work:

$$\text{grad}(f) = \lambda_1 \text{grad}(g_1) + \lambda_2 \text{grad}(g_2) + \dots + \lambda_m \text{grad}(g_m) \tag{13}$$

Construct as formula (14) showed Lagrange function

$$L(x_1, x_2, \dots, x_n; \lambda_1, \lambda_2, \dots, \lambda_m) = f(x_1, x_2, \dots, x_n) - \sum_{i=1}^m \lambda_i g_i(x_1, x_2, \dots, x_n) \tag{14}$$

Then conditional extremum point is in the corresponding point of all equation solutions as formula (15) show.

$$\begin{cases} \frac{\partial L}{\partial x_k} = \frac{\partial f}{\partial x_k} - \sum_{i=1}^m \lambda_i \frac{\partial g_i}{\partial x_k} = 0, \\ g_i = 0 \end{cases} \tag{15} \quad (k = 1, 2, \dots, n; i = 1, 2, \dots, m)$$

Conclusion 2: If given point  $x_0 = (x_1^0, x_2^0, \dots, x_n^0)$  and  $m$  pieces of constants  $\lambda_1, \lambda_2, \dots, \lambda_m$  meet for-

mula(15), it will have matrix as formula(16) show if the matrix is positive definite or negative definite matrix it will have extremum point that meets constraint condition.

$$\left[ \frac{\partial^2 L}{\partial x_k \partial x_l} (x_0, \lambda_1, \lambda_2, \dots, \lambda_m) \right]_{n \times n} \tag{16}$$

**Lagrange extremum method solution result and analysis**

In order to apply Lagrange conditional extremum theory solving angle extremum, it needs to sort out constraint condition (8), (9), (10) so as to get angle Lagrange function form, which is also input  $t_1$  and  $t_3$  analytic form as well as  $V$  and angle  $\theta_x, \theta_z$  relationships, taking back side line difficult defensive area constraint equation as an example, it can get as formula(17) showed Lagrange function form:

$$\begin{cases} Q = A + B - g(L - l_3 + l_1) \\ A = V \cdot \cos \theta_x \cdot [-V \cos \theta_z + \sqrt{(V \cos \theta_z)^2 + 2gh}] \\ B = \lambda [V \cdot \sqrt{1 - \cos^2 \theta_x - \cos^2 \theta_z} \cdot (-V \cos \theta_z + \sqrt{(V \cos \theta_z)^2 + 2gh}) + gl_b] \end{cases} \tag{17}$$

Then input  $u = \cos \theta_x, v = \cos \theta_z$  into formula(17) to solve as formula(18) showed equations, it can get  $u, v$  value, use the two values corresponding angle extremum as range value.

$$\begin{cases} \frac{\partial Q}{\partial u} = \frac{\partial A}{\partial u} + \frac{\partial B}{\partial u} = 0 \\ \frac{\partial Q}{\partial v} = \frac{\partial A}{\partial v} + \frac{\partial B}{\partial v} = 0 \\ \frac{\partial Q}{\partial \lambda} = \frac{\partial B}{\partial \lambda} = 0 \end{cases} \tag{18}$$

TABLE 2 : Parameters values table

Parameter	Parameter value	Parameter	Parameter value
$l_1$	1.00 m	$H$	2.43 m
$l_2$	0.80 m	$V$	22.50 m/s
$l_3$	1.20 m	$h$	3.50 m
$g$	9.80 m/s <sup>2</sup>	$L$	9.00 m

TABLE 3 : Angle extremum result table

Difficult defensive area classification	$\theta_x$ minimum	$\theta_x$ maximum	$\theta_z$ minimum	$\theta_z$ maximum
	value	value	value	value
Difficult defensive area one	0.00°	41.34°	71.82°	80.00°
Difficult defensive area two	41.40°	64.60°	73.80°	90.00°
Difficult defensive area three	0.00°	84.60°	0.00°	90.00°

Note: Difficult defensive area one, two, three respectively represents back row side line area, further lateral side line area, nearer lateral side line area.

Similarly, it can get two lateral side line difficult defensive area Lagrange function, and according to the same theory to solve corresponding angle extremum.

Take parameters values as TABLE 2 show; make solution on three kinds of difficult defensive area angle range.

Input above parameters into Lagrange function, it can solve as TABLE 3 showed three difficult defensive area corresponding  $\theta_x, \theta_z$  extremum.

Therefore, volleyball players, if they want to spike ball in expected location, they can control hitting angle  $\theta_x, \theta_z$  within the range as formula (19) show.

$$\begin{cases} \theta_x \in [0.00^\circ, 84.60^\circ] \\ \theta_y \in [0.00^\circ, 90.00^\circ] \end{cases} \quad (19)$$

## CONCLUSIONS

It analyzes emphatically forward spike technical features, detailed analyzes running-up process braking principle and states key action point to improve hitting in the air process whip speed transferring, which provides theoretical basis for volleyball players' technical motion correction; under reasonable conditional hypothesis, it established volleyball mass center movement kinematical equation, and on the basis of kinematical equation, it utilizes mathematical programming thought proposing three kinds of side line difficult defensive area landing point constraint condition; Lagrange conditional extremum solution principle and calculation steps provide theoretical basis for the paper programming function solution; By Lagrange conditional extremum solution principle and this paper established expected landing point programming function, it put forward hitting angle extremum solution algorithm, and gets reasonable hit-

ting angle range under giving reasonable parameters.

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