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Heteroscedasticity space stochastic frontier model and technical efficiency estimation based on cold chain logistics

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ABSTRACT

Introduced the effect of the stochastic frontier analysis framework, respectively, paragraphs consider random disturbance and technical inefficiency and both exist homoscedastic, build normal - in the form of truncated normal distribution heteroblastic stochastic frontier model for logistics management, makes a research on the homoscedastic normal - truncated gaussian model production technical parameters and the influence of technical efficiency estimation, which estimates parameters by using the method of maximum likelihood, and concluded that the technical efficiency estimates.

KEYWORDS

Normal - truncated normal homoscedastic space stochastic frontier model; Maximum likelihood estimation; Technical efficiency; Cold chain logistics.

THE STOCHASTIC DISTURBANCE v EXIST HETEROSCEDASTICITY

This section of the study are based on cross-sectional data, the research train of thought can be expanded to the panel data model. The first to explore the homoscedastic of a random disturbance item for production technology parameter estimation and the influence of the inference efficiency [1] .

Assuming $u \sim N^+(\mu_* l, \sigma_u^2, I); \eta \sim N(0, M_u); v = (I - \rho W_2)^{-1}\eta \sim N(0, M_\eta \Sigma)$; $M_\eta = \begin{pmatrix} \sigma_{\eta 1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\eta N}^2 \end{pmatrix}$;

thus the average is:

$$E(y) = (I - \lambda W_1)^{-1} x\beta - (I - \lambda W_1)^{-1} \left\{ \beta_0 - \left[\mu_* l + \sigma_u^2 \times \frac{\Phi^*(\mu_* l; 0, \sigma_u^2, I)}{\Phi(\mu_* l; 0, \sigma_u^2, I)} \right] \right\} \quad (1)$$

In the presence of homoscedastic of the v situation, when will be estimated based on the conditions of technical efficiency modal based on the $Mode(u|\varepsilon)$, technical efficiency estimation formula should be expressed as follows:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[\hat{\sigma}_u^2 (\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma})^{-1} (\hat{\varepsilon} + \hat{\mu}_* l) + \hat{\mu}_* l \right] \quad (2)$$

When the technical efficiency of estimates based on conditional expectation, the

basis of $E(u|\varepsilon)$, v is the homoscedastic model technical efficiency estimate expression is as follows:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[-\hat{u} - \hat{\Omega} \times \frac{\Phi^*\left(\hat{\mu}; 0, \hat{\Omega}\right)}{\Phi\left(\hat{\mu}; 0, \hat{\Omega}\right)} \right]$$

$$= \exp\left\{ \begin{array}{l} \hat{\sigma}_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} (\hat{\varepsilon} + \hat{\mu}_* l) - \hat{\sigma}_u^2 \left[I - \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} \right] \\ \dfrac{\Phi^*\left[-\hat{\sigma}_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} (\hat{\varepsilon} + \hat{\mu}_* l) - \mu_* l; 0, \sigma_u^2 \left[I - \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} \right] \right]}{\Phi\left[-\hat{\sigma}_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} (\hat{\varepsilon} + \hat{\mu}_* l) - \mu_* l; 0, \sigma_u^2 \left[I - \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} \right] \right]} \end{array} \right\} \quad (3)$$

As you can see type (3), in the normal-truncated gaussian model, the random disturbance item v homoscedastic to the influence of the technical efficiency is much more complex than the estimation way, also is unable to come up from theory to determine the impact on lower partial or upper partial [2] .

(1)Maximum likelihood estimation

Considering the random disturbance item exists v homoscedastic, and $\sigma_{\eta i}^2 = g_1(z_i; \delta_1)$, there are

$M_\eta = \begin{pmatrix} g_1(z_i; \delta_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_1(z_N; \delta_1) \end{pmatrix}$.The following model of maximum likelihood function is derived.

Due to the $u \sim N^+(\mu_* l, \sigma_u^2, I)$, while $v \sim N(0, M_\eta \Sigma)$,because were not associated with u and v , (u, v) joint distribution density function is available:

$$f(u, v) = \left(\frac{1}{\pi \sigma_u} \right)^N \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u} \right) \left| M_\eta \Sigma \right|^{-1/2} \exp\left[\frac{(u - \mu_* l)'}{2\sigma_u^2} - \left(\frac{v'(M_\eta \Sigma)^{-1} v}{2} \right) \right] \quad (4)$$

By $\varepsilon = v - u$, known $v \sim N(0, M_\eta \Sigma)$ joint distribution density function for;

$$f(u, \varepsilon) = \left(\frac{1}{\pi \sigma_u} \right)^N \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u} \right) \left| M_\eta \Sigma \right|^{-1/2} \exp\left[\frac{(u - \mu_* l)'(u - \mu_* l)}{2\sigma_u^2} - \left(\frac{(\varepsilon + u)'(M_\eta \Sigma)^{-1}(\varepsilon + u)}{2} \right) \right]$$

$$= \left(\frac{1}{\pi \sigma_u} \right)^N \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u} \right) \left| M_\eta \Sigma \right|^{-1/2} \exp\left[-\frac{1}{2} (\varepsilon + \mu_* l)' \left(\sigma_u^2 I + M_\eta \Sigma \right)^{-1} (\varepsilon + \mu_* l) \right] \quad (5)$$

$$\times \exp\left[-\frac{1}{2} (u - \mu_* l - \mu)' \Omega^{-1} (u - \mu_* l - \mu) \right]$$

Among them, $\mu = \sigma_u^2 \left(\sigma_u^2 I + M_\eta \Sigma\right)^{-1} (\varepsilon + \mu_* l), \Omega = \sigma_u^2 \left[I - \sigma_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} \right]$

$\Omega^{-1} = \frac{1}{\sigma_u^2} I + (M_\eta \Sigma)^{-1}$;

Will type (5) to u integrate, can obtain the distribution density function for ε ;

$$f(\varepsilon) = \int_0^\infty f(u, \varepsilon_t) du$$
$$= \left(\frac{1}{\pi\sigma_u}\right)^N \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u}\right) \left|M_\eta \Sigma\right|^{-1/2} \exp\left[-\frac{1}{2}(\varepsilon + \mu_* l)'\left(\sigma_u^2 I + M_\eta \Sigma\right)^{-1} (\varepsilon + \mu_* l)\right]$$
$$\times \int_0^\infty \exp\left[-\frac{1}{2}(u - \mu_* l - \mu)'\Omega^{-1}(u - \mu_* l - \mu)\right] du \qquad (6)$$
$$\left(\frac{2}{\sqrt{2\pi}}\right)^N \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u}\right) \left|\sigma_u^2 I + M_\eta \Sigma\right|^{-1/2} \exp\left[-\frac{1}{2}(\varepsilon + \mu_* l)'\left(\sigma_u^2 I + M_\eta \Sigma\right)^{-1} (\varepsilon + \mu_* l)\right] \Phi\left[\Omega^{-1/2}(\mu + \mu_* l)\right]$$

Due to $\varepsilon = (I - \lambda W_1) y - x\beta$, based on the type (6), logarithmic likelihood function of the model can be obtained as follows:

$$ll(\beta, \sigma_u, \delta_1, \lambda, \mu_*, \rho) = NIn(2) - \frac{N}{2} In(2\pi) + In\left[\Phi^{-1}\left(\frac{\mu_* l}{\sigma_u}\right)\right] - \frac{1}{2} In\left|\sigma_u^2 I + M_\eta \Sigma\right|$$
$$-\frac{1}{2}(\varepsilon + \mu_* l)'\left(\sigma_u^2 I + M_\eta \Sigma\right)^{-1} (\varepsilon + \mu_* l) + In\left\{\Phi\left[\Omega^{-\frac{1}{2}}(\mu + \mu_* l)\right]\right\} + In\left|\det(I - \lambda W_1)\right| \qquad (7)$$

Based on the logarithmic likelihood function type (7) to carry on the optimization, the model can be obtained the parameter estimation of $(\hat{\beta}, \hat{\sigma}_u, \hat{\delta}_1, \hat{\lambda}, \hat{\mu}_* \hat{\rho})$. According to $\hat{\sigma}_{\eta i}^2 = g_1\left(z_i; \hat{\delta}_1\right)$, the estimates of the known $\hat{\delta}_1$ to $\hat{\sigma}_{\eta i}^2$ after the estimate.

(2) Technical efficiency

In this section is based on the JLMS method, with normal - truncated distribution assumptions, random disturbance in homoscedastic space stochastic frontier model estimate technical efficiency [3] .Combination type (5) and (6), in ε known, the conditional distribution of u is:

$$f(u|\varepsilon) = \frac{f(u, \varepsilon_t)}{f(\varepsilon)}$$
$$= \frac{\left(\frac{1}{\pi\sigma_u}\right)^N \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u}\right) \left|M_\eta \Sigma\right|^{-1/2} \exp\left[-\frac{1}{2}(\varepsilon + \mu_* l)'\left(\sigma_u^2 I + M_\eta \Sigma\right)^{-1} (\varepsilon + \mu_* l)\right] \times \exp\left[-\frac{1}{2}(u - \mu_* l - \mu)'\Omega^{-1}(u - \mu_* l - \mu)\right]}{\left(\frac{2}{\sqrt{2\pi}}\right)^N \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u}\right) \left|\sigma_u^2 I + M_\eta \Sigma\right|^{-1/2} \exp\left[-\frac{1}{2}(\varepsilon + \mu_* l)'\left(\sigma_u^2 I + M_\eta \Sigma\right)^{-1} (\varepsilon + \mu_* l)\right] \Phi\left[\Omega^{-\frac{1}{2}}(\mu + \mu_* l)\right]} \quad (8)$$
$$= \frac{1}{(2\pi)^{N/2} \left|M_\eta \Sigma\right|^{1/2} \Phi(\Omega^{-\frac{1}{2}}\mu)} \exp\left[-\frac{1}{2}(u - \mu_* l - \mu)'\Omega^{-1}(u - \mu_* l - \mu)\right]$$

$$Mode(u|\varepsilon) = \mu + \mu_* l = -\sigma_u^2 \left(\sigma_u^2 I + M_\eta \Sigma\right)^{-1} (\varepsilon + \mu_* l) + \mu_* l \quad (9)$$

Therefore, once the point estimate of u , each production unit of technical efficiency estimation can be expressed as follows:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[-\hat{u} - \hat{\mu}_* l - \hat{\Omega} \times \frac{\Phi^*\left(\hat{\mu} + \hat{\mu}_* l; 0\hat{\Omega}\right)}{\Phi\left(\hat{\mu} + \hat{\mu}_* l; 0\hat{\Omega}\right)}\right]$$
$$= \exp\left\{\begin{array}{c}\hat{\sigma}_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} \left(\hat{\varepsilon} + \hat{\mu}_* l\right) - \hat{\mu}_* l - \hat{\sigma}_u^2 \left[I - \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1}\right] \\ \dfrac{\Phi^*\left[-\sigma_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} \left(\hat{\varepsilon} + \mu_* l\right) + \mu_* l; 0, \sigma_u^2 \left[I - \sigma_u^2 \left(\sigma_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1}\right]\right]}{\Phi\left[-\hat{\sigma}_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1} \left(\hat{\varepsilon} + \hat{\mu}_* l\right) + \mu_* l; 0, \hat{\sigma}_u^2 \left[I - \hat{\sigma}_u^2 \left(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma}\right)^{-1}\right]\right]}\end{array}\right\} \quad (10)$$

Or:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[\hat{\sigma}_u^2(\hat{\sigma}_u^2 I + \hat{M}_\eta \hat{\Sigma})^{-1}(\hat{\varepsilon} + \hat{\mu}_* l) - \hat{\mu}_* l\right] \quad (11)$$

TECHNICAL INEFFICIENCY u EXIST HETEROSCEDASTICITY

This section first when the model of technical inefficiency homoscedastic exists, the parameter estimation of production technology and concluded that the effects of the technical efficiency [4] .

Assuming $u \sim N^+(\mu_* l, M_\eta); \eta \sim N(0, \sigma_v^2 I); v = (I - \rho W_2)^{-1}\eta \sim N(0, \sigma_v^2 \Sigma)$; $M_\eta = \begin{pmatrix} \sigma_{\eta 1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\eta N}^2 \end{pmatrix}$,

The mean of the y are as follows:

$$E(y) = (I - \lambda W_1)^{-1} x\beta - (I - \lambda W_1)^{-1}\left\{\beta_0 - \left[\mu_* l + M_u \times \frac{\Phi^*(\mu_* l; 0, M_u)}{\Phi(\mu_* l; 0, M_u)}\right]\right\} \quad (12)$$

In the presence of homoscedastic of the u situation, when will be estimated based on the conditions of technical efficiency modal $Mode(u|\varepsilon)$ basis, technical efficiency estimation formula should be expressed as follows:

$$Mode(u|\varepsilon) = \mu + \mu_* l = -M_u(M_u + \sigma_v^2 \hat{\Sigma})^{-1}(\varepsilon + \mu_* l) + \mu_* l \quad (13)$$

When the technical efficiency of estimates based on the conditional expectation $E(u|\varepsilon)$, u presence of homoscedastic model technical efficiency estimate expression is as follows:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[-\hat{u} - \hat{\mu}_* l - \hat{\Omega} \times \frac{\Phi^*\left(\hat{\mu} + \hat{\mu}_* l; 0, \hat{\Omega}\right)}{\Phi\left(\hat{\mu} + \hat{\mu}_* l; 0, \hat{\Omega}\right)}\right]$$

$$= \exp\left\{\begin{array}{l} \hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon} + \hat{\mu}_* l\right) - \hat{\mu}_* l - \hat{M}_u\left[I - \hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\right] \\ \times \dfrac{\Phi^*\left[-\hat{M}_u\left(\hat{M}_u + \sigma_v^2\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon} + \mu_* l\right) + \mu_* l; 0, \hat{M}_u\left[I - \hat{M}_u\left(\hat{M}_u + \sigma_v^2\hat{\Sigma}\right)^{-1}\right]\right]}{\Phi\left[-\hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon} + \hat{\mu}_* l\right) + \hat{\mu}_* l; 0, \hat{M}_u\left[I - \hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\right]\right]} \end{array}\right\} \quad (14)$$

As you can see type (14), in the normal - truncated gaussian model, technology without a u homoscedastic to the efficiency and the influence of the technical efficiency is much more complex than the $Mode(u|\varepsilon)$ estimate way, also is unable to come up from theory to determine the impact on partial or partial. Therefore, in normal-truncated gaussian model, under the analysis framework of u homoscedastic will not only directly affect the production technology of parameter estimate, also reflected in the estimate of technical efficiency.

(1) Maximum likelihood estimation

Consider technology without u deposit in efficiency and homoscedastic, and $\sigma_{ui}^2 = g_2\left(z_i; \delta_2\right)$,there are, $M_u = \begin{pmatrix} g_2\left(z_1; \delta_2\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_2\left(z_N; \delta_2\right) \end{pmatrix}$.Based on the above assumptions, the following the model of maximum likelihood estimation is derived.

On the basis of the probability density function as $u \sim N^+(\mu_* l, M_u)$, u is:

$$f(u) = \left(\frac{2}{\sqrt{2\pi}}\right)^N \left|M_\eta\right|^{-1/2} \Phi^{-1}\left(\frac{\mu_* l}{\sigma_u}\right)\exp\left[-\frac{(u - \mu_* l)' M_u' (u - \mu_* l)}{2}\right] (15)$$

Due to u and v are not relevant, the joint distribution density function of (u, v) are as follows:

$$f(u,v)=\left(\frac{1}{\pi\sigma_u}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|\Sigma\right|^{-1/2}\exp\left[-\frac{(u-\mu_* l)' M_u^{-1}(u-\mu_* l)}{2}-\frac{v'\Sigma^{-1}v}{2}\right] \quad (16)$$

By a $\varepsilon = v-u$, the joint distribution density function of (u,v) are as follows:

$$f(u,\varepsilon)=\left(\frac{1}{\pi\sigma_u}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|\Sigma\right|^{-1/2}\exp\left[-\frac{-(u-\mu_* l)' M_u^{-1}(u-\mu_* l)}{2}-\frac{(\varepsilon+\mu)'\Sigma^{-1}(\varepsilon+\mu)}{2\sigma_v^2}\right]$$

$$=\left(\frac{1}{\pi\sigma_u}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|\Sigma\right|^{-1/2}\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)\right] \quad (17)$$
$$\times\exp\left[-\frac{1}{2}(u-\mu_* l-\mu)'\Omega^{-1}(u-\mu_* l-\mu)\right]$$

Among them, $\mu=-M_u(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l), \Omega=M_u\left[I-M_u(M_u+\sigma_v^2\Sigma)^{-1}\right]$, $\Omega^{-1}=M_u^{-1}+\frac{1}{\sigma_v^2}\Sigma^{-1}$.

Will type (17) to u integrate, can obtain the distribution density function of ε :

$$f(\varepsilon)=\int_0^\infty f(u,\varepsilon_t)du$$
$$=\left(\frac{1}{\pi\sigma_u}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|\Sigma\right|^{-1/2}\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)\right]$$
$$\times\int_0^\infty \exp\left[-\frac{1}{2}(u-\mu_* l-\mu)'\Omega^{-1}(u-\mu_* l-\mu)\right]du \quad (18)$$
$$=\left(\frac{2}{\sqrt{2\pi}}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u+\sigma_v^2\Sigma\right|^{-1/2}\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)\right]\Phi\left[\Omega^{-\frac{1}{2}}(\mu+\mu_* l)\right]$$

Because of $\varepsilon=(I-\lambda W_1)^{-1}y-x\beta$, based on the type (18), logarithmic likelihood function of the model can be obtained as follows:

$$ll(\beta,\sigma_v,\delta_2,\mu_*,\lambda,\rho)=NIn(2)-\frac{N}{2}In(2\pi)+In\left[\Phi^{-1}\left(\mu_* l' M_u\right)\right]-\frac{1}{2}In\left|M_u+\sigma_v^2\Sigma\right|$$
$$-\frac{1}{2}(\varepsilon+\mu_* l)'\left(M_u+\sigma_v^2\Sigma\right)^{-1}(\varepsilon+\mu_* l)+In\left\{\Phi\left[\Omega^{-\frac{1}{2}}(\mu+\mu_* l)\right]\right\}+In\left|\det(I-\lambda W_1)\right| \quad (19)$$

Based on the logarithmic likelihood function (19) for optimization, the model can be obtained the parameter estimation of $(\hat{\beta},\hat{\sigma}_v,\hat{\delta}_2,\hat{\mu}_*,\hat{\lambda},\hat{\rho})$, according to $\hat{\sigma}_{ui}^2=g_2\left(z_i;\hat{\delta}_2\right)$, in the known $\hat{\delta}_2$ available after estimate $\hat{\sigma}_{ui}^2$ estimate.

(2) Technical efficiency Combination type (17) and (18), distribution under the condition of known ε , u is:

$$f(u|\varepsilon)=\frac{f(u,\varepsilon_t)}{f(\varepsilon)}$$
$$=\frac{\left(\frac{1}{\pi\sigma_v}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|\Sigma\right|^{-1/2}\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)\right]\times\exp\left[-\frac{1}{2}(u-\mu_* l-\mu)'\Omega^{-1}(u-\mu_* l-\mu)\right]}{\left(\frac{2}{\sqrt{2\pi}}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u+\sigma_v^2\Sigma\right|^{-1/2}\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)\right]\Phi\left[\Omega^{-\frac{1}{2}}(\mu+\mu_* l)\right]} \quad (20)$$

Among them, $\mu=-M_u(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l), \Omega=M_u\left[I-M_u(M_u+\sigma_v^2\Sigma)^{-1}\right]$.

We can see by type (20), $f(u|\varepsilon)$ subject to N yuan truncated normal distribution $N^+(\mu_* l, M_\eta)$, and so its mean and all the number are:

$$E(u|\varepsilon)=\mu+\mu_* l+\Omega\times\frac{\Phi^*(\mu+\mu_* l;0,\Omega)}{\Phi(\mu+\mu_* l;0,\Omega)}$$
$$\times\frac{\Phi^*\left[-M_u(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)+\mu_* l;0,M_u\left[I-M_u(M_u+\sigma_v^2\Sigma)^{-1}\right]\right]}{\Phi\left[-M_u(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)+\mu_* l;0,M_u\left[I-M_u(M_u+\sigma_v^2\Sigma)^{-1}\right]\right]} \quad (21)$$

$$Mode(u|\varepsilon)=\mu+\mu_* l=-M_u(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)+\mu_* l \quad (22)$$

Therefore, once the point estimate of u, each production unit of technical efficiency estimation can be expressed as follows:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[-\hat{u} - \hat{\mu}_* l - \hat{\Omega} \times \frac{\Phi^*\left(\hat{\mu} + \hat{\mu}_* l; 0, \hat{\Omega}\right)}{\Phi\left(\hat{\mu} + \hat{\mu}_* l; 0, \hat{\Omega}\right)} \right]$$

$$= \exp\left\{ \begin{array}{l} \hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon} + \hat{\mu}_* l\right) - \hat{\mu}_* l - \hat{M}_u\left[I - \hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\right] \\ \times \dfrac{\Phi^*\left[-\hat{M}_u\left(\hat{M}_u + \sigma_v^2\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon} + \mu_* l\right) + \mu_* l; 0, \hat{M}_u\left[I - \hat{M}_u\left(\hat{M}_u + \sigma_v^2\hat{\Sigma}\right)^{-1}\right]\right]}{\Phi\left[-\hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon} + \hat{\mu}_* l\right) + \hat{\mu}_* l; 0, \hat{M}_u\left[I - \hat{M}_u\left(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma}\right)^{-1}\right]\right]} \end{array} \right\} \quad (23)$$

Or:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[\hat{M}_u(\hat{M}_u + \hat{\sigma}_v^2\hat{\Sigma})^{-1}(\hat{\varepsilon} + \hat{\mu}_* l) - \hat{\mu}_* l\right] \quad (24)$$

DIFFERRENT VARIANCE OF v AND u AT SAME TIME

This section is based on normal-truncated normal distribution form, under this kind of homoscedastic hypothesis model of maximum likelihood estimation and inference technology efficiency.

(1) Maximum likelihood estimation

Before for maximum likelihood estimation for the following assumptions:

$u \sim N^+(\mu_* l, M_u); \eta \sim N(0, M_u); v = (I - \rho W_2)^{-1}\eta \sim N(0, M_u\Sigma)$;

$$M_u = \begin{pmatrix} \sigma_{u1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{uN}^2 \end{pmatrix} = \begin{pmatrix} g_2(z_1;\delta_2) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_2(z_N;\delta_2) \end{pmatrix}; M_\eta = \begin{pmatrix} \sigma_{\eta 1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\eta N}^2 \end{pmatrix} = \begin{pmatrix} g_1(z_1;\delta_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_1(z_N;\delta_1) \end{pmatrix}$$

Then the model is derived based on the hypothesis of the maximum likelihood estimation.

Due to $u \sim N^+(\mu_* l, M_u)$, available u distribution density function can be see type (7). On the basis of the probability density function and $v \sim N(0, M_u\Sigma)$.

$$f(u,v) = \left(\frac{1}{\pi}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|M_\eta\Sigma\right|^{-1/2} \exp\left[-\frac{(u-\mu_* l)' M_u^{-1}(u-\mu_* l)}{2} - \frac{v'(M_\eta\Sigma)^{-1}v}{2}\right] \quad (25)$$

By a $\varepsilon = v - u$, the $f(u,\varepsilon)$ of the joint distribution density function is:

$$f(u,\varepsilon) = \left(\frac{1}{\pi}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|M_\eta\Sigma\right|^{-1/2} \times \exp\left[-\frac{(u-\mu_* l)' M_u^{-1}(u-\mu_* l)}{2} - \frac{(\varepsilon+u)'(M_\eta\Sigma)^{-1}(\varepsilon+u)}{2}\right] \quad (26)$$

$$= \left(\frac{1}{\pi}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|M_u\Sigma\right|^{-1/2} \exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u + M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)\right] \times \exp\left[-\frac{1}{2}(u-\mu_* l-\mu)'\Omega^{-1}(u-\mu_* l-\mu)\right]$$

Among them, $\mu = -M_u(M_u + M_\eta\Sigma)^{-1}(\varepsilon + \mu_* l), \Omega = M_u\left[I - M_u(M_u + M_\eta\Sigma)^{-1}\right]; \Omega^{-1} = M_u^{-1} + (M_\eta\Sigma)^{-1}$.

Will type (26) to u integrate, can obtain the distribution density function ε :

$$f(\varepsilon) = \int_0^\infty f(u,\varepsilon)du$$

$$= \left(\frac{1}{\pi}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|M_\eta\Sigma\right|^{-1/2} \exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u + M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)\right]$$

$$\times\int_0^\infty \exp\left[-\frac{1}{2}(u-\mu_* l-\mu)'\Omega^{-1}(u-\mu_* l-\mu)\right]du \left(\frac{2}{\sqrt{2\pi}}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u + M_\eta\Sigma\right|^{-1/2} \Phi\left[\Omega^{-\frac{1}{2}}(\mu+\mu_* l)\right] \quad (27)$$

$$\times\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u + M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)\right]$$

Based on the type (21), the logarithmic likelihood function of the model can be obtained:

$$ll(\beta,\delta_1,\delta_2,\mu_*,\lambda,\rho) = NIn(2) - \frac{N}{2}In(2\pi) + In\left[\Phi^{-1}\left(\mu_* l' M_u\right)\right] - \frac{1}{2}In\left|M_u + M_\eta \Sigma\right|$$
$$-\frac{1}{2}(\varepsilon+\mu_* l)'\left(M_u + M_\eta\Sigma\right)^{-1}(\varepsilon+\mu_* l) + In\left\{\Phi\left[\Omega^{-\frac{1}{2}}(\mu+\mu_* l)\right]\right\} + In\left|\det(I-\lambda W_1)\right| \quad (28)$$

By maximizing the logarithm likelihood function type (28), can get a normal - truncated gaussian model parameter estimation $(\hat{\beta},\hat{\delta}_1,\hat{\delta}_2,\hat{\mu}_*,\hat{\lambda},\hat{\rho})$. According to the $\hat{\sigma}_{ui}^2 = g_2\left(z_i;\hat{\delta}_2\right)$ and $\hat{\sigma}_{\eta i}^2 = g_1\left(z_i;\hat{\delta}_1\right)$, after known estimates of the $\hat{\delta}_1$ and $\hat{\delta}_2$ can be estimated value of $\hat{\sigma}_{\eta i}^2$ and $\hat{\sigma}_{ui}^2$.

(2) Technical efficiency

Combination type (26) and (27), distribution under the condition of known ε when u is:

$$f(u|\varepsilon) = \frac{f(u,\varepsilon_t)}{f(\varepsilon)}$$
$$= \frac{\left(\frac{1}{\pi}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u\right|^{-1/2}\left|M_\eta\Sigma\right|^{-1/2}\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u+M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)\right] \times\exp\left[-\frac{1}{2}(u-\mu_* l-\mu)'\Omega^{-1}(u-\mu_* l-\mu)\right]}{\left(\frac{2}{\sqrt{2\pi}}\right)^N \Phi^{-1}\left(\mu_* l' M_u\right)\left|M_u+M_\eta\Sigma\right|^{-1/2}\Phi\left[\Omega^{-\frac{1}{2}}(\mu+\mu_* l)\right]\exp\left[-\frac{1}{2}(\varepsilon+\mu_* l)'(M_u+M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)\right]}$$
$$= \frac{1}{(2\pi)^{N/2}\left|\Omega\right|^{1/2}\Phi\left[\Omega^{-\frac{1}{2}}(\mu+\mu_* l)\right]}\exp\left[-\frac{1}{2}(u-\mu_* l-\mu)'\Omega^{-1}(u-\mu_* l-\mu)\right] \quad (29)$$

Among them, $\mu = -M_u(M_u+M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l), \Omega = M_u\left[I - M_u(M_u+M_\eta\Sigma)^{-1}\right]$,You can see by type (29), $f(u|\varepsilon)$ obey truncated normal distribution N yuan $N^+(\mu+\mu_* l,\Omega)$, so the mean and the number are:

$$E(u|\varepsilon) = \mu+\mu_* l+\Omega\times\frac{\Phi^*(\mu+\mu_* l;0,\Omega)}{\Phi(\mu+\mu_* l;0,\Omega)} = -M_u(M_u+M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)+\mu_* l+M_u\left[I-M_u(M_u+\mu_* l\Sigma)^{-1}\right]$$
$$\times\frac{\Phi^*\left[-M_u(M_u+\sigma_v^2\Sigma)^{-1}(\varepsilon+\mu_* l)+\mu_* l;0,M_u\left[I-M_u(M_u+M_\eta\Sigma)^{-1}\right]\right]}{\Phi\left[-M_u(M_u+M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)+\mu_* l;0,M_u\left[I-M_u(M_u+M_\eta\Sigma)^{-1}\right]\right]} \quad (30)$$

$$Mode(u|\varepsilon) = \mu+\mu_* l = -M_u(M_u+M_\eta\Sigma)^{-1}(\varepsilon+\mu_* l)+\mu_* l \quad (31)$$

$f(u|\varepsilon)$ in front of the mean or the mode can be used as u point estimates, as a result, each production unit of technical efficiency estimation can be expressed as follows:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[-\hat{u}-\hat{\mu}_* l-\hat{\Omega}\times\frac{\Phi^*\left(\hat{\mu}+\hat{\mu}_* l;0,\hat{\Omega}\right)}{\Phi\left(\hat{\mu}+\hat{\mu}_* l;0,\hat{\Omega}\right)}\right]$$
$$= \exp\left\{\begin{array}{l}\hat{M}_u\left(\hat{M}_u+\hat{M}_\eta\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon}+\hat{\mu}_* l\right)-\hat{\mu}_* l-\hat{M}_u\left[I-\hat{M}_u\left(\hat{M}_u+\hat{M}_\eta\hat{\Sigma}\right)^{-1}\right] \\ \times\frac{\Phi^*\left[-\hat{M}_u\left(\hat{M}_u+\hat{M}_\eta\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon}+\mu_* l\right)+\mu_* l;0,\hat{M}_u\left[I-\hat{M}_u\left(\hat{M}_u+\hat{M}_\eta\hat{\Sigma}\right)^{-1}\right]\right]}{\Phi\left[-\hat{M}_u\left(\hat{M}_u+\hat{M}_\eta\hat{\Sigma}\right)^{-1}\left(\hat{\varepsilon}+\hat{\mu}_* l\right)+\hat{\mu}_* l;0,\hat{M}_u\left[I-\hat{M}_u\left(\hat{M}_u+\hat{M}_\eta\hat{\Sigma}\right)^{-1}\right]\right]}\end{array}\right\} \quad (32)$$

Or:

$$\hat{TE} = \exp(-\hat{u}) = \exp\left[\hat{M}_u(\hat{M}_u+\hat{M}_\eta\hat{\Sigma})^{-1}(\hat{\varepsilon}+\hat{\mu}_* l)-\hat{\mu}_* l\right] \quad (33)$$

CONCLUSIONS

(1)The random disturbance of homoscedastic parameter estimate impact is not big, the production technology by has been estimated to correct deviation, the intercept and slope coefficient of available unbiased estimation, but its impact on technical efficiency estimation cannot be ignored.

(2)Technical inefficiency of homoscedastic will not only directly affect the production technology parameter estimates, also reflected in the estimate of technical efficiency.

(3)When homoscedastic model existence, its impact on technical efficiency estimation based on conditional expectation than technical efficiency estimation based on conditional modal is much more complicated, both can't come up from theory to determine the error is under partial or partial, the result will depend on the specific empirical data.

(4)Set various forms heteroblastic model was derived the logarithm likelihood function and technical efficiency estimation.

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