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FRW COSMOLOGICAL MODEL WITH COSMIC STRING IN SCALE COVARIANT THEORY

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ABSTRACT

In the presence of Cosmic String homogeneous isotropic Friedmann Robertson Walker cosmological model are constructed in the scale covariant theory of gravitation. Using special law of variation for Hubble's parameter, a class of solutions is found. The physical properties of the model are also discussed.

Key words: Friedman cosmological model, Cosmic string, Hubble parameter.

INTRODUCTION

Alternative theories of gravity have been extensively studied in connection with their cosmological applications. Noteworthy among them are scalar-tensor theories of gravitation formulated by Lyra (1951), Brans and Dicke (1961), Nordvedt (1970), Sen (1957), Sen and Dunn (1971) and Saez and Ballester (1985). In Brans-Dicke theory there exists a variable gravitational parameter G. Canuto et al., (1977) formulated a scale covariant theory of gravitation which admits a variable 'G' and which is a viable alternative in general relatively (Wesson 1980 and Will 1984). In the scale covariant theory Einstein's field equations are valid in gravitational units whereas physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by a conformal transformation.

$$\bar{g}_{ab} = \phi^2 \left(X^c \right) g_{ab}, \qquad \ldots(1)$$

Where Latin indices take values 1, 2, 3, 4, bars denote gravitational units and unbar denotes atomic quantities. The gauge function 'f', in its most general formulation is a function of all space time co ordinates. The possibilities that have been considered for gauge function f are (Canuto et al., 1977).

$$\phi(t) = \left(\frac{t_0}{t} \right)^{\in}, \in = \pm 1, \pm \frac{1}{2}, \qquad \ldots(2)$$

Where t_0 is constant.

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The field equations in scale covariant theory are

$$R_{ab} - \frac{R}{2} g_{ab} + f_{ab}(\phi) = GT_{ab}, \qquad \ldots(3)$$

where $f_{ab}(\phi)$ is given by,

$$\phi^2 f_{ab} = -2\phi\phi_{a;b} + 4\phi_a\phi_b + g_{ab}\left(2\phi\phi_{;k}^{\ k} - \phi^k\phi_k\right) \qquad \ldots(4)$$

Here T_{ab} is the usual stress energy tensor of the matter. Here semicolon denotes covariant differentiation.

Also, is a consequence of the field equation (3).

$$T_{;b}^{ab} = 0, \qquad \ldots(5)$$

In recent years, there has been a considerable interest in cosmological models in Einstein's theory and in several alternative theories of gravitation with cosmic string source. Cosmic strings and domain walls are the topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe (Kibble 1976). The gravitational effects of cosmic strings have been extensively discussed by Vilenkin (1981), Gott (1985), Latelier (1983) and Satchel (1980) in general relativity. Relativistic string models in the context of Bianchi space times have been obtained by Krori et al., (1990), Banarjee et al., (1990), Tikekar and Patel (1990) and Bhattacharjee and Baruah (2001). String cosmological models in scalar-tensor theories of gravitation have been investigated by Sen (2000), Barros et al., (2001), Sen et al., (1997), Gundalach and Ortiz (1990), Barros and Romero (1995), Rahaman et al., (2003) and others. In particular, Reddy (2003), Reddy et al., (2006) and Reddy and Naidu (2007) have discussed some string cosmological models in Saez–Ballester scalar-tensor theory of gravitation. Venkateswarlu et al., (2008) have investigated Bianchi type I, II, VIII and IX string cosmological solutions in self creation theory of gravitation and recently Rao et al., (2008) obtained Bianchi type II, VIII and IX string cosmological models in Saez–Ballester scalar tensor theory of gravitation.

In this paper, we have studied FRW cosmological model in the presence of Cosmic String in the scale covariant theory of gravitation.

Metric and field equations

We consider the spatially homogeneous and isotropic space time given by the Friedmann metric

$$ds^2 = dt^2 - R^2(t)\left[\left(1 - kr^2\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \qquad \ldots(6)$$

where $k = +1,-1,0$ and the energy momentum tensor of cosmic string is

$$T_j^i = \rho u^i u_j - \lambda x^i x_j, \qquad \ldots(7)$$

where ρ is the rest energy density of the cloud of string with massive particles attached to them

$\rho = \rho_p + \lambda$, ρ_p, being the rest energy of the particle attached to the string and λ be the tension density of the system of string. u^i describe the cloud four velocity and x^i represent the direction of string and anomalism of u^i, x^i is given as

$u^i u_i = 1, x^i x_j = 0$ and $x^i x_j = -1$, …(8)

In the commoving co-ordinate system, equation (2) implies that

$T_1^1 = T_2^2 = 0$ and $T_3^3 = \lambda$ and $T_4^4 = \rho$, …(9)

and $T_j^i = 0$ for $i \neq j$.

Using commoving co-ordinate, the field equations (3) for the metric (6) and the energy -momentum tensor (7) can be written as,

$$\frac{2\ddot{R}}{R} + \frac{\left(\dot{R}^2 + k\right)}{R^2} = \frac{4\dot{R}}{R}\frac{\dot{\phi}}{\phi} + \frac{2\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2}, \quad \text{…(10)}$$

$$\frac{2\ddot{R}}{R} + \frac{\left(\dot{R}^2 + k\right)}{R^2} = G\lambda + \frac{4\dot{R}}{R}\frac{\dot{\phi}}{\phi} + \frac{2\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2}, \quad \text{…(11)}$$

$$\frac{3\left(\dot{R}^2 + k\right)}{R^2} = G\rho + \frac{6\dot{\phi}}{\phi}\frac{\dot{R}}{R} + \frac{3\dot{\phi}^2}{\phi^2}, \quad \text{…(12)}$$

and

$$\dot{\rho} + 3(\lambda + \rho)\frac{\dot{R}}{R} = 0, \quad \text{…(13)}$$

where a dot ($\dot{}$) indicates differentiation with respect to *t*.

Solution of field equations

We solve the field equations (10)-(13) by using the special law of variation for Hubble Parameter proposed by Berman (1983) as -

$$H = DR^{-m} \quad \text{…(14)}$$

where *H* is the Hubble's parameter defined by -

$$H = \frac{\dot{R}}{R}, \quad \text{…(15)}$$

and *D* and *m* ($\neq$0) are constants.

Using equations (14) and (15) , we obtain -

$$R = \left[m(At + B)\right]^{\frac{1}{m}}, \quad \text{…(16)}$$

where *A* and *B* are taken to be positive constants of integration.

The deceleration parameter is defined by -

$$q = \frac{-\ddot{R}R}{\dot{R}^2}. \quad \text{…(17)}$$

For the special law (16), equation (17) yields,

$$q = m - 1 \quad \text{…(18)}$$

Also taking Gauge function,

$$\phi = \alpha t^n, \qquad \text{...(19)}$$

where α is constant.

Thus, FRW model for cosmic string in scale covariant theory can be written as -

$$ds^2 = dt^2 - [m(At+B)]^{\frac{1}{m}}\left[\left(1-kr^2\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]. \qquad \text{...(20)}$$

Using equations (16) , (17) in equations (10)-(13), we obtain -

$$\lambda = \frac{1}{G}\left\{\frac{A^2(3-2m)}{[m(At+B)]^2} + \frac{k}{[m(At+B)]^{\frac{2}{m}}} - \frac{4nA}{tm(At+B)} - \frac{n(n-2)}{t^2}\right\}, \qquad \text{...(21)}$$

$$\rho = \frac{1}{G}\left\{\frac{3A^2}{[m(At+B)]^2} + \frac{3k}{[m(At+B)]^{\frac{2}{m}}} - \frac{6nA}{tm(At+B)} + \frac{3n^2}{t^2}\right\}. \qquad \text{...(22)}$$

Physical properties

The Physical quantities that are important in cosmology are Special volume V^3, expansion scalar θ, shear scalar σ^2, and Hubble parameter H which have the following expression.

$$V^3 = (At+B)^{\frac{1}{1+q}}, \qquad \text{...(23)}$$

$$\theta = \frac{A}{(1+q)(At+B)}, \qquad \text{...(24)}$$

$$\sigma^2 = \frac{4}{9}\frac{A}{(1+q)(At+B)}, \qquad \text{...(25)}$$

and

$$H = \frac{A}{(1+q)(At+B)}. \qquad \text{...(26)}$$

CONCLUSION

In this paper, we have considered the field equations of scale covariant theory of gravitation proposed by Canuto et al., (1977a) with FRW space-time in presence of Cosmic String. The field equations being highly non-linear, we have obtained a cosmological model using special law of variation of Hubble's parameter proposed by Bermann (1983). We observed that the model is expanding and the age of universe increases as the radius of universe increases.

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