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Invalidity of the former exact solution of the two-dimensional Ising model

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ABSTRACT

Earlier, the author showed that there were mistakes within the traditional solutions of the one-dimensional and two-dimensional Ising models. Here, strong final proof is presented that the previous exact solution of the two-dimensional Ising model was untenable.

KEYWORDS

Ising model;
Magnetic materials;
Nanostructures;
Magnetic properties;
Magnetic structure.

THEORY

Let us consider the two-dimensional Ising model in the absence of an external magnetic field[1-3]. A spin is placed on each lattice site. This is a one-dimensional unit vector assuming only the discrete values +1 and -1. The Hamiltonian of a lattice of *N* atoms is of the form:

$$\mathbf{H} = -\mathbf{J}\sum_{\mathbf{ij}} \mathbf{s_i s_j} \quad (1)$$

where s_i and s_j are the spins and *J* is the interaction energy between the spins of nearest neighbours. Positive *J* favours parallel and negative *J* antiparallel alignment of the spins. The sum in Eq. (1) is over all the nearest neighbour spins.

The traditional exact result for the spontaneous magnetisation per spin of the two-dimensional Ising model on the square lattice is:[2]

$$\sigma = (1+u)^{1/4}(1-u)^{-1/2}\left(1-6u+u^2\right)^{1/8}, \quad u = \exp(-4J\beta) \quad (2)$$

Here, $\beta = \frac{1}{kT}$. The temperature of the phase transition T_c in this case is given by:

$$\frac{J}{kT_c} = -\frac{1}{4}\ln\left(3-2\sqrt{2}\right) \approx 0.441 \quad (3)$$

At $T > T_c$, the magnetisation σ in Eq. (2) becomes complex. Therefore, Eq. (2) is not the exact solution. The exact solution must describe the system precisely according to the definition. At $J = 0$, the numerator in Eq. (2) becomes complex and the denominator is equal to zero. That is, the magnetisation is a complex number with infinite real and imaginary parts. This means that the mathematical model described by the former exact solution contains strong interactions, which are absent in the real Ising model. When $J = 0$, there is no interaction between the spins and the exact solution must describe that case correctly. This is in agreement with the conclusion made in Ref. 4, that in the previous exact solutions many redundant terms in the partition function were included.

The exact solutions of the two-dimensional Ising model were obtained in Ref. 4. The magnetisation of the square lattice is:

$$\sigma = \left[\frac{\sinh(2\beta J)}{\cosh(2\beta J) + 3} \right]^{0.5} \quad (4)$$

The magnetisation of the hexagonal lattice is:

$$\sigma = \left[\frac{\exp(1.5\beta J) - \exp(-0.5\beta J)}{\exp(1.5\beta J) + 3\exp(-0.5\beta J)} \right]^{0.5} \quad (5)$$

CONCLUSIONS

The above arguments prove conclusively that the traditional exact solutions of the two-dimensional Ising model were totally wrong. They included many redundant terms in the calculation of the partition function. An attempt to obtain the exact solutions of the one-dimensional, two-dimensional and three-dimensional Ising models was made in Ref. 4.

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