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## Establishment of input and output mathematical model based on vibratory excavating system of excavator

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### ABSTRACT

In excavator dynamics, the establishment of excavator vibratory excavating system is illustrated comprehensively. In this paper, based on excavator vibratory excavating system, a related mathematical model of the input and output system was set up. In this process, Through the establishment of excavator dynamics vibration model, the linear dynamic system was effectively analyzed, and its output mathematical model was refined also, leading to a efficient establishment of transfer function equation. Then an effective simulation analysis was conducted regarding the excavator dynamics vibration model, by which the frequency response characteristic curve of the excavator dynamics system was generalized and summarized and charted eventually. After that, relevant discussions were conducted regarding the establishing process of excavator model, and a specific list of model components was given to illustrate the function characteristics of the components, which provided effective theoretic supports for the establishment of the input and output mathematical model of the vibratory excavating system. Finally, the establishing process of input and output model was effectively refined according to the path planning of excavating process, which is the main thought and approach of the research process of this paper so that the mathematic model can be built through related model matrix and the scientificity and rationality of the research process can be emphasized.

### KEYWORDS

Excavator; Dynamics vibratory excavating system; Input and output; Model building; Illustrative example.



**INTRODUCTION**

According to the excavator dynamics theory, the building process of vibratory excavating system is based on the input and out mathematical model, also by which the vibratory excavating system can be effectively refined. In the process of research and discussion, this paper is made in combination of following aspects, including: building and analysis of excavator dynamics vibration model, building of excavator model, path planning of excavating process and illustrative example. Corresponding relations is calculated out from mathematical matrix, providing power data support for the establishment of input and output mathematical model. It is hoped that research process of this paper can provide effective theoretical basis of building corresponding mathematical model, as well as solid theoretical and data basis for the effective development of further research and exploration in the future.

**ESTABLISHMENT OF EXCAVATOR DYNAMICS VIBRATION MODEL**

During the process of operation, excavator makes effective excavation of corresponding soil in crossing over earthen trenches, forming complex model of two source components, namely, pylon of suspensory and soil, and interaction between them in turn contributes to effective excavation<sup>[1]</sup>. Effective research is made towards its performance of vibration in this process: assume linear dynamic system is formed between furrow opener and pylon brought by it; take uneven degree of ground,  $Z(t)$ , and esistance produced by stubble in soil,  $R(t)$ , as its output variables, towards which transformation is made in order to form angular oscillation,  $W(t)$ , on horizontal plane and treat  $W(t)$  as the output variable of excavator, thus, dynamics output mathematical model of excavator in this process is as follows:

$$\sum_{j,m=1}^k (a_{jm} \Delta \ddot{q}_m + b_{jm} \dot{\Delta q}_m + c_{jm} \Delta q_m) = \sum_{i=1}^n (d_{mi} \Delta \dot{f}_i + m_{mi} \Delta f_i)$$

From above formula, we can find out that  $\Delta q_m$ ,  $\dot{\Delta q}_m$  and  $\ddot{\Delta q}_m$  are respectively defined as displacement under generalized coordinates, the first and second order derivative, while  $\Delta f_i$  and  $\dot{\Delta f}_i$  external force and its derivative;  $a_{jm}$ ,  $b_{jm}$ ,  $c_{jm}$ ,  $d_{mi}$  and  $m_{mi}$  are normal constant coefficient; k then is generalized coordinates, and n represents specific coefficient of input function. In the process of building this model, number of generalized coordinate and input variable are both limited to one, with  $K=1$  and  $n=2$ , while  $\Delta q_1 = \Psi_c$ ;  $a_{ii} = J_c$ , which is taken as moment of inertia of excavator’s relative suspension shaft: formula is then drawn as follows:

$$J_c \ddot{\Psi}_c + b_{11} \dot{\Psi}_c + c_{11} \Psi_c = d_{11} \dot{z} + m_{11} z + d_{21} \dot{R} + m_{21} R$$

And a few arrangement leads to:

$$T_2^2 \ddot{\Psi}_c + T_1 \dot{\Psi}_c + \Psi_c = \Gamma_z \dot{z} + k_z z + \Gamma_R \dot{R} + k_R R$$

In this formula:

$$\begin{aligned} T_2^2 &= J_c / c_{11}; T_1 = b_{11} / c_{11}; \tau_2 = d_{11} / c_{11}; k_2 = m_{11} / c_{11}; \\ \tau_R &= d_{21} / c_{11}; k_R = m_{21} / c_{11}; \tau_z = d_{11} / c_{11}; k_z = m_{11} / c_{11}; \\ \tau_R &= d_{21} / c_{11}; k_R = m_{21} / c_{11} \end{aligned}$$

Transfer function equation of excavator’s micro-swing is as follows by conducting Laplace transform to formula(3):

$$\Psi_c(s) = W_z(s)z(s) + W_R(s)R(s)$$

In which, transfer function of  $W_z(s)$  and  $W_R(s)$  are:

$$W_z(s) = \frac{\tau_z + k_z}{T_2^2 s^2 + T_1 s + 1}$$

$$W_R(s) = \frac{\tau_R s + k_R}{T_2^2 s^2 + T_1 s + 1}$$

### ANALYSIS OF EXCAVATOR DYNAMICS VIBRATION MODEL

In simulation analysis and corresponding experimental study,  $W_z(s)$  can be simplified as follows:

$$W_z(s) = \frac{k_z}{T_2^2 s^2 + T_1 s + 1}$$

For value of coefficient, we can calculate by least square method, while for specific coefficient of excavator's transfer function, which can be easily looked up from TABLE 1. Substituting  $s$  in  $W_z(s)$  with  $jw$ , frequency characteristic of its system transforms to  $w_z(jw)$ : frequency characteristic determines the fundamental characteristic of system, and relevant analysis is developed towards which in order that performance of system can be presented. In terms of method for system frequency, it is mainly done by effectively analyzing frequency characteristics, as well as specific relationship between signal frequency  $w$  and  $w_z(jw)$ 's two basic components, i.e., amplitude frequency and phase frequency, so as to effectively build the specific internal relationship between structure parameter and system performance. Output overshoot of step increases as the maximum limit of amplitude frequency becomes larger and larger, which indicates good performance of this system; while increase in resonant frequency leads to larger and larger band width, which will accelerate the response speed of system.

In analyzing excavator dynamics vibration model, matrix laboratory programming method is adopted for conducting effective matrix programming, and effective establishment of system's frequency-response curves: Bird curve and Nyquist curve(Figure 1)<sup>[3]</sup>. From following figures, we can clearly find out that: in the process of operation, excavator's forward speed has changed significantly, increasing from 2165m/s to 4115m/s, as is the case with resonant frequency, increasing from 1415Hz to 1916Hz, and maximum limit of amplitude frequency also increases from initial value of 3712dB to 3811dB, so that stability of excavator's system is increased and its operation process gets corresponding effective protection.

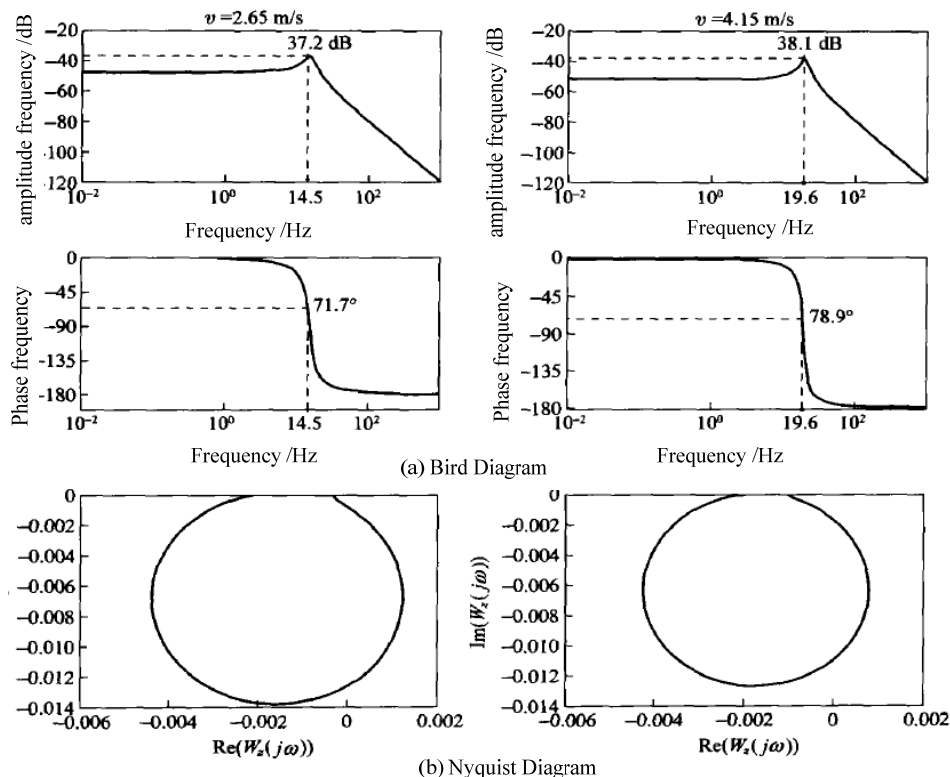


Figure 1 : Curve of excavator dynamical system's frequency-response characteristics under two different velocities

Excavator's dynamical system will produce certain fluctuation in the process of operation, having difference effect with respect to the work of excavator. And two effective measurements can be taken to effectively increase its vibration performance: one, effectively improve the suspension structure; two, install relevant hydraulic damper in the system, which can achieve better effects of anti-vibration. For the first solution, whose main principle lies in two aspects: substitute suspension mechanism with pomegranate connecting rod structure; install spring between structures and on each support link, thus transforming original suspension mechanism to elastic connection. In the process of this paper's study and discussion,

experiment is done by excavating a section of road and result of which is effectively analyzed, followed by effective arrangement with the list of its transfer function as is shown in TABLE 1. From data in TABLE 1, we can easily find out that, at the forward speed of 2165m/s and 4115m/s, damping before and after improvement are respectively 0136 and 0131, and 0115 and 0110: there is significant difference between two sets of data-damping is respectively improved by 214 and 311 times, thus, excavator’s vibration is effectively alleviated, at the same time, excavator uniformity is substantially improved as well<sup>[4]</sup>.

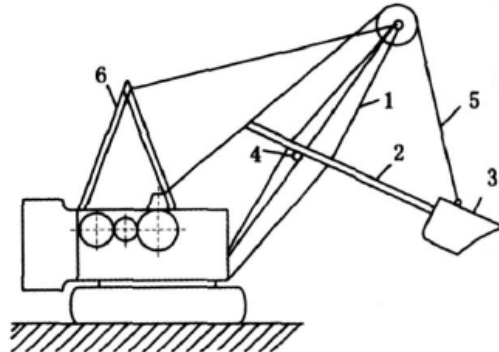
**TABLE 1 : Coefficient of transfer function**

Furrow opener	v/(m/s)	kz(°) / mm	T <sub>1</sub> / s	T <sub>2</sub> / s	Damping ratio ρ
Standard value	2.65	0.037	0.02	0.065	0.15
	4.15	0.025	0.01	0.050	0.10
Improved value	2.65	0.034	0.05	0.070	0.36
	4.15	0.025	0.04	0.065	0.31

$\rho = 0.5T_1/T_2$

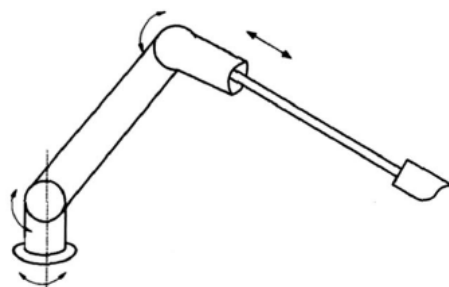
**ESTABLISHMENT OF EXCAVATOR MODEL**

Main constituent apparatus of mechanical excavator is divided into three important parts(Figure 2), and they are crowd shove working device, rotary device, chassis and walking device: crowd shove working device is mainly comprised of swing arm, hand shank, pushing gear and other important elements, whose bottom hinge is connected to its platform, while the top is effectively fixed through luffing cable of surrounding bracket-capable of staying at a fixed position, and its angle of inclination can be effectively changed through relevant regulation of luffing cable’s length. The rising operation of bucket is realized by effective pulling of cable through motor, while the fall depends on its own weight. In the process of excavation, pushing gear will push out the bucket arm, which then conducts effective control of its direction of rotation according to the command of bucket, either rise or fall<sup>[5]</sup>.



**Figure 2 : Composition structure of mechanical excavator**

Figure 3 is the specific diagram of excavator model in building process, which, from a simplified perspective, is a 3R-1P system, with four freedoms. And it is mainly comprised of rotary joint, rotary joint of swing arm with respect to chassis, swing arm, a rotary joint centered on push rod and other elements, among which bucket is the end effector of whole system.



**Figure 3 : Excavator model**

In the process of establishing this model, it is mainly about building 4×4 homogeneous transformation matrix for coordinates of connecting rod at each joint by Denavit-Hatenberg method, so as to further represent the relationship between them and coordinates of two sides. In the situation of this transformation, process of changing step by step is conducted, and can be effectively represented on reference coordinate system. The build process of connecting rod coordinates shown in Figure 4 indicates it directly, and coefficient of each connecting rod coordinates of excavator is also clearly shown in TABLE 2.

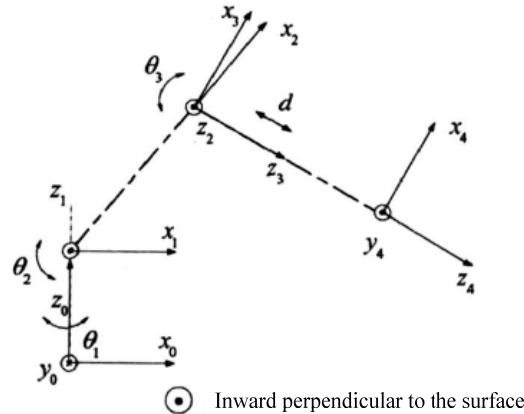


Figure 4 : Connecting rod coordinates of excavator

TABLE 2 : Rod parameter of excavator model

$i$	$\theta_i$	$d_i$	$a_i$	$\alpha_i$	Variable and range of connecting joint	
1	$\theta_1$	0	0	$90^\circ$	$\theta_1$	$-180^\circ \sim 180^\circ$
2	$\theta_2$	0	$l_1$	0	$\theta_2$	$36^\circ \sim 60^\circ$
3	$\theta_3$	0	0	$90^\circ$	$\theta_1$	$-45^\circ \sim 75^\circ$
4	0	$d$	0	0	$d$	1000 ~ 1900

PATH PLANNING OF EXCAVATION PROCESS

Excavating Trajectory

In keeping excavator at a constant working state, the excavating trajectory will inevitably be influenced by different factors, which in turn will cause negative effects on the efficiency of excavator. Constant of excavator can be kept under certain conditions according to original research data. And work efficiency of excavator continues to increase when excavating resistance is kept at a low value, at this point, the trajectory of excavator’s difference movement will be a logarithm helical curve, as is shown in Figure 5.

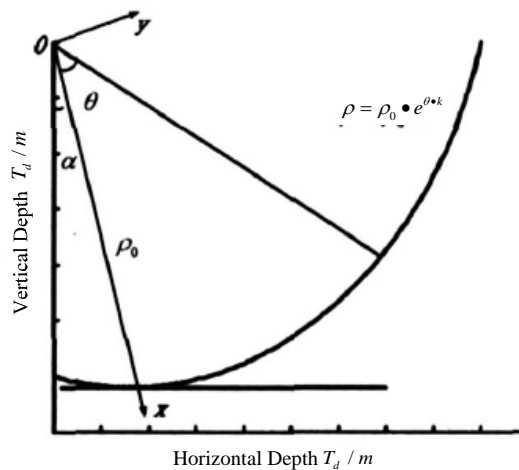


Figure 5 : Initial angle diagram

In the above formula, range of  $k$  is determined by bucket structure and angle of excavation. According to the characteristic of logarithm helical curve, what's normally called foundation-chewing phenomenon will occur if tip of bucket locates at a position perpendicular to the rod. And judging from Figure 5, the method of avoiding this phenomenon is as follows: take horizontal line as the tangent line of trajectory initial point when starting to develop excavation work, thus getting a original input angle  $\alpha$  in the actual process of excavation<sup>[6]</sup>.

Formula of trajectory derived from geometric relationship is specifically shown as follows:

$$\alpha = \frac{\pi}{2} - \arctan \frac{1}{k}$$

Trajectory of excavation can be determined from above formula as  $\rho = \rho_0 \cdot e^{\theta \cdot k}$ : define the axis of pushing rod as origin of coordinates, while  $\alpha$  is the starting point.

**Trajectory planning**

In the process of excavation, chassis and swing arm are at fixed locations; take  $\theta_1$  as 0 degree, substitute it into kinematics equation:

$$P_x = ds\theta_2c\theta_3 + dc\theta_2s\theta_3 + l_1c\theta_2$$

$$P_z = ds\theta_2s\theta_3 - dc\theta_2c\theta_3 + l_1s\theta_2$$

Excavation trajectory of polar coordinates under reference coordinate system:

$$(P_x - l_1 \cdot \cos \theta_2)^2 + (P_z - l_1 \cdot \sin \theta_2)^2 = \rho^2$$

$$\rho = \rho_0 \cdot e^{\theta \cdot k}$$

According to geometric relationship,

$$\theta_3 = \theta - \theta_2$$

In the formula,  $\theta$  is the polar angle of trajectory under system of polar coordinates, which is shown in Figure 5.

As is indicated in above formula, its equation set mainly include six variables, namely,  $\theta_3, d, \theta, \rho, P_x, P_y$ : in the planning process of excavator's trajectory, it's mainly about conducting effective operation with solution(s) of each variable, so as to effectively plan the work trajectory; deal with solution of equation set's variable effectively, and conclude them in the function of  $\theta_3, d$ . In this process, it is necessary to satisfy following formula for realizing trajectory of logarithm helical curve set in advance, and joint variable shall be based on this formula.

$$(\rho_0 e^{0.26(\theta_2 + \theta_3)})^2 = (-dc(\theta_2 + \theta_3) + l_1s\theta_2 - l_1s\theta_3)^2 + (ds(\theta_2 + \theta_3) + l_1c\theta_2 - l_1c\theta_3)^2$$

In the above formula,  $\theta_3$  serves as the independent variable, while function represented by  $d$  is expressed in  $d(\theta_3)$ . As it is very difficult to conduct verification of analytical solution with formula mentioned above, effective fitting then is made by the least square method for effectively calculating the analytical solution of  $d(\theta_3)$ . In the following process of discussion,  $\theta_3$  is taken as the independent variable of joint's interpolation function for two considerations: its movement can meet corresponding requirements of stability; angle of its joint can satisfy practical needs of trajectory's start and end points, at the same time, it's of necessity to maintain strong continuity of its movement-joint velocity of trajectory's start point and end point are kept constant, thus their velocities are maintained at zero, namely,

$$\theta(0) = \theta_0, \theta(t_f) = \theta_j$$

$$\dot{\theta}(0) = 0, \dot{\theta}(t_f) = 0$$

Interpolation function for satisfying demand of joint's continuous and stable movement can be derived as follows:

$$\theta(t) = \theta_0 + \frac{3}{t_f^2}(\theta_f - \theta_0)t^2 + \frac{2}{t_f^3}(\theta_f - \theta_0)t^3$$

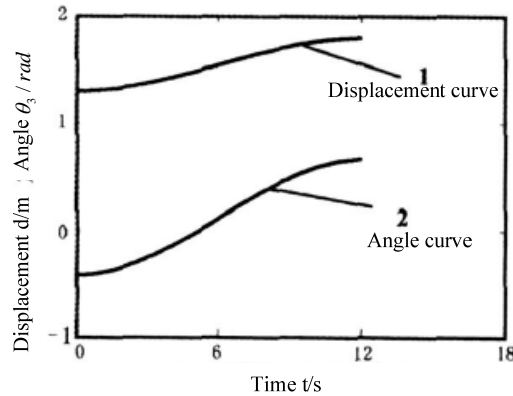
Take the derivative of  $\theta_3(t)$  and  $d(t)$ , and get the speed and acceleration of joint's movement.

**LLUSTRATIVE EXAMPLE**

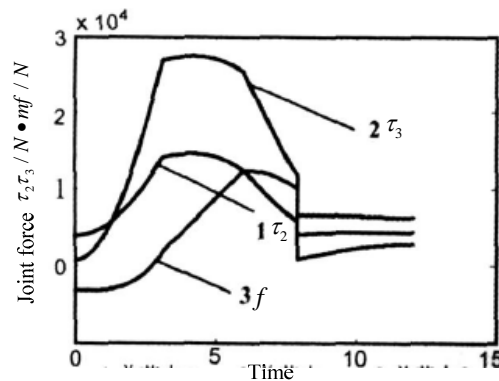
According to the specific structure and practical requirements of WS-005 mechanical excavator, take  $k = 0.26$ ,  $\psi = 0.3$ , with iron ore being the object.

Chassis and swing arm are fixed in the process of excavation; take  $\theta_1 = 0$  degree,  $\theta_2 = 0$  degree, initial input angle of bucket  $H_0 = A$  and output angle  $\theta_f = 105b$ , and time of the whole process  $t_f = 12s$ .

Simulation results of joint movement and joint force are got from above trajectory planning and dynamics analysis of excavator, as indicated in Figure 6 and Figure 7, and according to which, profile of each joint's driving force can be divided into three stages with the change of excavating resistance. As depth of excavation increases, driving force is required to increase rapidly, maximizing at the largest depth, thereafter, driving force of each joint decreases and stabilizes near the output angle. The lifting moment of excavator is the largest comparing to other joint forces. And mechanical excavator can be accurately controlled according to calculated results of dynamics, making joint move continuously and stably, reducing resistance of excavation and improving work efficiency.



**Figure 6 : Diagram of displacement and angle change**



**Figure 7 : Simulation diagram of joint force**

**CONCLUSION**

What's mentioned above is the relevant study and discussion process made with respect to input and output mathematical model of excavator's dynamics vibratory excavating system, and it's mainly about analysis and establishment of excavator model, trajectory planning of excavation process, making the research process more strongly targeted and producing positive effects on further exploration in the realm of excavator dynamics.

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