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Hybrid quantum-behaved particle swarm algorithm for nonlinear complementary problems

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ABSTRACT

Combining a multiplier penalty function method of dealing with constraints using the quantum particle swarm optimization (QPSO) algorithm, a hybrid QPSO algorithm is proposed for solving nonlinear complementary problems. This method utilizes the advantages of the QPSO and the multiplier penalty function method. The non-feasible particles produced in the iterative process are dealt with using the multiplier penalty function method to produce feasible particles. Numerical experiments show that the proposed algorithm is effective.

KEYWORDS

Nonlinear complementary problems;

Quantum particle swarm optimization;

Multiplier penalty function method.

INTRODUCTION

A classic nonlinear complementary problem (NCP) seeks a vector

$x = (x_1, x_2, \ldots, x_n) \in R^n$ [1]

subject to

$$x_i \geq 0,\ F_i(x) \geq 0,\ x_i F_i(x) = 0\ \ i = 1, 2, \ldots, n \qquad \textbf{(1)}$$

where $F_i(x) : R^n \to R$ is a continuously differentiable function. NCP has a wide range of applications in physics, mechanics, and engineering[4]. Therefore, the study of NCP numerical solution has drawn considerable attention[3]. Many algorithms are used for solving NCP, such as the interior point method, the non-smooth method, and the projector-like algorithm. The interior point method is used for monotone complementary problems with polynomial complexity bounds. The computation time of this algorithm is the polynomial function of the size in the worst case. However, the computer has difficulty calculating with such a method because the initial point is hard to find. The non-smooth Newton method transforms a complementary problem through the NCP function into the solution of equations. Three conditions, including equalities and inequalities, were transformed as equations. However, the degenerate solution in the Jacobi matrix of equations based on the differentiable NCP function is singular[9], thus, the Newton method does not possess local fast convergence. The projector-like algorithm has a simple format, small storage capacity, and easy computer implementation, which does not involve the solution of linear equations. However, given the short computation of each step, the shortcoming of the algorithm is that it exists in at most only linear convergence, the NCP can only find one of many solutions[2]. In general, these algorithms need to calculate the gradient and to given an initial point. Traditional optimization algorithms cannot find more optimal solutions if the solution is not unique.

In recent years, some random methods, such as the genetic, social cognitive optimization, and particle swarm optimization (PSO) algorithms, have been successfully applied to constrained optimization problems[5,7]. The PSO algorithm was proposed in 1995 by Kennedy and Eberhart[6]. This simple and fast-converging method does not require a continuous objective function and an initial point and gradient information. However, a widespread concern of domestic and foreign scholars over the PSO algorithm is that it is easy to fall into a local optimal solution in the late search. Sun et al proposed an improved algorithm, the quantum particle swarm optimization (QPSO) algorithm[8], which is based on a quantum perspective. This method is effective in improving the global search ability of all feasible solutions.

This paper transforms Problem (1) into the constrained optimization Problem (2):

$$\begin{aligned} &\min\ f(x) \\ &s.t\quad g_i(x) \le 0, i = 1, 2, \dots, n \end{aligned} \tag{2}$$

where $f(x) = \sum_{i=1}^{n} x_i F_i(x)$

and $g_i(x) = \left[-x_i, -F_i(x)\right]^T \le \vec{0}$.

The external point method is used to deal with the constraint conditions, and the QPSO algorithm is combined with the multiplier penalty function method. It is not in the objective function addition penalty term, but for the infeasible optimal particle, the multiplier penalty function method can deal with it. It can effectively overcome the multiplier penalty function method, which can search only for a local optimal solution. According to numerical experiments, the new algorithm has validity, versatility, and stability, and provides a new way of solving complementary problems. In this paper, the hybrid QPSO algorithm has the following advantages. First, it can avoid some difficulties in the initial point because QPSO initial particles are randomly generated and the algorithm does not require the initial point to be the certain interior point. Second, it does not require the solutions of Newton equations, involves small amounts of calculation, and avoids numerical difficulties caused by the degenerate solution. Lastly, the hybrid QPSO objective function does not require a continuous or differentiable function and fast convergence. Meanwhile, it can search for more solutions, that is, if the objective function optimal solution is unique.

PROCESSING TECHNOLOGY FOR PARTICLES OF VIOLATE CONSTRAINT CONDITIONS

$\Phi(x) = \sum_{i=1}^{n} \max\{0, g_i(x)\}$, where $\Phi(x)$ is $\Phi(x)$ the sum of all particle constraint violations. $\Phi(x) \ge 0$ and $\Phi(x) = 0$ if and only if x belongs to the feasible region. If $\Phi(x') > \varepsilon$ (ε for accuracy), the solution x' is the infeasible solution. Then, x' is the initial point in the QPSO algorithm. The multiple method is used to deal with it, the accuracy of $\hat{x}$ is satisfied to replace x', and the search for the optimal solution is continued.

The multiplier penalty function method is one of the most representative algorithms for solving optimization problems. For the objective function and constraint condition, the requirements are very low.

The steps of the multiplier penalty function method under the inequality constraint are as follows:

Step 1:

Initial data are selected, given an initial point $x^{(1)}$. σ_1 is the initial penalty factor, λ_1 is the initial multiplier, and ε is the permissible error. Let $k = 1$.

Step 2:

x^k is considered the initial point, solving unconstrained optimization problems as follows:

$$\min_{x \in R^n} L(x, \sigma_k, \lambda_k)$$

$$L(x,\sigma_k,\lambda_k) = f(x) + \frac{1}{2\sigma_k}\sum_{i=1}^{n}\left(\left[\max\left(0, \lambda_i^{(k)} + \sigma_k g_i(x)\right)\right]^2 - (\lambda_i^{(k)})^2\right)$$

$x^{(k+1)}$ is obtained if $\left\| g^{(-)}\left(x^{(k+1)}\right)\right\|_\infty \le \varepsilon$

Step 3:

If $\left\|g^{(-)}\left(x^{(k+1)}\right)\right\|_2 \le \frac{1}{4}\left\|g^{(-)}\left(x^{(k)}\right)\right\|_2$,

then step 4 is performed; if $\sigma_{k+1} = 10\sigma_k$, step 2 is repeated.

Step 4:

Through $\lambda_i^{(k+1)} = \max\left(0, \lambda_i^{(k)} + \sigma_k g_i(x^{(k)})\right)$, $\lambda^{(k+1)}$. is calculated. Let $k := k+1$, step 2 is repeated. $g^{(-)}(x) = \left(g_1^{(-)}(x),...,g_n^{(-)}(x)\right)^T$,

$g_i^{(-)}(x) = \min\left(0, g_i(x)\right)$ $i = 1,2,\ldots,n$.

HYBRID QUANTUM-BEHAVED PARTICLE SWARM ALGORITHM OF NCPS

QPSO Algorithm:

A previous study[8] on quantum mechanics proposed the QPSO algorithm based on a standard PSO. In PSO, particles meet different states of aggregation and can search the global optimization solution in the entire feasible region. Each particle in the particle swarm must converge to its random point $P_i, P_i = (P_{i_1}, P_{i_2}, \cdots, P_{i_D}), D$ is the particle dimension. The particle swarm moves according to Formulae (3) to (6).

$$X_i = P_i + \alpha \times |mbest - X_i| \times \ln(u^{-1}), u = rand1() \quad \textbf{(3)}$$

$$P_i = \varphi \cdot pbest_i + (1-\varphi) \times gbest, \varphi = rand2() \quad \textbf{(4)}$$

$$\alpha = 0.5 + 0.5 \times (iter_{\max} - t) / iter_{\max} \quad \textbf{(5)}$$

$$mbest = M^{-1}\sum_{i=1}^{M} pbest_i = (M^{-1}\sum_{i=1}^{M} pbest_{i_1}, \cdots, M^{-1}\sum_{i=1}^{M} pbest_{i_D}) \quad \textbf{(6)}$$

where X_i is the number *i* particle's position in the iteration,

$pbest_i$ is the self-local optimization of the number *i*, *gbest* is the global optimal solution; $mbest$ is the optimization center of particle i, $rand1(), rand2()$ are $[0,1]$ random numbers, α is the contraction expansion coefficient, *t* is the particle's current iteration generation, M is population size, and $iter_{\max}$ is the maximum iteration generation.

Hybrid algorithm:

The basic process of the hybrid QPSO algorithm for solving NCPs based on the multiplier penalty function method is as follows:

Step 1:

The current iteration generation is set, t = 1. To determine the size of the population , the dimension of the search space , generate the initial position and velocity of every particle in the entire search space. If a particle does not satisfy the constraint conditions, then we regard as the initial point and operate in the multiplier penalty function method. is obtained instead of , and the fitness value of is calculated.

Step 2:

The current position is replaced by the best position of the number *i*. is set to the optimal position in the population.

Step 3:

The following procedures are performed for all the particles in the swarm.

3.1 According to Formulae (3) to (6), the position of each particle is updated, and a new kind of particle group is generated.

3.2 If a particle $x_i(t)$ does not satisfy the constraint conditions, then is regarded as the initial point $(x_i(t))^*$ is obtained through the multiplier penalty function method to replace $x_i(t)$.

3.3 If the current particle's $x_i(t)$ fitness is better than that of $x_i(t)$, then the fitness of replaces that of $pbest_i$.

3.4 If the current particle's $x_i(t)$ fitness is better than the $gbest$ fitness value, then the fitness of replaces $x_i(t)$ that of .

Step 4:

let $t = t + 1$, the step 3 is performed again until an expected fitness value or maximum iteration generation is reached.

NUMERICAL RESULTS

A classical nonlinear complementation experiment[2,9] was selected to test the ability of the proposed algorithm:

$$F(x) = \begin{bmatrix} 3x_1^2 + 2x_1x_2 + 2x_2^2 + x_3 + 3x_4 - 6 \\ 2x_1 + x_2^2 + 10x_3 + 2x_4 - 2 \\ 3x_1 + x_1x_2 + 2x_2^2 + 2x_3 + 9x_4 - 9 \\ x_1^2 + 3x_2^2 + 2x_3 + 3x_4 - 3 \end{bmatrix}$$

The example often used in many nonlinear complementary problems and variation inequality problems in articles was applied. The parameters of the proposed algorithm were set as follows:

$M = 100$, $D = 4$, $iter_{\max} = 4000$, $\lambda^{(1)} = 0$, $\sigma_1 = 4$, and $\varepsilon = 10^{-3}$. The program was programmed by MATLAB 6.0 and was run on an ordinary PC (CPU, 2.00 GHz; memory, 512 MB). The algorithm was run 50 times. TABLE 1 shows the results of 10 runs.

TABLE 1 : Hybrid Quantum Particle Swarm algorithm results of 10 runs

	x_1	x_2	x_3	x_4
1	1.230784	0.001412	0.016422	0.508871
2	1.221801	0.000323	0.009425	0.500256
3	1.002321	0.001219	3.003415	0.008746
4	1.005031	0.000421	3.023378	0.001241
5	1.222452	0.002547	0.001203	0.500521
6	1.223784	0.000316	0.000473	0.504833
7	1.223323	0.000343	0.000231	0.508702
8	1.000213	0.000210	3.009847	0.000120
9	1.228945	0.003504	0.032342	0.500098
10	1.233375	0.000531	0.039359	0.504490

Some previous studies[2,9] used traditional algorithms, the gradient should be calculated, the initial point should be given, and a continuous objective function should be provided. By contrast, the algorithm proposed in this paper is the swarm intelligence optimization algorithm, which does not give the initial point and does not require calculation of the gradient nor a smooth objective function. Furthermore, this algorithm can obtain all the standard solutions of the objective function at the same time (TABLE 1), compared with traditional algorithms, which cannot obtain all the solutions of the original problem simultaneously. The solution $\left(\sqrt{6}/2, 0, 0, 1/2\right)^T$ was obtained seven times, whereas the solution $(1, 0, 3, 0)^T$ was obtained three times.

CONCLUSIONS

In this paper, the hybrid QPSO was proposed to solve NCPs based on the multiplier penalty function method. The proposed method mainly uses the multiplier penalty function to deal with infeasible particle algorithms in iteration. The multiplier penalty function method was introduced to deal with the constraint conditions and to reduce the complexity of the fitness function and the computation time. The proposed algorithm is a new way of solving NCPs and linear complementary problems. It also expands the application of the swarm intelligence optimization algorithm to more fields. According to the numerical experiments, the new algorithm is effective for NCPs, with a higher calculation accuracy and success rate.

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